

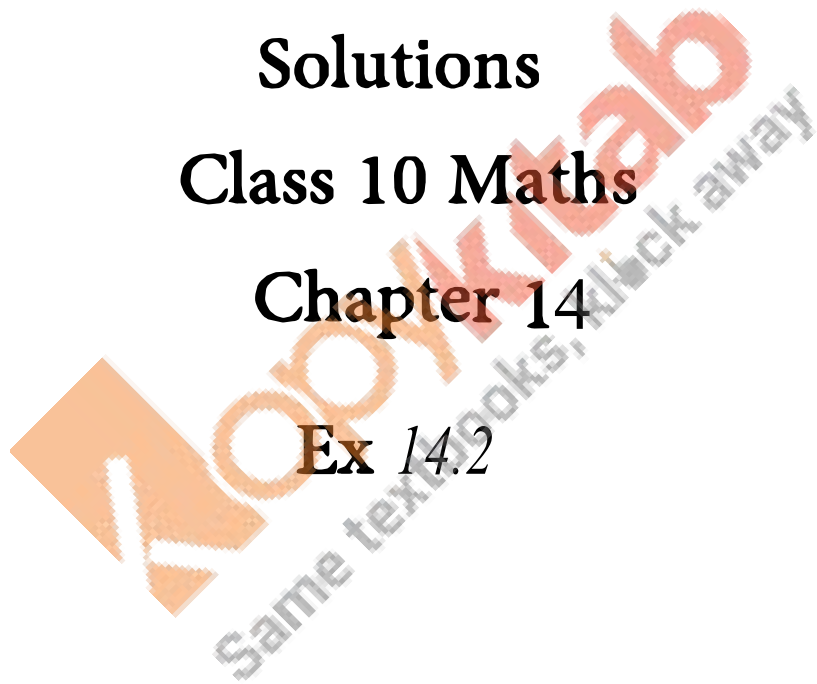
RD SHARMA

Solutions

Class 10 Maths

Chapter 14

Ex 14.2



1. Find the distance between the following pair of points:

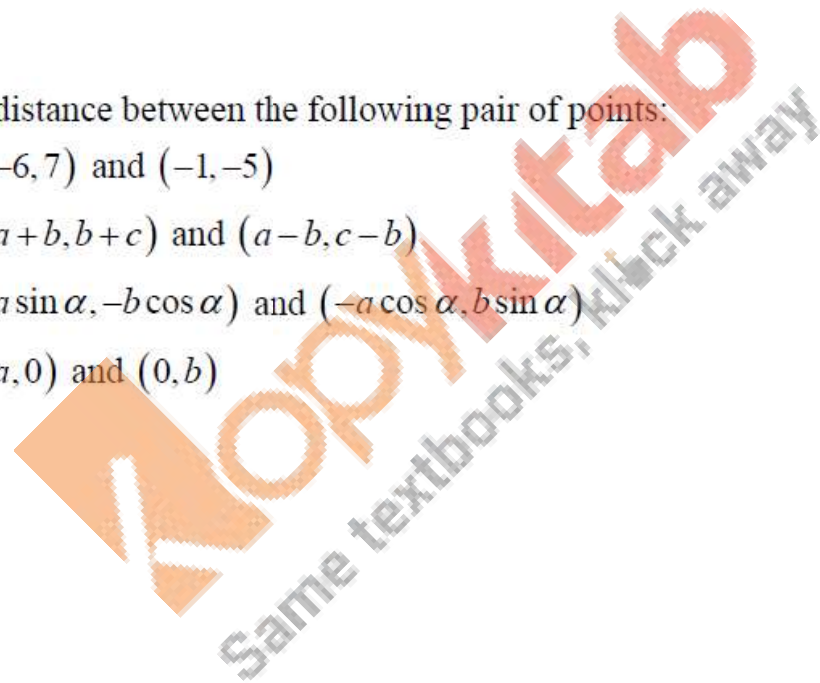
(i) $(-6, 7)$ and $(-1, -5)$

(ii) $(a + b, b + c)$ and $(a - b, c - b)$

(iii) $(a \sin \alpha, -b \cos \alpha)$ and $(-a \cos \alpha, b \sin \alpha)$

(iv) $(a, 0)$ and $(0, b)$

Sol:



(i) We have $P(-6, 7)$ and $Q(-1, -5)$

Here,

$$x_1 = -6, y_1 = 7 \text{ and}$$

$$x_2 = -1, y_2 = -5$$

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$PQ = \sqrt{[-1 - (-6)]^2 + (-5 - 7)^2}$$

$$PQ = \sqrt{(-1 + 6)^2 + (-5 - 7)^2}$$

$$PQ = \sqrt{(5)^2 + (-12)^2}$$

$$PQ = \sqrt{25 + 144}$$

$$PQ = \sqrt{169}$$

$$PQ = 13$$

(ii) we have $P(a+b, b+c)$ and $Q(a-b, c-b)$ here,

$$x_1 = a+b, y_1 = b+c \text{ and } x_2 = a-b, y_2 = c-b$$

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$PQ = \sqrt{[a-b - (a+b)]^2 + (c-b - (b+c))^2}$$

$$PQ = \sqrt{(a-b-a-b)^2 + (c-b-b-c)^2}$$

$$PQ = \sqrt{(-2b)^2 + (-2b)^2}$$

$$PQ = \sqrt{4b^2 + 4b^2}$$

$$PQ = \sqrt{8b^2}$$

$$PQ = \sqrt{4 \times 2b^2}$$

$$PQ = 2\sqrt{2}b$$

(iii) we have $P(a \sin \alpha, -b \cos \alpha)$ and $Q(-a \cos \alpha, b \sin \alpha)$ here

$$x_1 = a \sin \alpha, y_1 = -b \cos \alpha \text{ and}$$

$$x_2 = -a \cos \alpha, y_2 = b \sin \alpha$$

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$PQ = \sqrt{(-a \cos \alpha - a \sin \alpha)^2 + [-b \sin \alpha - (-b \cos \alpha)]^2}$$

$$PQ = \sqrt{(-a \cos \alpha)^2 + (-a \sin \alpha)^2 + 2(-a \cos \alpha)(-a \sin \alpha) + (b \sin \alpha)^2 + (-b \cos \alpha)^2 - 2(b \sin \alpha)(-b \cos \alpha)}$$

$$PQ = \sqrt{a^2 \cos^2 \alpha + a^2 \sin^2 \alpha + 2a^2 \cos \alpha \sin \alpha + b^2 \sin^2 \alpha + b^2 \cos^2 \alpha + 2b^2 \sin \alpha \cos \alpha}$$

$$PQ = \sqrt{a^2 (\cos^2 \alpha + \sin^2 \alpha) + 2a^2 \cos \alpha \sin \alpha + b^2 (\sin^2 \alpha + \cos^2 \alpha) + 2b^2 \sin \alpha \cos \alpha}$$

$$PQ = \sqrt{a^2 \times 1 + 2a^2 \cos \alpha \sin \alpha + b^2 \times 1 + 2b^2 \sin \alpha \cos \alpha} \quad [\because \sin^2 \alpha + \cos^2 \alpha = 1]$$

$$PQ = \sqrt{a^2 + b^2 + 2a^2 \cos \alpha \sin \alpha + 2b^2 \sin \alpha \cos \alpha}$$

$$PQ = \sqrt{(a^2 + b^2) + 2 \cos \alpha \sin \alpha (a^2 + b^2)}$$

$$PQ = \sqrt{(a^2 + b^2)(1 + 2 \cos \alpha \sin \alpha)}$$

(iv) We have $P(a, 0)$ and $Q(0, b)$

Here,

$$x_1 = a, y_1 = 0, x_2 = 0, y_2 = b,$$

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$PQ = \sqrt{(0 - a)^2 + (b - 0)^2}$$

$$PQ = \sqrt{(-a)^2 + (b)^2}$$

$$PQ = \sqrt{a^2 + b^2}$$

Same textbook is available away

2. Find the value of a when the distance between the points $(3, a)$ and $(4, 1)$ is $\sqrt{10}$.

Sol:

We have $P(3, a)$ and $Q(4, 1)$

Here,

$$x_1 = 3, y_1 = a$$

$$x_2 = 4, y_2 = 1$$

$$PQ = \sqrt{10}$$

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\Rightarrow \sqrt{10} = \sqrt{(4 - 3)^2 + (1 - a)^2}$$

$$\Rightarrow \sqrt{10} = \sqrt{(1)^2 + (1 - a)^2}$$

$$\Rightarrow \sqrt{10} = \sqrt{1 + 1 + a^2 - 2a} \quad \left[\because (a - b)^2 = a^2 + b^2 - 2ab \right]$$

$$\Rightarrow \sqrt{10} = \sqrt{2 + a^2 - 2a}$$

Squaring both sides

$$\Rightarrow (\sqrt{10})^2 = (\sqrt{2 + a^2 - 2a})^2$$

$$\Rightarrow 10 = 2 + a^2 - 2a$$

$$\Rightarrow a^2 - 2a + 2 - 10 = 0$$

$$\Rightarrow a^2 - 2a - 8 = 0$$

Splitting the middle term.

$$\Rightarrow a^2 - 4a + 2a - 8 = 0$$

$$\Rightarrow a(a - 4) + 2(a - 4) = 0$$

$$\Rightarrow (a - 4)(a + 2) = 0$$

$$\Rightarrow a = 4, a = -2$$

3. If the points $(2, 1)$ and $(1, -2)$ are equidistant from the point (x, y) from $(-3, 0)$ as well as from $(3, 0)$ are 4.

Sol:

We have $P(2, 1)$ and $Q(1, -2)$ and $R(X, Y)$

Also, $PR = QR$

$$PR = \sqrt{(x - 2)^2 + (y - 1)^2}$$

$$\Rightarrow PR = \sqrt{x^2 + (2)^2 - 2 \times x \times 2 + y^2 + (1)^2 - 2 \times y \times 1}$$

$$\Rightarrow PR = \sqrt{x^2 + 4 - 4x + y^2 + 1 - 2y}$$

$$\Rightarrow PR = \sqrt{x^2 + 5 - 4x + y^2 - 2y}$$

$$QR = \sqrt{(x-1)^2 + (y+2)^2}$$

$$\Rightarrow PR = \sqrt{x^2 + 1 - 2x + y^2 + 4 + 4y}$$

$$\Rightarrow PR = \sqrt{x^2 + 5 - 2x + y^2 + 4y}$$

$$\because PR = QR$$

$$\Rightarrow \sqrt{x^2 + 5 - 4x + y^2 - 2y} = \sqrt{x^2 + 5 - 2x + y^2 + 4y}$$

$$\Rightarrow x^2 + 5 - 4x + y^2 - 2y = x^2 + 5 - 2x + y^2 + 4y$$

$$\Rightarrow x^2 + 5 - 4x + y^2 - 2y = x^2 + 5 - 2x + y^2 + 4y$$

$$\Rightarrow -4x + 2x - 2y - 4y = 0$$

$$\Rightarrow -2x - 6y = 0$$

$$\Rightarrow -2(x + 3y) = 0$$

$$\Rightarrow x + 3y = \frac{0}{-2}$$

$$\Rightarrow x + 3y = 0$$

Hence proved.

4. Find the values of x, y if the distances of the point (x, y) from $(-3, 0)$ as well as from $(3, 0)$ are 4.

Sol:

We have $P(x, y), Q(-3, 0)$ and $R(3, 0)$

$$PQ = \sqrt{(x+3)^2 + (y-0)^2}$$

$$\Rightarrow 4 = \sqrt{x^2 + 9 + 6x + y^2}$$

Squaring both sides

$$\Rightarrow (4)^2 = (\sqrt{x^2 + 9 + 6x + y^2})^2$$

$$\Rightarrow 16 = x^2 + 9 + 6x + y^2$$

$$\Rightarrow x^2 + y^2 = 16 - 9 - 6x$$

$$\Rightarrow x^2 + y^2 = 7 - 6x \quad \dots\dots\dots(1)$$

$$PR = \sqrt{(x-3)^2 + (y-0)^2}$$

$$\Rightarrow 4 = \sqrt{x^2 + 9 - 6x + y^2}$$

Squaring both sides

$$(4)^2 = \left(\sqrt{x^2 + 9 - 6x + y^2} \right)^2$$

$$\Rightarrow 16 = x^2 + 9 - 6x + y^2$$

$$\Rightarrow x^2 + y^2 = 16 - 9 + 6x$$

$$\Rightarrow x^2 + y^2 = 7 + 6x \quad \dots\dots\dots(2)$$

Equating (1) and (2)

$$7 - 6x = 7 + 6x$$

$$\Rightarrow 7 - 7 = 6x + 6x$$

$$\Rightarrow 0 = 12x$$

$$\Rightarrow x = 0$$

Equating (1) and (2)

$$7 - 6x = 7 + 6x$$

$$\Rightarrow 7 - 7 = 6x + 6x$$

$$\Rightarrow 0 = 12x$$

$$\Rightarrow x = 0$$

Substituting the value of $x = 0$ in (2)

$$x^2 + y^2 = 7 + 6x$$

$$0 + y^2 = 7 + 6 \times 0$$

$$y^2 = 7$$

$$y = \pm\sqrt{7}$$

