Algebraic Identities RD Sharma Class 9 Solutions Chapter 4 Exercise 4.5

1. i

We have,

$(3x+2y+2z)(9x^2+4y^2+4z^2-6xy-4yz-6zx)$

$=(3x+2y+2z)\left((3x)^2+(2y)^2+(2z)^2-3x\times2y-2y\times2z-2z\times3x\right)$

$=(3x)^3+(2y)^3+(2z)^3-3\times3x\times2y\times2z \qquad \left[\because a^3+b^3+c^3-3abc=(a+b+c)(a^2+b^2+c^2-ab-bc-ca)\right]$

$=27x^3+8y^3+8z^3-36xyz$

$\therefore (3x+2y+2z)(9x^2+4y^2+4z^2-6xy-4yz-6zx)=27x^3+8y^3+8z^3-36xyz$

1. ii

We have,

$(4x-3y+2z)(16x^2+9y^2+4z^2+12xy+6yz-8zx)$

$=(4x+(-3y)+2z)\left[(4x)^2+(-3y)^2+(2z)^2-(4x)(-3y)-(-3y)(2z)-(2z)(4x)\right]$

$=(4x)^3+(-3y)^3+(2z)^3-3(4x)(-3y)(2z) \qquad \left[\because a^3+b^3+c^3-3abc=(a+b+c)(a^2+b^2+c^2-ab-bc-ca)\right]$

$=64x^3-27y^3+8z^3+72xyz$

$\therefore (4x-3y+2z)(16x^2+9y^2+4z^2+12xy+6yz-8zx)=64x^3-27y^3+8z^3+72xyz$

1. iii

We have,

$(2a-3b-2c)(4a^2+9b^2+4c^2+6ab-6bc+4ca)$

$=(2a+(-3b)+(-2c))\left((2a)^2+(-3b)^2+(-2c)^2-(2a)(-3b)-(-3b)(-2c)-(-2c)(2a)\right)$

$=(2a)^3+(-3b)^3+(-2c)^3-3(2a)(-3b)(-2c)$

$\left[\because a^3+b^3+c^3-3abc=(a+b+c)(a^2+b^2+c^2-ab-bc-ca)\right]$

$=8a^3-27b^3-8c^3-36abc$

$\therefore (2a-3b-2c)(4a^2+9b^2+4c^2+6ab-6bc+4ca)=8a^3-27b^3-8c^3-36abc$

1. iv

We have,

$(3x-4y+5z)(9x^2+16y^2+25z^2+12xy-15zx+20yz)$

$=(3x+(-4y)+5z)\left((3x)^2+(-4y)^2+(5z)^2-(3x)(-4y)-(-4y)(5z)-(5z)(3x)\right)$

$=(3x)^3+(-4y)^3+(5z)^3-3(3x)(-4y)(5z)$

$\left[\because a^3+b^3+c^3-3abc=(a+b+c)(a^2+b^2+c^2-ab-bc-ca)\right]$

$=27x^3-64y^3+125z^3+180xyz$

$\therefore (3x-4y+5z)(9x^2+16y^2+25z^2+12xy-15zx+20yz)=27x^3-64y^3+125z^3+180xyz$

2.

We know that

$x^3+y^3+z^3-3xyz=(x+y+z)(x^2+y^2+z^2-xy-yz-zx)$

$\Rightarrow x^3+y^3+z^3-3xyz=(x+y+z)[(x^2+y^2+z^2)-(xy+yz+zx)]$ --- (1)

It follow from the above identity that we require the values of $x+y+z, x^2+y^2+z^2$, and $xy+yz+zx$ to get the value of $x^3+y^3+z^3-3xyz$. The values of $x+y+z$ and $xy+yz+zx$ are known to us. So we require the value of $x^2+y^2+z^2$.

Now,

$(x+y+z)^2=x^2+y^2+z^2+2(xy+yz+zx)$

$\Rightarrow (8)^2=x^2+y^2+z^2+2(20)$ $[\because x+y+z=8 \text{ and } xy+yz+zx=20]$

$\Rightarrow 64=x^2+y^2+z^2+40$

$\Rightarrow x^2+y^2+z^2=64-40=24$

Substituting the values of $x^2+y^2+z^2, x+y+z$ and $xy+yz+zx$ in equation (1), we get,

$x^3+y^3+z^3-3xyz=8\times(24-20)$

$=8\times 4$

$=32$

$\therefore x^3+y^3+z^3-3xyz=32$

3.

We know that

$a^3+b^3+c^3-3abc=(a+b+c)(a^2+b^2+c^2-ab-bc-ca)$

$\Rightarrow a^3+b^3+c^3-3abc=(a+b+c)[(a^2+b^2+c^2)-(ab+bc+ca)]$ --- (1)

It follows from the above identity that we require the values of $a+b+c, a^2+b^2+c^2$, and $ab+bc+ca$ to get the value of $a^3+b^3+c^3-3abc$. The values of $a+b+c$ and $ab+bc+ca$ are known to us. So we require the value of $a^2+b^2+c^2$.

Now,

$(a+b+c)^2=a^2+b^2+c^2+2(ab+bc+ca)$

$\Rightarrow (9)^2=a^2+b^2+c^2+2\times 26$ $[\because a+b+c=9 \text{ and } ab+bc+ca=26]$

$\Rightarrow 81=a^2+b^2+c^2+52$

$\Rightarrow a^2+b^2+c^2=81-52=29$

Substituting the values of $a^2+b^2+c^2$ in (1), we get,

$a^3+b^3+c^3-3abc=9(29-26)$ $(\because a+b+c=9 \text{ and } ab+bc+ca=26)$

$=9\times 3$

$=27$

$\therefore a^3+b^3+c^3-3abc=27$

4.

We know that,

$a^3+b^3+c^3-3abc=(a+b+c)(a^2+b^2+c^2-ab-bc-ca)$

$\Rightarrow a^3+b^3+c^3-3abc=[a+b+c][(a^2+b^2+c^2)-(ab+bc+ca)]$ ---(1)

It follows from the above identity that we require the values of $a+b+c$, $a^2+b^2+c^2$, and $ab+bc+ca$ to get the value of $a^3+b^3+c^3-3abc$. The values of $a+b+c$ and $a^2+b^2+c^2$ are known to us. So we require the value of $ab+bc+ca$.

Now,

$(a+b+c)^2=a^2+b^2+c^2+2(ab+bc+ca)$

$\Rightarrow (9)^2=35+2(ab+bc+ca)$ $\qquad [\because a+b+c=9 \text{ and } a^2+b^2+c^2=35]$

$\Rightarrow 81=35+2(ab+bc+ca)$

$\Rightarrow 2(ab+bc+ca)=81-35=46$

$\Rightarrow ab+bc+ca=\frac{46}{2}=23$

Substituting the values of $ab+bc+ca$ in (1), we get,

$a^3+b^3+c^3-3abc=9(35-23)$ $\qquad [\because a+b+c=9 \text{ and } a^2+b^2+c^2=35]$

$=9\times12$

$=108$

$\therefore a^3+b^3+c^3-3abc=108$

5. i

Let $a=25$, $b=-75$ and $c=50$

Then,

$a+b+c=25-75+50$

$=0$

$\therefore a^3+b^3+c^3=3abc$

$\Rightarrow (25)^3+(-75)^3+(50)^3=3\times25\times(-75)\times50$

$=-75\times75\times50$

$=-5625\times50$

$=-281250$

$\therefore 25^3-75^3+50^3=-281250$

5. ii

Let $a = 48, b = -30$ and $c = -18$

Then,
$a + b + c = 48 - 30 - 18$
$= 0$

$\therefore a^3 + b^3 + c^3 = 3abc$
$\Rightarrow (48)^3 + (-30)^3 + (-18)^3 = 3 \times (48) \times (-30) \times (-18)$
$= 144 \times 540$
$= 77760$

$\therefore 48^3 - 30^3 - 18^3 = 77760$

5. iii

Let $a = \frac{1}{2}, b = \frac{1}{3}$ and $c = \frac{-5}{6}$

Then,
$a + b + c = \frac{1}{2} + \frac{1}{3} - \frac{5}{6}$
$= \frac{3+2}{6} - \frac{5}{6}$
$\Rightarrow a + b + c = \frac{5}{6} - \frac{5}{6} = 0$

$\therefore a^3 + b^3 + c^3 = 3abc$
$\Rightarrow \left(\frac{1}{2}\right)^3 + \left(\frac{1}{3}\right)^3 + \left(\frac{-5}{6}\right)^3 = 3 \times \left(\frac{1}{2}\right) \times \left(\frac{1}{3}\right) \times \left(\frac{-5}{6}\right)$
$= \frac{-5}{12}$

$\therefore \left(\frac{1}{2}\right)^3 + \left(\frac{1}{3}\right)^3 - \left(\frac{5}{6}\right)^3 = \frac{-5}{12}$

5. iv

Let $a = 0.2, b = -0.3,$ and $c = 0.1$

Then,
$a + b + c = 0.2 - 0.3 + 0.1$
$= 0.3 - 0.3$
$\Rightarrow a + b + c = 0$

$\therefore a^3 + b^3 + c^3 = 3abc$
$\Rightarrow (0.2)^3 + (-0.3)^3 + (0.1)^3 = 3 \times (0.2) \times (-0.3) \times (0.1)$
$= -0.018$