RD Sharma Class 10 Solutions Chapter 16 Surface Areas and Volumes VSAQS

Question 1.

The radii of the bases of a cylinder and a cone are in the ratio 3 : 4 and their heights are in the ratio 2 : 3. What is the ratio of their volumes ? Solution:

Radii of the bases of a cylinder and a cone = 3:4 and ratio in their heights = 2:3 Let r_1 , r_2 be the radii and h_1 and h_2 be their heights heights of the cylinder and cone respectively,

then
$$\frac{r_1}{r_2} = \frac{3}{4}$$

and $\frac{h_1}{h_2} = \frac{2}{3}$
Now $\frac{\text{Volume of cylinder}}{\text{Volume of cone}}$
 $= \frac{\pi r_1^2 h_1}{\frac{1}{3} \pi r_2^2 h_2}$
 $= \frac{3r_1^2 h_1}{r_2^2 h_2} = 3\left(\frac{r_1}{r_2}\right)^2 \times \left(\frac{h_1}{h_2}\right)$
 $= 3 \times \left(\frac{3}{4}\right)^2 \times \frac{2}{3}$
 $= 3 \times \frac{9}{16} \times \frac{2}{3} = \frac{9}{8}$

:. Ratio in their volumes = 9:8

Question 2.

If the heights of two right circular cones are in the ratio 1 : 2 and the perimeters of their bases are in the ratio 3 : 4. What is the ratio of their volumes ? Solution:

Ratio in the heights of two cones =1:2 and ratio in the perimeter of their bases = 3:4 Let r₁, r₂ be the radii of two cones and ht and h2 be their heights

 $\therefore \frac{h_1}{h_2} = \frac{1}{2}$ and $\frac{2\pi r_1}{2\pi r_2} = \frac{3}{4}$ $\Rightarrow \frac{r_1}{r_2} = \frac{3}{4}$ Now $\frac{\text{Volume of first cone}}{\text{Volume of second cone}} = \frac{\frac{1}{3}\pi r_1^2 h_1}{\frac{1}{3}\pi r_2^2 h_2}$ $=\frac{r_1^2 h_1}{r_2^2 h_2} = \left(\frac{r_1}{r_2}\right)^2 \times \frac{h_1}{h_2}$ $=\left(\frac{3}{4}\right)^2 \times \frac{1}{2} = \frac{9}{16} \times \frac{1}{2} = \frac{9}{32}$ \therefore Ratio in their volumes = 9 : 32

Question 3.

If a cone and sphere have equal radii and equal volumes what is the ratio of the diameter of the sphere to the height of the cone? Solution:

Let r be the radius of a cone, then radius of sphere = r Let h be the height of cone

Now volume of cone = volume of sphere

$$\Rightarrow \frac{1}{3}\pi r^{2}h = \frac{4}{3}\pi r^{3}$$

$$\Rightarrow h = 4r = 2 (2r) \qquad \left(\text{Dividing by } \frac{1}{3}\pi r^{2}\right)$$

$$\Rightarrow \frac{2r}{h} = \frac{1}{2} \Rightarrow \frac{\text{diameter of sphere}}{\text{height of cone}} = \frac{1}{2}$$

$$\therefore \text{ Ratio} = 1:2$$

Question 4.

A cone, a hemisphere and a cylinder stand on equal bases and have the same height. What is the ratio of their volumes? Solution:

Let r and h be the radius and heights of a cone, a hemisphere and a cylinder \therefore Volume of cone = (13) πr^2h

Volume of hemisphere = $(23) \pi r^3$

Volume of cylinder = $\pi r^2 h$

$$\therefore \frac{1}{3}\pi r^2h: \frac{2}{3}\pi r^3: \pi r^2h$$

$$= \frac{1}{3}h: \frac{2}{3}r:h$$

$$= h: 2r: 3h = h: 2h: 3h$$
(:: $r = h$ in hemisphere)
$$= 1: 2: 3$$

Question 5.

The radii of two cylinders are in the ratio 3 : 5 and their heights are in the ratio 2 : 3. What is the ratio of their curved surface areas ? Solution:

Radii of two cylinders are in the ratio = 3:5 and ratio in their heights = 2:3

Let r_1 , r_2 be the radii and h_1 , h_2 be the heights of the two cylinders respectively, then

$$\frac{r_1}{r_2} = \frac{3}{5}$$
 and $\frac{h_1}{h_2}$

Now Curved surface of first cylinder Curved surface of second cylinder

h

$$= \frac{2\pi r_1 h_1}{2\pi r_2 h_2}$$
$$= \frac{r_1 h_1}{r_2 h_2} = \frac{r_1}{r_2} \times$$
$$= \frac{3}{5} \times \frac{2}{3} = \frac{2}{5}$$

 \therefore Their ratio = 2 : 5

Question 6.

Two cubes have their volumes in the ratio 1 : 27. What is the ratio of their surface areas ?

Solution:

Ratio in the volumes of two cubes = 1 : 27

Let a_1 and a_2 be the sides of the two cubes respectively then volume of the first area = a_1^3

and volume of second cube = a_{2^3}

$$\therefore \ \frac{a_1^3}{a_2^3} = \frac{1}{27} \Rightarrow \left(\frac{a_1}{a_2}\right)^3 = \left(\frac{1}{3}\right)^3$$

 $\Rightarrow \frac{a_1}{a_2} = \frac{1}{3}$

 $\frac{\text{Surface area of the first cube}}{\text{Surface area of the second cube}}$

$$= \frac{a_1^2}{a_2^2} = \left(\frac{a_1}{a_2}\right)^2 = \left(\frac{1}{3}\right)^2 = \frac{1}{9}$$

 \therefore The ratio in them = 1 : 9

Question 7.

Two right circular cylinders of equal volumes have their heights in the ratio 1 : 2. What is the ratio of their radii ?

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Solution:

Ratio the heights of two right circular cylinders = 1:2 Let r_1, r_2 be their radii and h_1, h_2 be their

heights respectively then $\frac{n_1}{h_2}$

· Their volumes are equal

$$\therefore \frac{\pi r_1^2 h_1}{\pi r_2^2 h_2} = \frac{1}{1} \Longrightarrow \left(\frac{r_1}{r_2}\right)^2 \times \frac{h_1}{h_2} = 1$$
$$\left(\frac{r_1}{r_2}\right)^2 \times \frac{1}{2} = 1 \Longrightarrow \left(\frac{r_1}{r_2}\right)^2 = \frac{2}{1} \Longrightarrow \frac{r_1}{r_2} = \frac{\sqrt{2}}{1}$$

 \therefore Ratio of their radii are = $\sqrt{2}$: 1

Question 8.

If the volumes of two cones are in the ratio 1 : 4, and their diameters are in the ratio 4 : 5, then write the ratio of their weights.

Solution:

Volumes of two cones are in the ratio =1:4 and their diameter are in the ratio = 4:5 Let r_1 and r_2 be the radii and h_1 , h_2 be their

heights of two cones respectively, then

$$\frac{2r_1}{2r_2} = \frac{4}{5} \implies \frac{r_1}{r_2} = \frac{4}{5}$$

and $\frac{\frac{1}{3}\pi r_1^2 h_1}{\frac{1}{3}\pi r_2^2 h_2} = \frac{1}{4} \implies \left(\frac{r_1}{r_2}\right)^2 \left(\frac{h_1}{h_2}\right) = \frac{1}{4}$
$$\implies \left(\frac{4}{5}\right)^2 \times \frac{h_1}{h_2} = \frac{1}{4} \implies \frac{16}{25} \times \frac{h_1}{h_2} = \frac{1}{4}$$

$$\implies \frac{h_1}{h_2} = \frac{1}{4} \times \frac{25}{16} = \frac{25}{64}$$

:. Ratio in their heights = 25 : 64

Question 9.

A sphere and a cube have equal surface areas. What is the ratio of the volume of the sphere to that of the cube ?

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Solution:

Surface areas of a sphere and a cube are equal

Let r be the radius of sphere and a be the edge of cube,

Then $4\pi r^2 = 6a^2 \Longrightarrow \frac{r^2}{a^2} = \frac{6}{4\pi} = \frac{3}{2\pi}$ $\Rightarrow \frac{r}{a} = \sqrt{\frac{3}{2\pi}}$ Now Volume of sphere $=\frac{\frac{4}{3}\pi r^3}{r^3}=\frac{4\pi r^3}{2r^3}$ $=\frac{4\pi}{3}\left(\frac{r}{a}\right)^3=\frac{4\pi}{3}\times\left(\sqrt{\frac{3}{2\pi}}\right)^3$ Athonks, Hischaway $=\frac{4\pi}{3}\times\sqrt{\left(\frac{27}{8\pi^3}\right)}=\frac{4\pi}{3}\times\frac{3\sqrt{3}}{2\pi\sqrt{2\pi}}$ $\frac{2\sqrt{3}}{\sqrt{2}\sqrt{\pi}} = \frac{\sqrt{2}\sqrt{3}}{\sqrt{\pi}} = \sqrt{\frac{6}{\pi}}$ \therefore Ratio = $\sqrt{\frac{6}{\pi}}$: 1 = $\frac{\sqrt{6}}{\sqrt{\pi}}$: 1 $=\sqrt{6}:\sqrt{\pi}$

Question 10.

What is the ratio of the volume of a cube to that of a sphere which will fit inside it? Solution:

A sphere is fit inside the cube Side of a cube = diameter of sphere Let a be the side of cube and r be the radius of the sphere, then

$$2r = a \Rightarrow \frac{a}{r} = \frac{2}{1}$$
Now $\frac{\text{Volume of cube}}{\text{Volume of sphere}} = \frac{a^3}{\frac{4}{3}\pi r^3}$

$$= \frac{3a^3}{4\pi r^3} = \frac{3}{4\pi} \times \left(\frac{a}{r}\right)^3$$

$$= \frac{3}{4\pi} \left(\frac{2}{1}\right)^3 = \frac{3 \times 8}{4\pi \times 1} = \frac{6}{\pi}$$

 \therefore Their ratio = 6 : π

Question 11.

What is the ratio of the volumes of a cylinder, a cone and a sphere, if each has the same diameter and same height ?

Solution:

Diameters (or radii), and heights of a cylinder a cone and a sphere are equal, Let r and h be the radius and height be the cone cylinder, cone and sphere respectively, thus their volumes will be

$$\pi r^{2}h: \frac{1}{3}\pi r^{2}h: \frac{4}{3}\pi r^{3}$$

$$\Rightarrow h: \frac{1}{3}h: \frac{4}{3}r$$

$$\Rightarrow 3h: h: 4h$$

$$\Rightarrow 3: 1: 4$$
(:: r = h in sphere)

Note: If we take volume of hemisphere instead of sphere, then the ratio will be 3:1:2

Question 12.

A sphere of maximum volume is cut-out from a solid hemisphere of radius r. What is the ratio of the volume of the hemisphere to that of the cut-out sphere? Solution:

r is the radius of a hemisphere, then

the diameter of the sphere which is cut out of the hemisphere will be r

 \therefore Its radius = $\frac{r}{2}$ Now $\frac{\text{Volume of the hemisphere}}{\text{Volume of the sphere (cutout)}}$ $=\frac{\frac{2}{3}\pi r^{3}}{\frac{4}{3}\pi \left(\frac{r}{2}\right)^{3}}=\frac{\frac{2}{3}\pi r^{3}}{\frac{4}{3}\pi \frac{r^{3}}{8}}$ $=\frac{\frac{2}{3}\pi r^{3}}{\frac{1}{2\times 2}\pi r^{3}}=\frac{2}{3}\times\frac{6}{1}=\frac{4}{1}$:. Ratio = 4 : 1

Question 13.

A metallic hemisphere is melted and recast in the shape of a cone with the same base radius R as that of the hemisphere. If H is the height of the cone, then write thooks the value of (HR).

Solution:

R is the radius of a hemisphere 2

 \therefore Volume = $\frac{2}{3}\pi R^3$

Now radius of cone formed from it = Rand height = H

 \therefore Volume = $\frac{1}{3}\pi R^2 H$

$$\frac{\frac{1}{3}\pi R^{2}H}{\frac{2}{3}\pi R^{3}} = 1 \quad (\because \text{ Their volumes is same})$$

$$\Rightarrow \frac{H}{2R} = 1 \Rightarrow \frac{H}{R} = \frac{2}{1}$$
$$\therefore \frac{H}{R} = 2$$

Question 14.

A right circular cone and a right circular cylinder have equal base and equal height. If the radius of the base and height are in the ratio 5 : 12, write the ratio of the total surface area of the cylinder to that of the cone.

Solution:

Radius and height of a cone and a cylinder be r and h respectively

 \therefore Radius : Height = 5 : 12 \Rightarrow r : h = 5 : 12

But
$$\frac{r}{h} = \frac{5}{12} \Rightarrow r = \frac{5}{12} h$$

Now total surface area of cylinder

$$=2\pi rh+2\pi r^2$$

 $= 2\pi r \left(h + r \right)$

and total surface area of cone = $\pi r l + \pi r^2$ = $\pi r (l + r)$

$$\therefore \frac{2\pi r(h+r)}{\pi r(l+r)} = \frac{2(h+r)}{l+r} = \frac{2(h+r)}{\sqrt{h^2 + r^2 + r}}$$

$$= \frac{2h+2r}{\sqrt{h^2 + r^2 + r}} + \frac{2h+2\times\frac{5}{12}h}{\sqrt{h^2 + \left(\frac{5}{12}h\right)^2 + \frac{5}{12}h}}$$

$$= \frac{2h + \frac{5}{6}h}{\sqrt{h^2 + \frac{25h^2}{144} + \frac{5}{12}h}} = \frac{\frac{17}{6}h}{\sqrt{\frac{169h^2}{144} + \frac{5}{12}h}}$$

$$= \frac{\frac{17}{6}h}{\frac{13}{12}h + \frac{5}{12}h} = \frac{\frac{17}{6}h}{\frac{18}{12}h} = \frac{17}{6} \times \frac{12h}{18h}$$

$$= \frac{34}{18} = \frac{17}{9}$$

 \therefore Their ratio = 17 : 9

Question 15.

A cylinder, a cone and a hemisphere are of equal base and have the same height.

What is the ratio of their volumes ? Solution:

Let r and h be the radii and heights of the cylinder cone and hemisphere respectively, then

Volume of cylinder = $\pi r^2 h$ Volume of cone = $(13) \pi r^2 h$ Volume of hemisphere = $(23) \pi r^3$

$$\pi r^{2}h: \frac{1}{3}\pi r^{2}h: \frac{2}{3}\pi r^{3}$$

$$\Rightarrow h: \frac{1}{3}h: \frac{2}{3}r \qquad \text{(Dividing by } \pi r^{2}\text{)}$$

$$\Rightarrow h: \frac{1}{3}h: \frac{2}{3}h \qquad (\because r = h \text{ in hemisphere})$$

$$1 \quad 2$$

$$\Rightarrow 1: \frac{1}{3}: \frac{2}{3} \Rightarrow 3: 1: 2$$

Question 16.

same textbooks white The radii of two cones are in the ratio 2 : 1 and their volumes are equal. What is the ratio of their heights ?

Solution:

Radii of two cones are in the ratio = 2:1

Let $r_{\scriptscriptstyle 1}, r_{\scriptscriptstyle 2}$ be the radii of two cones and $h_{\scriptscriptstyle 1}, h_{\scriptscriptstyle 2}$ be their heights respectively,

Then
$$\frac{r_1}{r_2} = \frac{2}{1}$$

Now $\frac{\text{Volume of first cone}}{\text{Volume of the second cone}}$
 $= \frac{\frac{1}{3}\pi r_1^2 h_1}{\frac{1}{3}\pi r_2^2 h_2}$
 $= \frac{r_1^2 h_1}{r_2^2 h_2} = \left(\frac{r_1}{r_2}\right)^2 \times \left(\frac{h_1}{h_2}\right)$
 $= \left(\frac{2}{1}\right)^2 \times \frac{h_1}{h_2} = \frac{4h_1}{h_2}$

: Their volumes are equal

$$\therefore \ \frac{4h_1}{h_2} = 1$$

$$\Rightarrow \frac{h_1}{h_2} = \frac{1}{4}$$

 \therefore Their ratio is = 1 :

Question 17.

Two cones have their heights in the ratio 1 : 3 and radii 3:1. What is the ratio of their volumes ? Solution:

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Ratio in heights of two cones = 1:3 and ratio in their ratio = 3:1 Let r_1 , r_2 be their radii and h_1 , h_2 be their heights, then

 $\frac{r_1}{h_2} = \frac{3}{1}$ and $\frac{h_1}{h_2} = \frac{1}{3}$ Now Volume of first cone Volume of second cone $=\frac{\frac{1}{3}\pi {v_1}^2 h_1}{\frac{1}{3}\pi {v_2}^2 h_2} = \left(\frac{r_1}{r_2}\right)^2 \times \frac{h_1}{h_2}$ $(3)^2$ 1 0 1 2

$$= \left(\frac{3}{1}\right) \times \frac{1}{3} = \frac{3}{1} \times \frac{1}{3} = \frac{3}{1}$$

 \therefore Their ratio = 3 : 1

Question 18.

Same textbooks white A hemisphere and a cone have equal bases. If their heights are also equal, then what is the ratio of their curved surfaces? Solution:

Bases of a hemisphere and a cone are equal and their heights are also equal Let r and h be their radii and heights respectively

 \therefore r = h₁

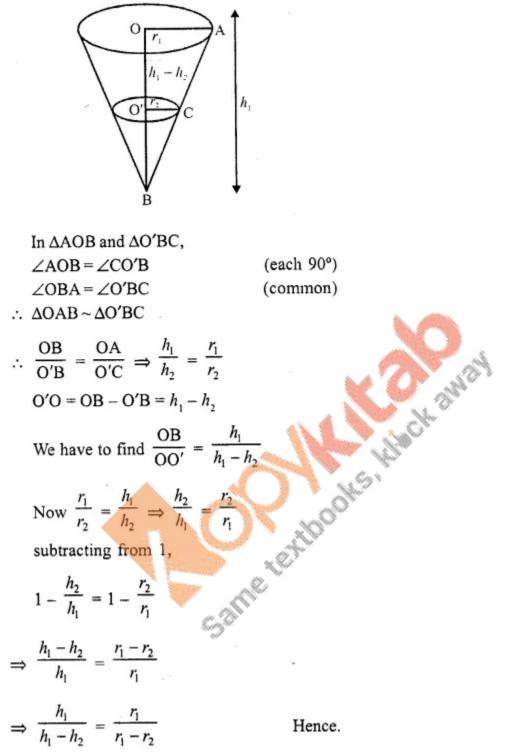
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Curved surface of 1	hemisphere
Now Curved surface	of cone
$=\frac{2\pi r^2}{\pi rl}=\frac{2r^2}{rl}=\frac{2r}{l}=$	$\frac{2r}{\sqrt{r^2 + h^2}}$
	$\left(\because l=\sqrt{r^2+h^2}\right)$
$=\frac{2r}{\sqrt{r^2+r^2}}$	$(\because r = h)$
$=\frac{2r}{\sqrt{2r^2}}=\frac{2r}{\sqrt{2}.r}=\frac{2}{\sqrt{2}}$	
$=\frac{\sqrt{2}}{1}$	(Dividing by √2)
\therefore Their ratio = $\sqrt{2} = 1$	Hacker
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Question 19.

If r_1 and r_2 denote the radii of the circular bases of the frustum of a cone such that $r_1 > r_2$ then write the ratio of the height of the cone of which the frustum is a part to the height of the frustum.

Solution:

 r_1 , r_2 are the radii of the bases of a frustum and $r_1 > r_2$ Let h_1 be the height of cone and h_2 be the height of smaller cone : Height of frustum = $h_1 - h_2$

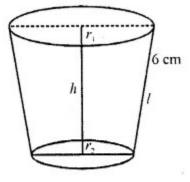


Question 20.

If the slant height of the frustum of a cone is 6 cm and the perimeters of its circular bases are 24 cm and 12 cm respectively. What is the curved surface area of the frustum ?

Solution:

Slant height of a frustum (I) = 6 cm Perimeter of upper base (P_1) = 24 cm and perimeter of lower base $(P_2) = 12 \text{ cm}$



$$\therefore \text{ Upper radius } (r_1) = \frac{P_1}{2\pi} = \frac{24 \times 7}{2 \times 22} = \frac{42}{11} \text{ cm}$$

and lower radius $(r_2) = \frac{P_2}{2\pi} = \frac{12 \times 7}{2 \times 22} = \frac{21}{11}$ cm

Now slant surface area = $\pi [r_1 + r_2] \times l$

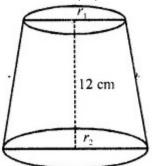
$$= \frac{22}{7} \left[\frac{42}{11} + \frac{21}{11} \right] \times 6 \text{ cm}^2$$
$$= \frac{22}{7} \left[\frac{63}{11} \right] \times 6 = 108 \text{ cm}^2$$

Question 21.

If the areas of circular bases of a frustum of a cone are 4 cm² and 9 cm² respectively and the height of the frustum is 12 cm. What is the volume of the frustum? ×C.

Solution:

In a frustum, Area of upper base $(A_1) = 4 \text{ cm}^2$ and area of lower base $(A_2) = 9 \text{ cm}^2$



$$\therefore \text{ Radius of upper base } (r_1) = \sqrt{\frac{\text{Area}}{\pi}} = \sqrt{\frac{4}{\pi}}$$

$$=\frac{2}{\sqrt{\pi}}$$
 cm

and radius of lower base (r_2)

$$=\sqrt{\frac{A}{\pi}}=\sqrt{\frac{9}{\pi}}=\frac{3}{\sqrt{\pi}}$$

Height (h) = 12 cm

:. Volume = $\frac{\pi}{3} (r_1^2 + r_1 r_2 + r_2^2) \times h$

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$$= \frac{\pi}{3} \left[\left(\frac{2}{\sqrt{\pi}} \right)^2 + \frac{2}{\sqrt{\pi}} \times \frac{3}{\sqrt{\pi}} + \left(\frac{3}{\pi} \right)^2 \right] \times 12 \text{ cm}^3$$
$$= \frac{\pi}{3} \left[\frac{4}{\pi} + \frac{6}{\pi} + \frac{9}{\pi} \right] \times 12$$
$$= 4\pi \left[\frac{19}{\pi} \right] = 76 \text{ cm}^3$$

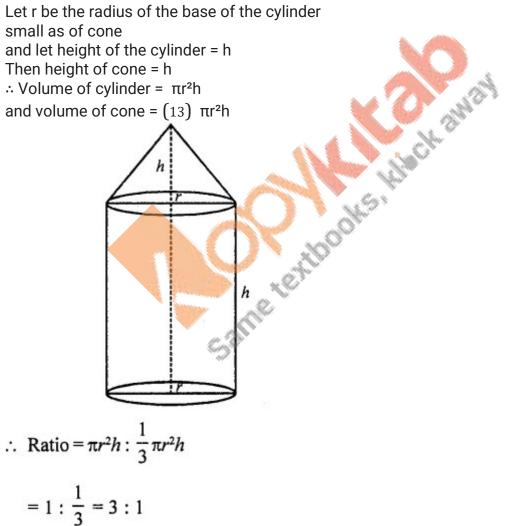
Question 22.

The surface area of a sphere is 616 cm². Find its radius. Solution: Surface area of a sphere = 616 cm² Let r be the radius, then

$$4\pi r^2 = 616 \Rightarrow \frac{4 \times 22}{7} r^2 = 616$$
$$\Rightarrow r^2 = \frac{616 \times 7}{4 \times 22} = 49 = (7)^2$$
$$\therefore r = 7$$
$$\therefore \text{ Radius} = 7 \text{ cm}$$

Question 23.

A cylinder and a cone are of the same base radius and of same height. Find the ratio of the value of the cylinder to that of the cone. [CBSE 2009] Solution:



Question 24.

The slant height of the frustum of a cone is 5 cm. If the difference between the radii of its two circular ends is 4 cm, write the height of the frustum. [CBSE 2010] Solution:

Slant height of frustum (I) = 5 cm

Difference between the upper and lower radii = 4 cm Let h be height and upper radius r_1 and lower radius = r_2

$$\therefore r_1 - r_2 = 4$$

and $l = \sqrt{h^2 + (r_1 - r_2)^2} \implies l^2 = h^2 + (r_1 - r_2)^2$
(5)² = h² + (4)²
$$\implies 25 = h^2 + 16 \implies h^2 = 25 - 16 = 9 = (3)^2$$

$$\therefore h = 3$$

Hence height = 3 cm

Question 25.

Volume and surface area of a solid hemisphere are numerically equal. What is the diameter of hemisphere?

Solution:

Volume of hemisphere = Surface area of hemisphere (given)

e of hemisphere = Surface area of hemisphere (given)

$$\frac{2}{3}\pi r^3 = 2\pi r^2 \Rightarrow \frac{1}{3}r = 1 \Rightarrow r = 3$$

Diameter of hemisphere = $2r = 2(3) = 6$ cm

:. Diameter of hemisphere = 2r = 2(3) = 6 cm