Exercise 16.1

1.

Sol:

Given that a solid sphere f radius $(r_1) = 8cm$

With this sphere we have to make spherical balls of radius $(r_2) = 1cm$

Since we don't know no of balls let us assume that no of balls formed be 'n'

We know that

$$\boxed{Volume\ of\ \ sphere = \frac{4}{3}\pi r^2}$$

Volume of solid sphere should be equal to sum of volumes of n spherical balls

$$n \times \frac{4}{3}\pi(1)^3 = \frac{4}{3}\pi r^3$$

$$n = \frac{\frac{4}{3}\pi(8)^3}{\frac{4}{3}\pi(1)^3}$$

$$n = 8^3$$

$$\boxed{n = 512}$$

$\therefore$ hence 512 no of balls can be made of radius 1cm from a solid sphere of radius $8cm$

2.

Sol:

Given that a metallic block which is rectangular of diameter $11dm \times 1m \times 5dm$

Given that diameter of each bullet is 5cm

$$\boxed{Volume\ of\ \ sphere = \frac{4}{3}\pi r^2}$$

Dimensions of rectangular block = $11dm \times 1m \times 5dm$

Since we know that $1dm = 10^{-1}m$

$$11 \times 10^{-1} \times 1 \times 5 \times 10^{-1} = 55 \times 10^{-2} m^3 \qquad \text{.........(1)}$$

Diameter of each bullet $= 5cm$

Radius of bullet $(r) = \frac{d}{2} = \frac{5}{2} = 2 \cdot 5cm$

$$= 25 \times 10^{-2} m$$

So volume $= \frac{4}{3}\pi\left(25 \times 10^{-2}\right)^3$

Volume of rectangular block should be equal sum of volumes of n spherical bullets
Let no of bullets be ‘n’
Equating (1) and (2)

$$55\times10^{-2} = n = \frac{4}{3}\pi\left(25\times10^{-2}\right)^3$$

$$\frac{55\times10^{-2}}{\frac{4}{3}\times\frac{22}{7}\left(25\times10^{-2}\right)^3} = n$$

$$n = 8400$$

$$\boxed{\therefore\ No\ of\ bullets\ found\ were\ 8400}$$

3.

Sol:

Given that a spherical ball of radius 3cm

We know that Volume of a sphere $= \frac{4}{3}\pi r^2$

So its volume $(v) = \frac{4}{3}\pi(3)^2$

Given that ball is melted and recast into three spherical balls

Radii of first ball $(v_1) = \frac{4}{3}\pi(1\cdot5)^3$

Radii of second ball $(v_2) = \frac{4}{3}\pi(2)^3$

Radii of third ball ____________?

Volume of third ball $= \frac{4}{3}\pi r^3 = v_3$

Volume of spherical ball is equal to volume of 3 small spherical balls

$$\Rightarrow \frac{4}{3}\pi r^2 + \frac{4}{3}\pi(1\cdot5)^3 + \frac{4}{3}\pi(2)^3 = \frac{4}{3}\pi(3)^3$$

$$\Rightarrow r^2 + (1\cdot5)^3 + (2)^3 = (3)^3$$

$$\Rightarrow r^3 = 3^3 - 1\cdot5^3 - 2^3$$

$$\Rightarrow r = (15\cdot6)\frac{1}{3}$$

$$\Rightarrow r = 2\cdot5cm$$

Diameter $(d) = 2r = 2\times2\cdot5 = 5cm$

$$\boxed{\therefore\ Diameter\ of\ third\ ball = 5cm.}$$

4.

Sol:

Given that $2\cdot 2dm^3$ of grass is to be drawn into a cylindrical wire $0.25cm$ in diameter

Given diameter of cylindrical wire $=0.25cm$

Radius of wire $(r)=\frac{d}{2}=\frac{0\cdot 25}{2}=0\cdot 125cm$

$=0\cdot 125\times 10^{-2}m.$

We have to find length of wire?

Let length of wire be 'h' $\quad (\because 1cm=10^{-2}m)$

$\boxed{Volume\ of\ Cylinder=\pi r^2 h}$

Volume of brass of $2\cdot 2dm^3$ is equal to volume of cylindrical wire

$\frac{22}{7}\left(0\cdot 125\times 10^{-2}\right)h=2\cdot 2\times 10^{-3}$

$\Rightarrow h=\frac{2\cdot 2\times 10^{-3}\times 7}{22\left(0\cdot 125\times 10^{-2}\right)^2}$

$\Rightarrow h=448m$

$\boxed{\therefore\ Length\ of\ cylindrical\ wire=448m}$

5.

Sol:

Given that diameter of solid cylinder $=2cm$

Given that solid cylinder is recast to hollow cylinder

Length of hollow cylinder $=16cm$

External diameter = 20cm

Thickness $=2\cdot 5mm=0\cdot 25cm$

$\boxed{Volume\ of\ solid\ cylinder=\pi r^2 h}$

Radius of cylinder $=1cm$

So volume of solid cylinder $=\pi(1)^2 h$ ……(i)

Let length of solid cylinder be h

$\boxed{Volume\ of\ hollow\ cylinder=\pi h\left(R^2-r^2\right)}$

Thickness $=R-r$

$0\cdot 25=10-r$

$\Rightarrow$ Internal radius $=9\cdot 75cm$

So volume of hollow cylinder $=\pi\times 16(100-95\cdot 0625)$ ….(2)

Volume of solid cylinder is equal to volume of hollow cylinder.

(1) = (2)

Equating equations (1) and (2)

$\pi(1)^2 h = \pi \times 16(100 - 95\cdot 06)$

$\frac{22}{7}(1)^2 \times h = \frac{22}{7} \times 16(4\cdot 94)$

$\boxed{h = 79\cdot 04cm}$

$\therefore$ Length of solid cylinder $= 79cm$

6.

Sol:

Given that diameter is equal to height of a cylinder

So $h = 2r$

$\boxed{Volume\ of\ \ cylinder = \pi r^2 h}$

So volume $= \pi r^2 (2r)$

$= 2\pi r^3$

Volume of each vessel $= \pi r^2 h$

Diameter $= 42cm$

Height $= 21cm$

Diameter $(d) = 2r$

$2r = 42$

$r = 21$

$\therefore$ Radius $= 21cm$

Volume of vessel $= \pi(21)^2 \times 21$ ……….(2)

Since volumes are equal

Equating (1) and (2)

$\Rightarrow 2\pi r^3 = \pi(21)^2 \times 21 \times 2$ ($\because$ 2 identical vessels)

$\Rightarrow r^3 = \frac{\pi(21)^2 \times 21 \times 2}{2 \times \pi}$

$\Rightarrow r^3 = (21)^3$

$\Rightarrow r = 21 \Rightarrow \boxed{d = 42cm}$

$\therefore$ Radius of cylindrical vessel $= 21cm$

Diameter of cylindrical vessel $= 42cm$.

7.

Sol:

Given that 50 circular plates each with diameter $=14cm$

Radius of circular plates $\left(r\right)=7cm$

Thickness of plates $=0\cdot 5$

Since these plates are placed one above other so total thickness of plates $=0\cdot 5\times 50$
$=25cm.$

$\boxed{Total\ surface\ area\ of\ \ a\,cylinder=2\pi rh+2\pi r^2}$

$=2\pi rh+2\pi r^2$

$=2\pi r\left(h+r\right)$

$=2\times\dfrac{22}{7}\times 7\left(25+7\right)$

$\boxed{T.S.A=1408cm^2}$

$\therefore$ Total surface area of circular plates is $1408cm^2$

8.

Sol:

Given that 25 circular plates each with radius (r) = 10.5cm

Thickness $=1\cdot 6cm$

Since plates are placed one above other so its height becomes $=1\cdot 6\times 25=40cm$

$\boxed{Volume\ of\ cylinder=\pi r^2h}$

$=\pi\left(10\cdot 5\right)^2\times 40$

$=13860cm^3$

$\boxed{Curved\ \ surface\ area\ of\ \ a\ cylinder=2\pi rh}$

$=2\times\pi\times 10\cdot 5\times 40$

$=2\times\dfrac{22}{7}\times 10\cdot 5\times 40$

$=2640cm^2$

$\therefore$ Volume of cylinder $=13860cm^3$

Curved surface area of a cylinder $=2640cm^2$

9.

Sol:

Diameter of circular pond $=40m$

Radius of pond(r) $=20m.$

Thickness $=2m$

Depth $=20cm=0\cdot 2m$

Since it is viewed as a hollow cylinder

$$\boxed{Thickness\ (t)=R-r}$$

$2=R-r$

$2=R-20$

$R=22m$

$$\boxed{\therefore Volume\ of\ hollow\ cylinder=\pi\left(R^2-r^2\right)h}$$

$=\pi\left(22^2-20^2\right)h$

$=\pi\left(22^2-20^2\right)\times 0\cdot 2$

$=\pi(84)\times 0.2$

$$\boxed{\therefore Volume\ of\ hollow\ cylinder=52\cdot \pi m^3}$$

$\therefore 52\cdot 77m^3$ of gravel is required to have path to a depth of 20cm.

10.

Sol:

Let as assume well is a solid right circular cylinder

Radius of cylinder $(r)=\frac{3\cdot 5}{2}=1\cdot 75m$

Height (or) depth of well $=16m.$

$$\boxed{Volume\ of\ right\ circular\ cylinder=\pi r^2 h}$$

$=\frac{22}{7}\times(1\cdot 75)^2\times 16$ ……..(1)

Given that length of platform $(l)=27.5m$

Breath of platform $(b)=7cm$

Let height of platform be xm

$$\boxed{Volume\ of\ rectangle=lbh}$$

$=27\cdot 5\times 7\times x=192\cdot 5x$ ………(2)

Since well is spread evenly to form platform

So equating (1) and (2)

$V_1=V_2$

$\Rightarrow \frac{22}{7}(1\cdot 75)^2\times 16=192\cdot 5x$

$\Rightarrow x=0\cdot 8m$

$\boxed{\therefore \text{ } Height \text{ } of \text{ } platform(h) = 80cm.}$

11.

Sol:

Let us assume well as a solid circular cylinder

Radius of circular cylinder $= \frac{2}{2} = 1m$

Height (or) depth of well $= 14m$

$\boxed{Volume \text{ } of \text{ } solid \text{ } circular \text{ } cylindeer = \pi r^2 h}$

$= \pi(1)^2 14$ …..(1)

Given that height of embankment (h) = 40cm

Let width of embankment be 'x' m

Volume of embankment $= \pi r^2 h$

$= \pi\left(\left(1+x^2\right)-1\right)^2 \times 0\cdot 4$ …..(2)

Since well is spread evenly to form embankment so their volumes will be same so equating (1) and (2)

$\Rightarrow \pi(1)^2 \times 14 = \pi\left(\left(1+x\right)^2 - 1\right)^2 \times 0\cdot 4$

$\Rightarrow x = 5m$

$\boxed{\therefore Width \text{ } of \text{ } embankment \text{ } of \text{ } (x) = 5m}$

12.

Sol:

Given that side of cube =9cm

Given that largest cone is curved from cube

Diameter of base of cone = side of cube

$\Rightarrow 2x = 9$

$\Rightarrow r = \frac{9}{2}cm$

Height of cone = side of cube

$\Rightarrow$ Height of cone (h) = 9cm

$\boxed{Volume \text{ } of \text{ } l\arg est \text{ } cone = \frac{1}{3}\pi r^2 h}$

$= \frac{1}{3} \times \pi t\left(\frac{9}{2}\right)^2 \times 9$

$=\frac{\pi}{12}\times 9^3$

$=190\cdot 92cm^3$

$\therefore$ Volume of largest cone $(v)=190\cdot 92cm^3$

13.

Sol:

36cm, 43.27 cm

14.

Sol:

Given length of rectangular surface $=6cm$

Breath of rectangular surface $=4cm$

Height (h) $1cm$

$\boxed{Volume\ of\ a\ flat\ rectangular\ surface = lbh}$

$=6000\times 400\times 1$

Volume $=240000cm^3$ __________(1)

Given radius of cylindrical vessel $=20cm$

Let height off cylindrical vessel be h_1

Since rains are transferred to cylindrical vessel.

So equating (1) with (2)

$\boxed{Volume\ of\ cylindrical\ vessel = \pi r_1^2 h_1}$

$=\frac{22}{7}(20)^2\times h_1$ __________(2)

$24000=\frac{22}{7}(20)^2\times h_1$

$\Rightarrow \boxed{h_1=190\cdot 9cm}$

$\therefore$ height of water in cylindrical vessel $=190\cdot 9cms$

15.

Sol:

Given base radius of conical flask be r

Height of conical flask is h

$\boxed{Volume\ of\ cone = \frac{1}{3}\pi r^2 h}$

So its volume $=\frac{1}{3}\pi r^2 h$ ________(1)

Given base radius of cylindrical flask is ms.

Let height of flask be h_1

$$\boxed{Volume\ of\ cylinder = \pi r^2 h_1}$$

So its volume $= \frac{22}{7}(mr)^2 h_1$ ________(2)

Since water in conical flask is poured in cylindrical flask their volumes are same

(1) = (2)

$$\Rightarrow \frac{1}{3}\pi r^2 h = \pi (mr)^2 \times h_1$$

$$\Rightarrow \boxed{h_1 = \frac{h}{3m^2}}$$

$\therefore$ Height of water in cylindrical flask $= \frac{h}{3m^2}$

16.

Sol:

Given length of rectangular tank = 15m

Breath of rectangular tank = 11m

Let height of rectangular tank be h

$$\boxed{Volume\ of\ rectangular\ tank = lbh}$$

Volume $= 15 \times 11 \times h$ ______(1)

Given radius of cylindrical tank $(r) = \frac{21}{2} m$

Length/height of tank = 5m

$$\boxed{Volume\ of\ cylindrical\ tank = \pi r^2 h}$$

$= \pi \left(\frac{21}{2}\right)^2 \times 5$ ______(2)

Since volumes are equal

Equating (1) and (2)

$$15 \times 11 \times h = \pi \left(\frac{21}{2}\right)^2 \times 5$$

$$\Rightarrow h = \frac{\frac{22}{7} \times \left(\frac{21}{2}\right)^2 \times 5}{15 \times 11}$$

$$\Rightarrow \boxed{h = 10 \cdot 5m}$$

$\therefore$ Height of tank $= 10 \cdot 5m.$

17.

Sol:

Given that internal radius of hemisphere bowl $=90m$

$$\boxed{Volume\ of\ \ hemisphere=\frac{4}{3}\pi r^{3}}$$

$$=\frac{2}{3}\times\pi(9)^{3} \qquad \_\_\_\_\_\_(1)$$

Given diameter of cylindrical bottle $=3cm$

Radius $=\frac{3}{2}cm$

Height $=4cm$

$$\boxed{Volume\ of\ \ cylindrical=\pi r^{2}h}$$

$$=\pi\left(\frac{3}{2}\right)^{2}\times 4 \qquad \_\_\_\_\_\_(2)$$

Volume of hemisphere bowl is equal to volume sum of n cylindrical bottles

(1) = (2)

$$\frac{2}{3}\pi(9)^{3}=\pi\left(\frac{3}{2}\right)^{2}\times 4\times n$$

$$\Rightarrow n=\frac{\frac{2}{3}\pi(9)^{3}}{\pi\left(\frac{3}{2}\right)^{2}\times 4}$$

$$\Rightarrow \boxed{n=54}$$

$\therefore$ No of bottles necessary to empty the bottle $=54$.

18.

Sol:

Internal diameter of hollow spherical shell $=6cm$

Internal radius of hollow spherical shell $=\frac{6}{3}=3cm$

External diameter of hollow spherical shell $=10\,cm$

External radius of hollow spherical shell $=\frac{10}{2}=5\,cm$

Diameter of cylinder $=14\ cm$

Radius of cylinder $= \frac{14}{2} = 7cm$

Let height of cylinder $= xcm$

According to the question

Volume of cylinder = Volume of spherical shell

$$\Rightarrow \pi(7)^2 x\times = \frac{4}{3}\pi\left(5^3 - 3^3\right)$$

$$\Rightarrow 49x\times = \frac{4}{3}(125-27)$$

$$\Rightarrow 49x\times = \frac{4}{3}\times 98$$

$$x = \frac{4\times 98}{3\times 49} = \frac{8}{3}cm$$

$\therefore$ Height off cylinder $= \frac{8}{3}cm$

19.

Sol:

Given internal diameter of hollow sphere (r) $= 4cm$

External diameter $(R) = 8cm$

$Volume\ of\ \ hollow\ sphere = \frac{4}{3}\pi\left(R^2 - r^2\right)$

$$= \frac{4}{3}\pi\left(B^2 - 4^2\right) \qquad \text{\_\_\_\_(1)}$$

Given diameter of cone $= 8cm$

Radius of cone $= 4cm$

Let height of cone be h

$Volume\ of\ \ cone = \frac{1}{3}\pi r^2 h$

$$= \frac{1}{3}\times\pi(4)^2 h \qquad \text{\_\_\_\_(2)}$$

Since hollow sphere is melted into a cone so there volumes are equal

(1) = (2)

$$\Rightarrow \frac{4}{3}\pi(64-16) = \frac{1}{3}\pi(4)^2 h$$

$$\Rightarrow \frac{\frac{4}{3}\pi(48)}{\frac{1}{3}\pi(16)} = h$$

$$\Rightarrow \boxed{h = 12cm}$$

$\therefore$ Height of cone $= 12cm$

20.

Sol:

Given that radius of a cylindrical tube (r) $= 12cm$

Level of water raised in tube $(h) = 6 \cdot 75cm$

$$\boxed{Volume\ of\ \ cylinder = \pi r^2 h}$$

$$= \pi(12)^2 \times 6 \cdot 75cm^3$$

$$= \frac{22}{7}(12)^2\, 6 \cdot 25cm^3 \qquad \text{.........(1)}$$

Let ‘r’ be radius of a spherical ball

$$\boxed{Volume\ of\ \ sphere = \frac{4}{3}\pi r^3} \qquad \text{.........(2)}$$

To find radius of spherical balls

Equating (1) and (2)

$$\pi \times (12)^2 \times 6 \cdot 75 = \frac{4}{3}\pi r^3$$

$$r^3 = \frac{\pi \times (12)^2 \times 6 \cdot 75}{\frac{4}{3} \times \pi}$$

$$r^3 = 729$$

$$r^3 = 9^3$$

$$\boxed{r = 9cm}$$

$\therefore$ Radius of spherical ball $(r) = 9cm$

21.

Sol:

Given that length of a rectangular tank (r) = 80m

Breath of a rectangular tank (b) $= 50m$

Total displacement of water in rectangular tank

By 500 persons $= 500 \times 0 \cdot 04 m^3$

$= 20 m^3$ ______(1)

Let depth of rectangular tank be h

$\boxed{Volume\ of\ \ rectangular\ \tan k = lbh}$

$= 80 \times 50 \times h m^3$ ______(2)

Equating (1) and (2)

$\Rightarrow 20 = 80 \times 50 \times h$

$\Rightarrow 20 = 4000h$

$\Rightarrow \dfrac{20}{4000} = h$

$\Rightarrow h = 0 \cdot 005m$

$\boxed{h = 0 \cdot 5cm}$

$\therefore$ Rise in level of water in tank $(h) = 0 \cdot 05cm.$

22.

Sol:

Given that radius of a cylindrical jar $(r) = 6cm$

Depth/height of cylindrical jar $(h) = 2cm$

Let no of balls be 'n'

$\boxed{Volume\ of\ a\ cylinder = \pi r^2 h}$

$V_1 = \dfrac{22}{7} \times (6)^2 \times 2cm^3$ ....(1)

Radius of sphere $1 \cdot 5cm$

$\boxed{So\ volume\ of\ \ sphere = \dfrac{4}{3}\pi r^3}$

$V_2 = \dfrac{4}{3} \times \dfrac{22}{7} (1 \cdot 5)^3\ cm^3$ ....(2)

Volume of cylindrical jar is equal to sum of volume of n spheres

Equating (1) and (2)

$\dfrac{22}{7} \times (6)^2 \times 2 = n \times \dfrac{4}{3} \times \dfrac{22}{4} (1 \cdot 5)^3$

$n = \dfrac{v_1}{v_2} \Rightarrow n = \dfrac{\dfrac{22}{7} \times (6)^2 \times 2}{\dfrac{4}{3} \times \dfrac{22}{7} (1 \cdot 5)^3}$

$\boxed{n = 16}$

$\therefore$ No of spherical balls $(n) = 16$

23.

Sol:

Given that internal radii of hollow sphere (r) = 2cm

External radii of hollow sphere $(R) = 4cm$

$$\boxed{Volume\ of\ \ hollow\ sphere = \frac{4}{3}\pi\left(R^2 - r^2\right)}$$

$v_1 = \frac{4}{3} \times \pi\left(4^2 - 2^2\right)$ ....(1)

Given that sphere is melted into a cone

Base radius of cone $= 4cm$

Let slant height of cone be l

Let height of cone be h

$l^2 = r^2 + h^2$

$l^2 = 16 + h^2$ ....(3)

$$\boxed{Volume\ of\ \ cone = \frac{1}{3}\pi r^2 h}$$

$v_2 = \frac{1}{3}\pi(4)^2 h$ ....(2)

$v_1 = v_2$ Equating (1) and (2)

$\frac{4}{3}\pi\left(4^2 - 2^2\right) = \frac{1}{3}\pi(4)^2 h$

$$\frac{\frac{4}{3}\pi(16-4)}{\frac{1}{3}\pi(16)} = h$$

$h = 14cm$

Substituting 'h' value in (2)

$l^2 = 16 + h^2$

$l^2 = 16 + 14^2$

$l^2 = 16 + 196$

$\boxed{l = 14 \cdot 56cm}$

$\therefore$ Slant height of cone $= 14 \cdot 56cm$

24.

Sol:

Given that internal diameter of hollow hemisphere $(r) = \frac{21}{2} cm = 10 \cdot 5 cm$

External diameter $(R) = \frac{25 \cdot 2}{2} = 12 \cdot 6 cm$

Total surface area of hollow hemisphere

$= 2\pi R^2 + 2\pi r^2 + \pi\left(R^2 - r^2\right)$

$= 2\pi(12 \cdot 6)^2 + 2\pi(10 \cdot 5)^2 + \pi\left(12 \cdot 6^2 - 10 \cdot 5^2\right)$

$= 997 \cdot 51 + 692 \cdot 72 + 152 \cdot 39$

$= 1843 \cdot 38 cm^2$

Given that cost of painting $1cm^2$ of surface $= 10\, ps$

Total cost for painting $1843 \cdot 38 cm^2$

$= 1843 \cdot 38 \times 10\, ps$

$= 184 \cdot 338\, Rs.$

$\therefore$ Total cot to paint vessel all over $= 184 \cdot 338\, Rs.$

25.

Sol:

Given that radius of cylindrical tube $(r_1) = 12cm$

Let height of cylindrical tube (h)

$\boxed{Volume\ of\ a\ cylinder = \pi r_1^2 h}$

$v_1 = \pi(12)^2 \times h$ ……(1)

Given spherical ball radius $(r_2) = 9cm$

$\boxed{Volume\ of\ sphere = \frac{4}{3}\pi r_2^3}$

$v_2 = \frac{4}{3} \times \pi \times 9^3$ ……(2)

Equating (1) and (2)

$v_1 = v_2$

$\pi(12)^2 \times h = \frac{4}{3} \times \pi \times 9^3$

$h = \frac{\frac{4}{3} \times \pi \times 9^3}{\pi(12)^2}$

$h = 6 \cdot 75 cm$

Level of water raised in tube (h) $= 6 \cdot 75 cm$

26.

Sol:

Given height of a hollow cylinder = 14cm

Let internal and external radii of hollow

Cylinder be 'r' and R

Given that difference between inner and outer

Curved surface $= 88cm^2$

Curved surface area of cylinder (hollow)

$= 2\pi(R-r)h\ cm^2$

$\Rightarrow 88 = 2\pi(R-r)h$

$\Rightarrow 88 = 2\pi(R-r)14$

$\Rightarrow R-r=1$ ……(1)

Volume of cylinder (hollow) $= \pi(R^2-r^2)h\ cm^3$

Given volume of a cylinder $= 176cm^3$

$\Rightarrow \pi(R^2-r^2)h = 176$

$\Rightarrow \pi(R^2-r^2)\times 14 = 176$

$\Rightarrow R^2-r^2=4$

$\Rightarrow (R+r)(R-r)=4$

$\Rightarrow R+r=4$ ……(2)

$R-r=1$

$R+r=4$

$2R \quad =5$

$2R=5 \Rightarrow \boxed{R=\frac{5}{2}=2\cdot 5cm}$

Substituting 'R' value in (1)

$\Rightarrow R-r=1$

$\Rightarrow 2\cdot 5-r=1$

$\Rightarrow 2\cdot 5-1=r$

$\Rightarrow \boxed{r=1\cdot 5cm}$

$\therefore$ Internal radii of hollow cylinder $=1\cdot 5cm$

External radii of hollow cylinder $=2\cdot 5cm$

27.

Sol:

Let radius of a sphere be r

$\boxed{Curved\ surface\ area\ of\ sphere = 4\pi r^2}$

$S_1 = 4\pi r^2$

Let radius of cylinder be $'r'cm$

Height of cylinder be $'2r'cm$

$\boxed{Curved\ surface\ area\ of\ cylinder = 2\pi rh}$

$S_2 = 2\pi r(2r) = 4\pi r^2$

S_1 and S_2 are equal. Hence proved

So curved surface area of sphere = surface area of cylinder

28.

Sol:

Given diameter of a sphere (d) $= 9cm$

Radius (r) $= \frac{9}{2} = 4\cdot 5cm$

$\boxed{Volume\ of\ a\ sphere = \frac{4}{3}\pi r^3}$

$V_1 = \frac{4}{3} \times \pi \times 4\cdot 5^3 = 381\cdot 70cm^3$ ......(1)

Since metallic sphere is melted and made into a cylindrical wire

$\boxed{Volume\ of\ a\ cylinder = \pi r^2 h}$

Given radius of cylindrical wire $(r) = \frac{2mm}{2}$

$= 1mm = 0\cdot 1cm$

$V_2 = \pi (0\cdot 1)^2 h$ ......(2)

Equating (1) and (2)

$V_1 = V_2$

$\Rightarrow 381\cdot 703 = \pi (0\cdot 1)^2 h$

$\Rightarrow \boxed{h = 12150cm}$

$\therefore$ Length of wire (h) $= 12150cm$

29.

Sol:

Given that radius of each of smaller ball $= \frac{1}{4}$ Radius of original ball.

Let radius of smaller ball be r.
Radius of bigger ball be $4r$

Volume of big spherical ball $= \frac{4}{3}\pi r^3 \quad (\because r = 4r)$

$$V_1 = \frac{4}{3}\pi(4r)^3 \qquad \ldots\ldots(1)$$

$$\boxed{Volume\ of\ each\ small\ ball = \frac{4}{3}\pi r^3}$$

$$V_2 = \frac{4}{3}\pi r^3 \qquad \ldots\ldots(2)$$

Let no of balls be 'n'

$$n = \frac{V_1}{V_2}$$

$$\Rightarrow n = \frac{\frac{4}{3}\pi(4r)^3}{\frac{4}{3}\pi(r)^3}$$

$$\Rightarrow n = 4^3 = 64$$

$$\boxed{\therefore No\ of\ small\ balls = 64}$$

Curved surface area of sphere $= 4\pi r^2$

Surface area of big ball $(S_1) = 4\pi(4r)^2 \quad \ldots\ldots(3)$

Surface area of each small ball $(S_1) = 4\pi r^2$

Total surface area of 64 small balls

$(S_2) = 64 \times 4\pi r^2 \quad \ldots\ldots(4)$

By combining (3) and (4)

$$\Rightarrow \frac{S_2}{3} = 4$$

$$\Rightarrow \boxed{S_2 = 4s}$$

$\therefore$ Total surface area of small balls is equal to 4 times surface area of big ball.

30.

Sol:

Given that height of a tent = 77dm

Height of cone $= 44dm$

Height of a tent without cone $= 77 - 44 = 33dm$

$= 3 \cdot 3m$

Given diameter of cylinder (d) = 36m

Radius $(r) = \frac{36}{2} = 18m$

Let 'l' be slant height of cone

$l^2 = r^2 + h^2$

$l^2 = 18^2 + 3\cdot3^2$

$l^2 = 324 + 10\cdot89$

$l^2 = 334\cdot89$

$l = 18\cdot3$

Slant height of cone $l = 18\cdot3$

Curved surface area of cylinder $(S_1) = 2\pi rh$

$= 2\times\pi\times18\times4\cdot4m^2$ …….(1)

Curved surface area of cone $(S_2) = \pi rl$

$= \pi\times18\times18\cdot3m^2$ …….(2)

Total curved surface of tent $= S_1 + S_2$

T.C.S.A $= S_1 + S_2$

$= 1532\cdot46m^2$

Given cost canvas per $m^2 = Rs\ 3\cdot50$

Total cost of canvas per $1532\cdot46\times3\cdot50$

$= 1532\cdot46\times3\cdot50$

$= 5363\cdot61$

$\therefore$ Total cost of canvas $= Rs$ 5363.61

31.

Sol:

Given radius of metal spheres $= 2cm$

$$\boxed{Volume\ of\ sphere(v) = \frac{4}{3}\pi r^3}$$

So volume of each metallic sphere $= \frac{4}{3}\pi(2)^3\ cm^3$

Total volume of 16 spheres $(v_1) = 16\times\frac{4}{3}\pi(2)^3\ cm^3$ …(1)

Volume of rectangular box $= lbh$

$V_2 = 16\times8\times8cm^3$ …(2)

Subtracting (2) – (1) we get volume of liquid

$\Rightarrow V_2 - V_1 =$ Volume off liquid

$\Rightarrow 16\times 8\times 8-\frac{4}{3}\pi\left(2\right)^{3}\times 16$

$\Rightarrow 1024-536\cdot 16=488cm^{3}$

$\therefore$ Hence volume of liquid $=488cm^{3}$

32.

Sol:

Given radius of cylinder $\left(r\right)=7cm$

Height of cylinder $\left(h\right)=14cm$

Largest sphere is curved out from cylinder

Thus diameter of sphere = diameter of cylinder

Diameter of sphere $\left(d\right)=2\times 7=14cm$

Volume of a sphere $=\frac{4}{3}\pi r^{3}$

$=\frac{4}{3}\times\pi\left(7\right)^{3}$

$=\frac{1372\pi}{3}$

$=1436\cdot 75cm^{3}$

$\therefore$ Volume of sphere $=1436\cdot 75cm^{3}$

33.

Sol:

Given radius of sphere $=3cm$

Volume of a sphere $=\frac{4}{3}\pi r^{3}$

$=\frac{4}{3}\times\pi\times 3^{3}cm^{3}$ …..(1)

Given sphere is melted and recast into a right circular cone

Given height of circular cone $=3cm.$

Volume of right circular cone $=\pi r^{2}h\times\frac{1}{3}$

$=\frac{\pi}{3}\left(r\right)^{2}\times 3cm^{2}$ …..(1)

Equating 1 and 2 we get

$$\frac{4}{3}\pi \times 3^3 = \frac{1}{3}\pi (r)^2 \times 3$$

$$r^2 = \frac{\frac{4}{3}\pi \times 3^3}{\pi}$$

$$r^2 = 36cm$$

$$\boxed{r = 6cm}$$

$\therefore$ Radius of base of cone $(r) = 6cm$

34.

Sol:

Given that area of cuboid $= 160cm^2$

Level of water increased in vessel $= 2cm$

Volume of a vessel $= 160 \times 2cm^3$ ……(1)

Volume of each sphere $= \frac{4}{3}\pi r^3 cm^3$

Total volume of 3 spheres $= 3 \times \frac{4}{3}\pi r^3 cm^3$ ……(2)

Equating (1) and (2) ($\because$ Volumes are equal $V_1 = V_2$)

$$160 \times 2 = 3 \times \frac{4}{3}\pi r^3$$

$$r^3 = \frac{160 \times 2}{3 \times \frac{4}{3}\pi}$$

$$r^3 = \frac{320}{4\pi}$$

$$\boxed{r = 2 \cdot 94cm}$$

$\therefore$ Radius of sphere $= 2 \cdot 94cm$

35.

Sol:

Given diameter of copper rod $(d_1) = 1cm$

Radius $(r_1) = \frac{1}{2} = 0 \cdot 5cm$

Length of copper rod $(h_1) = 8cm$

$$\boxed{Volume\ of\ \ cylinder = \pi r_1^2 h_1}$$

$V_1 = \pi(0\cdot5)^2 \times 8cm^3$ ……(1)

$V_2 = \pi r_2^2 h_2$

Length of wire $(h_2) = 18m = 1800cm$

$V_2 = \pi r_2^2 (1800) cm^3$ …….(2)

Equating (1) and (2)

$V_1 = V_2$

$\pi(0\cdot5)^2 \times 8 = \pi r_2^2 (1800)$

$\frac{\pi(0\cdot5)^2 \times 8}{\pi(1800)} = r_2^2$

$\boxed{r_2 = 0\cdot033cm}$

$\therefore$ Radius thickness of wire $= 0\cdot033cm.$

36.

Sol:

Given diameter of internal surfaces of a hollow spherical shell $= 10cm$

Radius $(r) = \frac{10}{2} = 5cm.$

External radii $(R) = \frac{6}{2} = 3cm$

$\boxed{Volume\ of\ \ a\ spherica\ shell\ (hollow) = \frac{4}{3}\pi\left(R^2 - r^2\right)}$

$V_1 = \frac{4}{3}\pi\left(5^2 - 3^2\right)cm^3$ ………(1)

Given length of solid cylinder $(h) = \frac{8}{3}$

Let radius of solid cylinder be $'r'$

$\boxed{Volume\ of\ \ a\ cylinder = \pi r^2 h}$

$V_2 = \pi r^2\left(\frac{8}{3}\right)cm^3$ ………(2)

$V_1 = V_2$

Equating (1) and (2)

$\Rightarrow \frac{4}{3}\pi(25-9) = \pi r^2\left(\frac{8}{3}\right)$

$$\Rightarrow \frac{\frac{4}{3}\pi(16)}{\pi\left(\frac{8}{3}\right)} = r^2$$

$$\Rightarrow r^2 = 49cm$$
$$\Rightarrow r = 7cm$$
$$d = 2r = 14cm$$
$$\boxed{\therefore Diameter\ of\ cylinder = 14cm}$$

37.

Sol:

(i) Given that radius of cone $(r_1) = 4cm$

Height of cone $(h_1) = 3cm$

Slant height of cone $(l_1) = 5cm$

Volume of cone $(V_1) = \frac{1}{3}\pi r_1^2 h_1$

$= \frac{1}{3}\pi(4)^2(3) = 16\pi cm^3$

(ii) Given radius of second cone $(r_2) = 3cm$

Height of cone $(h_2) = 4cm$

Slant height of cone $(l_2) = 5cm$

Volume of cone $(V_2) = \frac{1}{3}r_2^2 h_2$

$= \frac{1}{3}\pi(3)^2(4) = 12\pi cm^3$

Difference in volumes of two cones $(V) = V_1 - V_2$

$V = 16\pi - 12\pi$

$V = 4\pi cm^3$

Curved surface area of first cone $(S_1) = \pi r_1 l_1$

$S_1 = \pi(4)(5) = 20\pi cm^2$

Curved surface area of first cone $(S_1) = \pi r_1 l_1$

$S_1 = \pi(4)(5) = 20\pi cm^2$

Curved surface area of second cone $(S_2) = \pi r_2 l_2$

$S_1 = \pi(3)(5) = 15\pi cm^2$

$S_1 = 20\pi cm^2 S_2 = 15\pi cm^2$

38.

Sol:

Given that dimensions of a cuboid $11cm \times 10cm \times 75cm$

So its volume $(V_1) = 11cm \times 10cm \times 7cm$

$= 11 \times 10 \times 7cm^3$ …….(1)

Given diameter (d) $= 1\cdot 75cm$

Radius $(r) = \frac{d}{2} = \frac{1\cdot 75}{2} = 0\cdot 875cm$

Thickness $(h) = 2mm = 0\cdot 2cm$

$\boxed{Volume \;\; of \;\; a \, cylinder = \pi r^2 h}$

$V_2 = \pi(0\cdot 875)^2 (0\cdot 2) cm^3$ …….(2)

$V_1 = V_2 \times n$

Since volume of a cuboid is equal to sum of n volume of 'n' coins

$n = \frac{V_1}{V_2}$

$n = no$ of coins

$n = \frac{11 \times 10 \times 7}{\pi(0\cdot 875)^2 (0\cdot 2)}$

$\boxed{n = 1600}$

$\therefore$ No of coins $(n) = 1600$,

39.

Sol:

Given that inner radius of a well $(a) = 4m$

Depth of a well $(h) = 14m$

$\boxed{Volume \; of \; a \; cylinder = \pi r^2 h}$

$V_1 = \pi(4)^2 \times 14cm^3$ ……….(1)

Given well is spread evenly to form an embankment

Width of an embankment $= 3m$

Outer radii of a well (R) $= 4 + 3 = 7m.$

$$\boxed{Volume\ of\ a\ hollow\ cylinder = \pi\left(R^2 - r^2\right)\times hm^3}$$

$V_2 = \pi\left(7^2 - 4^2\right)\times hm^3$ ……….(2)

Equating (1) and (2)

$V_1 = V_2$

$\Rightarrow \pi(4)^2 \times 14 = \pi(49-16)\times h$

$$\Rightarrow h = \frac{\pi(4)^2 \times 14}{\pi(33)}$$

$$\boxed{h = 6\cdot 78m}$$

40.

Sol:

Given that water is flowering with a speed $= 10km/hr$

In 30 minutes length of flowering standing water $= 10\times\frac{30}{60}km$

$= 5km = 5000m.$

Volume of flowering water in 30 minutes

$V = 5000 \times width \times depth\ m^3$

Given width of canal $= 1\cdot 5m$

Depth of canal $= 6m$

$V = 5000\times 1\cdot 5\times 6m^3$

$$\boxed{V = 45000m^3}$$

Irrigating area in 30 minutes if 8cm of standing water is desired $= \frac{45000}{0\cdot 08}$

$= \frac{45000}{0\cdot 08} = 562500m^2$

$$\boxed{\therefore\ Irrigated\ area\ in\ 30\min utes = 562500m^2}$$

41.

Sol:

$\frac{9}{8}\ m$

42.

Sol:

Given diameter of well = 3m

Radius of well $= \frac{3}{2} m = 4$

Depth of well $(b) = 14m$

With of embankment $= 4m$

$\therefore$ Radius of outer surface of embankment $= 4 + \frac{3}{2} = \frac{11}{2} m$

Let height of embankment $= hm$

$\boxed{Volume\ of\ embankment\,(V_1) = \pi\left(r_2^2 - r_1^2\right)h}$

($\because$ it is viewed as a hollow cylinder)

$$V_1 = \pi\left(\left(\frac{11}{2}\right)^2 - \left(\frac{3}{2}\right)\right)^2 \times h - m^3 \qquad \text{.....(1)}$$

$\boxed{Volume\ of\ earth\ dugout\ (V_2) = \pi r_1^2 h}$

$$V_2 = \pi\left(\frac{3}{2}\right)^2 \times 14\ m^3 \qquad \text{.....(2)}$$

Given that volumes (1) and (2) are equal

So $V_1 = V_2$

$$\Rightarrow \left(\left(\frac{11}{2}\right)^2 - \left(\frac{3}{2}\right)^2\right) \times h = \pi\left(\frac{3}{2}\right)^2 \times 14$$

$$\Rightarrow \left(\frac{121}{4} - \frac{9}{4}\right) h = \frac{9}{4} \times 14$$

$$\Rightarrow \boxed{h = \frac{9}{8} m}$$

$\therefore$ Height of embankment $(h) = \frac{9}{8} m.$

43.

Sol:

Given height of cone $(h) = 28cm$

Given surface area of Sphere $= 616cm^2$

We know surface area of sphere $= 4\pi r^2$

$\Rightarrow 4\pi r^2 = 616$

$\Rightarrow r^2 = \frac{616 \times 7}{4 \times 22}$

$\Rightarrow r^2 = 49$

$\Rightarrow \boxed{r = 7cm}$

$\therefore$ Radius of sphere $(r) = 7cm$

Let r_1 be radius of cone

Given volume of cone $=$ Volume of sphere

$$\boxed{Volume\ of\ \ cone = \frac{1}{3}\pi\left(r^2\right)h}$$

$$V_1 = \frac{1}{3}\pi\left(r_1\right)^2 \times 28cm^3 \qquad \text{..........(1)}$$

$$\boxed{Volume\ of\ sphere = \left(V_2\right) = \frac{4}{3}\pi r^3}$$

$$V_2 = \frac{4}{3}\pi\left(7\right)^3 cm^3 \qquad \text{..........(1)}$$

(1) = (2) $\Rightarrow V_1 = V_2$

$$\Rightarrow \frac{1}{3}\pi\left(r_1\right)^2 \times 28 = \frac{4}{3}\pi\left(7\right)^3$$

$$\Rightarrow r_1^2 = 49$$

$$r_1 = 7cm$$

Radius of cone $\left(r_1\right) = 7cm$

$$\boxed{Diameter\ of\ \ base\ of\ \ cone\left(d_1\right) = 2 \times 7 = 14cm}$$

44.

Sol:

Given height of a hollow cylinder $= 14cm$

Let internal and external radii of hollow

Cylinder be ‘r’ an ‘R’

Given that difference between inner and outer curved surface $= 88cm^2$

$$\boxed{Curved\ \ surface\ area\ of\ \ hollow\ cylinder = 2\pi\left(R - r\right)h}$$

$$\Rightarrow 88 = 2\pi\left(R - 0\right)h$$

$$\Rightarrow 88 = 2\pi\left(R - r\right)14$$

$$\Rightarrow R - r = 1 \qquad \text{........(1)}$$

$$\boxed{Volume\ of\ \ hollow\ cylinder = \pi\left(R^2 - r^2\right)h\ cm^3}$$

Given volume of cylinder $= 176cm^3$

$$\Rightarrow \pi\left(R^2 - r^2\right)h = 176$$

$$\Rightarrow \pi\left(R^2 - r^2\right) \times 14 = 176$$

$\Rightarrow R^2 - r^2 = 4$

$\Rightarrow (R+r)(R-r) = 4$

$\Rightarrow R + 4 = 4$ ……..(2)

By using (1) and (2) equations and solving we get

$R - r = 1$ ..(1)

$R + r = 4$ ..(2)

$2R = 5$

$\Rightarrow \boxed{R = \frac{5}{2} = 2\cdot 5cm}$

Substituting 'R' value in (1)

$\Rightarrow R - r = 1$

$\Rightarrow 2\cdot 5 - r = 1$

$\Rightarrow 2\cdot 5 - 1 = r$

$\Rightarrow \boxed{r = 1\cdot 5cm}$

External radii of hollow cylinder $(R) = 2\cdot 5cm$

Internal radii of hollow cylinder $(r) = 1\cdot 5cm$

45.

Sol:

Given that volume of a hemisphere $= 2424\frac{1}{2}cm^3$

Volume of a hemisphere $= \frac{2}{3}\pi r^3$

$\Rightarrow \frac{2}{3}\pi r^3 = 2425\frac{1}{2}$

$\Rightarrow \frac{2}{3}\pi r^3 = \frac{4841}{2}$

$\Rightarrow r^3 = \frac{4851\times 3}{2\times 2\times \pi}$

$\Rightarrow r^3 = \frac{4851\times 3}{4\pi}$

r^3

$r = 10\cdot 50cm$

$\therefore$ Radius of hemisphere $= 10\cdot 5cm$

Curved surface area of hemisphere $= 2\pi r^2$

$= 2\pi (10\cdot 5)^2$

$= 692 \cdot 72$

$\Rightarrow 693 xm^2$

$\therefore$ curved surface area off hemisphere $= 693 cm^2$

46.

Sol:

Given that height of cylindrical bucket $(h) = 32cm$

Radius $(r) = 18cm$

Volume of cylinder $= \pi r^2 h$

$= \frac{22}{7}(18)^2 \times 32 cm^3$ ....(1)

Given height of conical heap $= 24cm$

Let radius of conical heap be r_1

Slant height of conical heap be l_1

$\Rightarrow l_1^2 = r_1^2 + h_1^2$

$\Rightarrow r_1^2 = l_1^2 + h_1^2$

$\Rightarrow r_1^2 = l_1^2 - (24)^2$ ....(2)

Volume of cone $= \frac{1}{3}\pi r^2 h$

So its volume $= \frac{1}{3}\pi \Rightarrow r_1^2 h_1$

$= \frac{1}{3} \times \frac{22}{7} \times r_1^2 \times 24$

$= \frac{22}{7} \times r_1^2 \times 8 cm^3$ ....(3)

So equating (1) and (3)

(1) = (3)

$\Rightarrow \frac{22}{7}(18)^2 \times 32 = \frac{22}{7} \times r_1^2 \times 8$

$\Rightarrow \frac{(18)^2 \times 32}{8} = r_1^2$

$\Rightarrow r_1^2 = 1296$

$\Rightarrow \boxed{r_1 = 36cm}$

Radius of conical heap is 36cm

Substituting r_1 in (2)

$\Rightarrow r_1^2 = l_1^2 - (24)^2$

$\Rightarrow 1296 = l_1^2 - 576$

$\Rightarrow 1296 + 576 = l_1^2$

$\Rightarrow 1872 = l_1^2$

$\Rightarrow \boxed{l_1 = 43\cdot 26cm}$

$\therefore$ Slant height of conical heap $= 43\cdot 26cm$

Exercise 16.2

47.

Sol:

Given diameter of cylinder $24m$

Radius $(r) = \frac{24}{2} = 12m$

Given height of cylindrical part $(h_1) = 11m$

$\therefore$ Height of cone part $(h_2) = 5m$

Vertex of cone above ground $= 11 + 5 = 16m$

Curved surface area of cone $(S_1) = \pi rl$

$= \frac{22}{7} \times 12 \times l$

Let l be slant height of cone

$\Rightarrow l = \sqrt{r^2 + h_2^2}$

$\Rightarrow l = \sqrt{12^2 + 5^2} = 13m$

$l = 13m$

$\therefore$ Curved surface area of cone $(5) = \frac{22}{7} \times 12 \times 13m^2$ ……….(1)

Curved surface area of cylinder $(S_2) = 2\pi rh$

$S_2 = 2\pi(12)(11)m^2$ ………..(2)

To find area of canvas required for tent

$S = S_1 + S_2 = (1) + (2)$

$S = \frac{22}{7} \times 12 \times 13 + 2\pi(12)(11)$

$S = 490 + 829\cdot 38$

$S = 1320 m^2$

$\therefore$ Total canvas required for tent $(S) = 1320 m^2$

48.

Sol:

Given radius of cylinder $(a) = 2 \cdot 5 m$

Height of cylinder $(h) = 21 m$

Slant height of cylinder $(l) = 8 m$

Curved surface area of cone $(S_1) = \pi r l$

$S_1 = \pi (2 \cdot 5)(8) cm^2$ ………(1)

Curbed surface area of a cone $= 2\pi r h + \pi r^2$

$S_2 = 2\pi (2 \cdot 5)(21) + \pi (2 \cdot 5)^2 cm^2$ ………(2)

$\therefore$ Total curved surface area = (1) + (2)

$S = S_1 + S_2$

$S = \pi (2 \cdot 5)(8) + 2\pi (2 \cdot 5)(21) + \pi (2 \cdot 5)^2$

$S = 62 \cdot 831 + 329 \cdot 86 + 19 \cdot 63$

$S = 412 \cdot 3 m^2$

$\therefore$ Total curved surface area $= 412 \cdot 3 m^2$

Volume of a cone $= \frac{1}{3} \pi r^2 h$

$V_1 = \frac{1}{3} \times \pi (2 \cdot 5)^2 h \ cm^3$ ………..(3)

Let 'h' be height of cone

$l^2 = r^2 + h^2$

$\Rightarrow l^2 - r^2 = h^2$

$\Rightarrow h = \sqrt{l^2 - r^2}$

$\Rightarrow h = \sqrt{8^2 - 25^2}$

$\Rightarrow h = 23 \cdot 685 m$

Subtracting 'h' value in (3)

Volume of a cone $(V_1) = \frac{1}{3} \times \pi (2 \cdot 5)^2 (23 \cdot 685) cm^2$ ………(4)

Volume of a cylinder $(V_2) = \pi r^2 h$

$= \pi (2 \cdot 5)^2 21 m^3$ ………..(5)

Total volume = (4) + (5)

$V = V_1 + V_2$

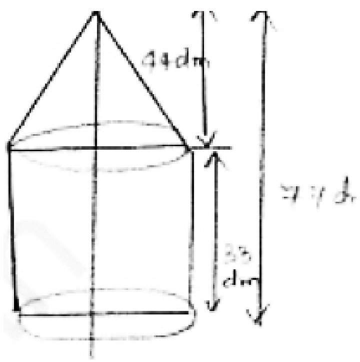
$\Rightarrow V = \frac{1}{3} \times \pi (2 \cdot 5)^2 (23 \cdot 685) + \pi (2 \cdot 5)^2 = 1$

$\Rightarrow V = 461 \cdot 84 m^2$

Total volume $(V) = 461 \cdot 84 m^2$

49.

Sol:



Given that height of a tent $= 77dm$

Height of a surmounted cone $= 44dm$

Height of cylinder part $= 77 - 44$

$= 33dm = 3.3m$

Given diameter of cylinder (d) $= 26m$

Radius $(r) = \frac{36}{2} = 18m.$

Let 'l' be slant height of cone

$\Rightarrow l^2 = r^2 + h^2$

$\Rightarrow l^2 = 18^2 + 3 \cdot 3^2$

$\Rightarrow l^2 = 824 + 10 \cdot 89$

$\Rightarrow l = 18 \cdot 3$

$\therefore$ Slant height of cone $(1) = 18 \cdot 3$

Curved surface area of cylinder $(S_1) = 2\pi rh$

$= 2 \times \pi \times 18 \times 4 \cdot 4 m^2$ ………..(1)

Curved surface area of cone $(S_2) = \pi rh$

$= \pi \times 18 \times 18 \cdot 3 m^2$ ………..(2)

Total curved surface of tent $= S_1 + S_2$

$S = S_1 + S_2$

$S = 1532 \cdot 46 m^2$

$\therefore$ Total curved surface area $(S) = 12 = 1532 \cdot 46 m^2$

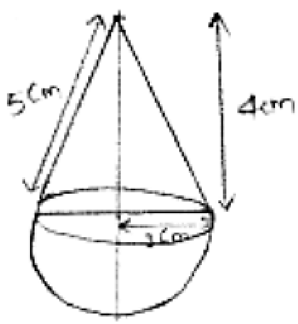
50.

Sol:

Given height of cone $(h) = 4 cm$

Diameter of cone $(d) = 6 cm$

$\therefore$ Radius (r) $= \frac{6}{2} = 3 cm$



Let 'l' be slant height of cone

$l = \sqrt{r^2 + h^2}$

$= \sqrt{3^2 + 4^2} = 5 cm$

$l = 5 cm$

$\therefore$ Slant height of cone $(l) = 5 cm.$

Curved surface area of cone $(S_1) = \pi r l$

$S_1 = \pi (3)(5) = 47 \cdot 1 cm^2$

Curved surface area of hemisphere $(S_2) = 2\pi r^2$

$S_2 = 2\pi (3)^2 = 56 \cdot 52 cm^2$

$\therefore$ Total surface area $(s) = 6_1 + S_2$

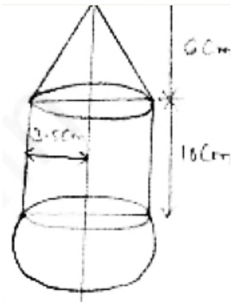
$= 47 \cdot 1 + 56 \cdot 52$

$= 103 \cdot 62 cm^2$

$\therefore$ Curved surface area of toy $= 103 \cdot 62 cm^2$

51.

Sol:



Given radius of common base $= 3\cdot 5cm$

Height of cylindrical part $(h) = 10cm$

Height of conical part $(h) = 6cm$

Let $'l'$ be slant height of cone

$l = \sqrt{r^2 + h^2}$

$l = \sqrt{(3\cdot 5)^2 + 6^2}$

$l = 48\cdot 25cm$

Curved surface area of cone $(S_1) = \pi rl$

$= \pi(3\cdot 5)(48\cdot 25)$

$= 76\cdot 408cm^2$

Curved surface area of cylinder $(S_2) = 2\pi rh$

$= 2\pi(3\cdot 5)(10)$

$= 220cm^2$

Curved surface area of hemisphere $(S) = S_1 + S_2 + S_3$

$= 76\cdot 408 + 220 + 77$

$= 373\cdot 408cm^2$

$\therefore$ Total surface area of solid $(S) = 373\cdot 408cm^2$

Cost of canvas per $m^2 = Rs\ 3\cdot 50$

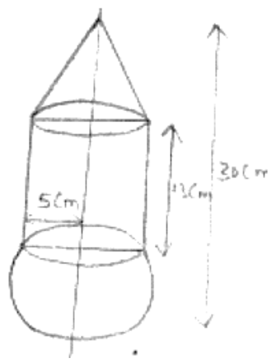
Cost of canvas for $1532\cdot 46m^2 = 1532\cdot 46 \times 3\cdot 50$

$= 5363\cdot 61Rs$

$\therefore$ Cost of canvas required for tent $= Rs\ 5363\cdot 61pr$

52.

Sol:



$S_1 = 2\pi(2)(13)$

$S_1 = 408 \cdot 2cm^2$

Curved surface area of cone $(S_2) = \pi rl$

Let l be slant height of cone

$l = \sqrt{r^2 + h^2}$

$h = 30 - 13 - 5 = 12cn$

$\Rightarrow l = \sqrt{12^2 + 5^2} = 13cm$

$l = 13cm$

$\therefore$ Curved surface area of cone $(S_2) = \pi(5)(13)$

$= 204 \cdot 1cm^2$

Curved surface area of hemisphere $(S_3) = 2\pi r^2$

$= 2\pi(5)^2$

$= 2\pi(25) = 50\pi = 157cm^2$

$S_3 = 157cm^2$

Total curved surface area $(S) = S_1 + S_2 + S_3$

$S = 408 \cdot 2 + 204 \cdot 1 + 157$

$S = 769 \cdot 3cm^2$

$\therefore$ Surface area of toy $(S) = 769.3cm^2$

53.

Sol:

Given radius of cylindrical tube $(r) = 5cm.$

Height of cylindrical tube $(h) = 9 \cdot 8cm$

Volume of cylinder $= \pi r^2 h$

$V_1 = \pi(5)^2(9 \cdot 8) = 770cm^3$

Given radius of hemisphere $(r) = 3 \cdot 5cm$

Height of cone $(h) = 5cm$

Volume of hemisphere $= \frac{2}{3}\pi r^3$

$= \frac{2}{3} \times \pi (3 \cdot 5)^3 = 89 \cdot 79cm^3$

Volume of cone $= \frac{1}{3}\pi r^2 h$

$= \frac{\pi}{3}(3 \cdot 5)^2 5 = 64 \cdot 14cm^3$

Volume of cone + volume of hemisphere $(V_2) = 39 \cdot 79 + 64 \cdot 14 = 154cm^3$

54.

Sol:

Given radius of cylindrical base $= 20m$

Height of cylindrical part $(h) = 4 \cdot 2m.$

Volume of cylindrical $= \pi r^2 h_1$

$V_1 = \pi (20)^2 \, 4 \cdot 2 = 5280m^3$

Volume of cone $= \frac{1}{3}\pi r^2 h_2$

Height of conical part $(h_2) = 2 \cdot 1m$

$V_2 = \frac{\pi}{3}(20)^2 (2 \cdot 1) = 880m^3$

Volume of tent $(v) = V_1 + V_2$

$V = 5280 + 880$

$V = 6160m^3$

$\therefore$ Volume of tent $(v) = V_1 + V_2$

$V = 5280 + 880$

$V = 6160m^3$

$\therefore$ Volume of tent $(v) = 6160m^3$

55.

Sol:

Given base diameter of cylinder $= 21cm$

Radius $(r) = \frac{21}{2} = 11 \cdot 5cm$

Height of cylindrical part $(h) = 18cm$

Height of conical part $(h_2) = 9cn$

Volume of cylinder $= \pi r^2 h_1$

$V_1 = \pi (11 \cdot 5)^2 18 = 7474 \cdot 77cm^3$

Volume of cone $= \frac{1}{3}\pi r^2 h_2$ ($\because$ 2 conical end)

$V_2 = \frac{1}{3}\pi (11 \cdot 5)^2 (9) \times 2$

$V_2 = \frac{1}{3}\pi (1190 \cdot 25) = 2492 \cdot 25cm^3$

Volume of tank = volume of cylinder + volume of cone

$V = V_1 + V_2$

$V = 7474 \cdot 77 + 2492 \cdot 85$

$V = 9966 \cdot 36cm^3$

Volume of water left in tube = Volume of cylinder – Volume of hemisphere and cone

$V = V_1 - V_2$

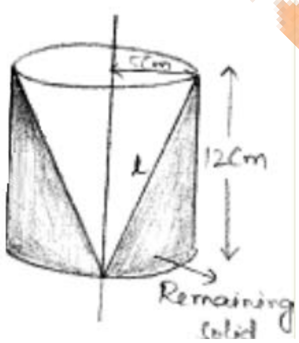
$= 770 - 154$

$= 616cm^3$

$\therefore$ Volume of water left in tube $= 616cm^3$

56.

Sol:



Given base radius of cylinder $(r) = 5cm$

Height of cylinder $(h) = 12cm$

Let 'l' be slant height of cone

$l = \sqrt{r^2 + h^2}$

$= \sqrt{5^2 + 12^2}$

$l = 13cm$

$\therefore$ Height and base radius of cone and cylinder are same

Total surface area of remaining part $(s) = 2\pi rh + \pi r^2 + \pi rl$

$= 2\pi(5)(12) + \pi(5)^2 + \pi(5)(13)$

T.S.A $= 210\pi cm^2$

Volume of remaining part = Volume of cylinder – Volume of cone

$\Rightarrow V = \pi r^2 h - \frac{1}{3}\pi r^2 h$

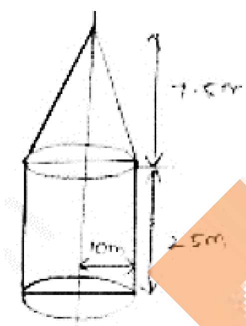
$\Rightarrow V = \pi(5)^2(12) - \frac{1}{3}\pi(5)^2(12)$

$\Rightarrow V = 200\pi cm^3$

$\therefore$ Volume of remaining part $(v) = 200\pi cm^3$

57.

Sol:



Given radius of cylinder $(r) = \frac{20}{2} = 10m$

Height of a cylinder $(h_1) = 2 \cdot 5m$

Height of cone $(h_2) = 7 \cdot 5m$

Let 'l' be slant height of cone

$l = \sqrt{r^2 + h_2^2}$

$l = \sqrt{10^2 + 7 \cdot 5^2}$

$\Rightarrow l = 12 \cdot 5m$

Volume of cylinder $(V_1) = \pi r^2 h$

$V_1 = \pi(10)^2(2 \cdot 5)$ ……..(1)

Volume of cone $(V_2) = \frac{1}{3}\pi r^2 h_2$

$= \frac{1}{3}\pi(10)^2(7\cdot 5)m^3$ ……..(2)

Total capacity of tent = (1) + (2)

$V = V_1 + V_2$

$V = \pi(10)^2 2\cdot 5 + \frac{1}{3}\pi(10)^2 7\cdot 5$

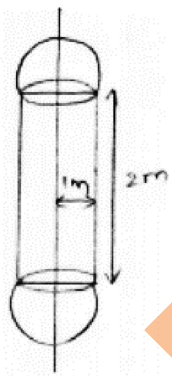
$V = 250\pi + 250\pi$

$V = 500\pi cm^3$

$\therefore$ Total capacity of tent $= 500\pi cm^2$

58.

Sol:



Given height of cylinder $(h) = 2m$

Diameter of hemisphere $(d) = 2m$

Radius $(r) = 1m$

Volume of a cylinder $= \pi r^2 h$

$V_1 = \pi(1)^2(2)cm^3$ ……..(1)

Volume of hemisphere $= \frac{2}{3}\pi r^3$

Since at ends of cylinder hemisphere are attached

Volumes of 2 hemispheres

$= 2\times\frac{2}{3}\pi(1)^2 cm^2$ ………(2)

Volumes of boiler = (1) + (2)

$V = V_1 + V_2$

$V = 2\times\frac{2}{3}\pi(1)^2 + \pi(1)^2(2)$

$V = \frac{220}{21}m^3$

$\therefore$ Volumes of boiler $= \frac{220}{21}m^3$

59.

Sol:

Given radius of hemisphere $(r) = \frac{3\cdot5}{2} = 1\cdot75m$

Height of cylinder $(h) = \frac{14}{3}m$

Volume of cylinder $= \pi r^2 h$

$= \pi(1\cdot75)^2\left(\frac{14}{3}\right)cm^3$ ……..(1)

Volume of hemisphere $= \frac{2}{3}\pi r^3$

$= \frac{2}{3}\times\pi(1\cdot75)^3 cm^3$ ……..(2)

Volume of vessel = (1) + (2)

$V = V_1 + V_2$

$V = \pi r^2 h + \frac{2}{3}\pi r^3$

$V = \pi(1\cdot75)^2\left(\frac{14}{3}\right) + \frac{2}{3}\pi(1\cdot75)^2$

$V = 56m^3$

$\therefore$ Volumes of vessel $(v) = 56m^3$

Internal surface area of solid $(s) = 2\pi rh + 2\pi r^2$

S = Surface area of cylinder + surface are of hemisphere

S = $2\pi(1\cdot75)\left(\frac{14}{3}\right) + 2\pi(1\cdot75)^2$

$S = 70\cdot51m^2$

$\therefore$ Internal surface area of solid $(s) = 70\cdot51m^2$

60.

Sol:

Given radius of hemispherical ends $= 7cm$

Height of body $(h+2r) = 104cm.$

Curved surface area of cylinder $= 2\pi rh$

$= 2\pi(7)h$ ....(1)

$\Rightarrow h + 2x = 104$

$\Rightarrow h = 104 - 2(r)$

$\Rightarrow h = 90cm$

Substitute 'h' value in (1)

Curved surface area of cylinder $= 2\pi(7)(90)$

$= 3948 \cdot 40cm^2$ ....(2)

Curved surface area of 2 hemisphere $= 2(2\pi r^2)$

$= 2(2 \times \pi \times 7^2)$

$= 615 \cdot 75cm^3$ ....(3)

Total curved surface area = (2) + (3)

$= 3958 \cdot 40 + 615 \cdot 75 = 4574 \cdot 15cm^2 = 45 \cdot 74dm^2$

Cost of polishing for $1dm^2 = Rs10$

Cost of polishing for $45 \cdot 74dm^2 = 45 \cdot 74 \times 10$

$= Rs\ 457 \cdot 4$

61.

Sol:

Given height of cylindrical vessel $(h) = 42cm$

Inner radius of a vessel $(r_1) = \frac{14}{2}cm = 7cm$

Outer radius of a vessel $(r_2) = \frac{16}{2} = 8cm$

Volume of a cylinder $= \pi(r_2^2 - r_1^2)h$

$= \pi(8^2 - 7^2)42$

$= \pi(64 - 49)42$

$= 15 \times 42 \times \pi$

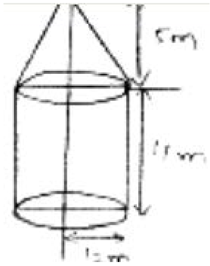
$= 630\pi$

$= 1980cm^3$

Volume of a vessel $= 1980cm^2$

62.

Sol:



Given internal radius of cylindrical road

Roller $\left(r_1\right) = \frac{54}{2} = 27cm$

Given thickness of road roller $\left(\frac{1}{b}\right) = 9cm$

Let order radii of cylindrical road roller be R

$\Rightarrow t = R - r$

$\Rightarrow 9 = R - 27$

$\Rightarrow R = 9 + 27 = 36cm$

$R = 36cm$

Given height of cylindrical road roller $\left(h\right) = 1m$

$h = 100cm.$

Volume of iron $= \pi h\left(R^2 - r^2\right)$

$= \pi\left(36^2 - 27^2\right) \times 100$

$= 1780 \cdot 38cm^3$

Volume of iron $= 1780 \cdot 38cm^3$

Mass of $1cm^3$ of iron $= 7 \cdot 8gm$

Mass of $1780 \cdot 38cm^3$ of iron $= 1780 \cdot 38 \times 7 \cdot 8$

$= 1388696 \cdot 4gm$

$= 1388 \cdot 7kg$

$\therefore$ Mass of roller $\left(m\right) = 1388 \cdot 7kg$

63.

Sol:

Given radius of hemisphere and cylinder (r)

$= \frac{14}{2} = 7cm$

Given total height of vessel $= 13cm$

$(h+r)=13cm$

Inner surface area of vessel $=2\pi r(h+r)$

$=2\times\pi\times7(13)$

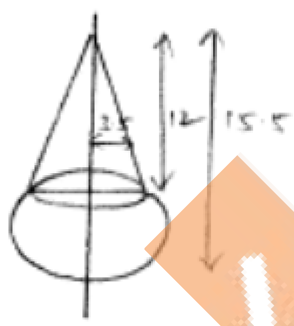
$=182\pi$

$=572cm^2$

$\therefore$ Inner surface area of vessel $=572cm^2$

64.

Sol:



Given radius of cone $(r)=3\cdot5cm$

Total height of toy $(h)=15\cdot5cm$

Length of cone $(l)=15\cdot5-3\cdot5$

$=12cm$

$\therefore$ Length of cone $(l)=12cm$

Curved surface area of cone $=\pi rl$

$S_1=\pi(3\cdot5)(12)$

$S_1=131\cdot94cm^2$ ……(1)

Curved surface area of hemisphere $=2\pi r^2$

$S_2=2\pi(3\cdot5)^2$

$S_2=76\cdot96cm^2$ ……(2)

$\therefore$ Total surface of toy = (1) + (2)

$S = S_1 + S_2$

$S = 181 \cdot 94 + 76 \cdot 96$

$S = 208 \cdot 90$

$S = 209 cm^2$

$\therefore$ Total surface area of toy $= 209 cm^2$

65.

Sol:

Let inner radius of pipe be r_1

Radius of outer cylinder be r_2

Length of cylinder $(h) = 14 cm.$

Surface area of hollow cylinder $= 2\pi h (r_2 - r_1)$

Given surface area of cylinder $= 44 m^2$

66.

Sol:

Given radius of cylinder $(r_1) = \frac{12}{2} = 6 cm$

Given radius of hemisphere $(r_2) = \frac{6}{2} = 3 cm.$

Given height of cylinder $(h) = 15 cm..$

Height of cones $(l) = 12 cm.$

Volume of cylinder $= \pi r_1^2 h$

$= \pi (6)^2 (15) cm^3$ ……(1)

Volume of each cone = volume of cone + volume of hemisphere

$= \frac{1}{3}\pi r_2^2 l + \frac{2}{3}\pi r_2^3$

$= \frac{1}{3}\pi (3)^2 (12) + \frac{2}{3}\pi (3)^3 cm^3$ ……..(2)

Let number of cones be ‘n’

n(Volume of each cone) = volume of cylinder

$n\left(\frac{1}{3}\pi (3)^2 (12) + \frac{2}{3}\pi (3)^3\right) = \pi (6)^2 15$

$$\Rightarrow n = \frac{\pi(6)^2 15}{\frac{1}{3}\pi(3)^2(12) + \frac{2}{3}\pi(3)^3}$$

$$\Rightarrow n = \frac{540}{5} = 10$$

$$\Rightarrow 2\pi h(r_2 - r_1) = 44$$

$$\Rightarrow 2\pi(14)(r_2 - r_1) = 44$$

$$\Rightarrow 28\pi(r_2 - r_1) = 44$$

$$\Rightarrow (r_2 - r_1) = \frac{44}{28\pi}$$

$$\Rightarrow (r_2 - r_1) = \frac{1}{2} \qquad \ldots\ldots\ldots\ldots(1)$$

Given volume of a hollow cylinder $= 99cm^3$

Volume of a hollow cylinder $= \pi h(r_2^2 - r_1^2)$

$$\Rightarrow \pi h(r_2^2 - r_1^2) = 99$$

$$\Rightarrow 14\pi(r_2^2 - r_1^2) = 99$$

$$\Rightarrow 14\pi(r_1 + r_2)(r_2 - r_1) = 99$$

$$\Rightarrow 14\pi(r_1 + r_2)(1) = 99$$

$$\Rightarrow 14\pi(r_1 + r_2) = 99$$

$$\Rightarrow (r_1 + r_2) = \frac{9}{2} \qquad \ldots\ldots\ldots\ldots(2)$$

Equating (1) and (2) equations we get

$$r_1 + r_2 = \frac{9}{2}$$

$$-r_1 + r_2 = \frac{1}{2}$$

$$2r_2 = 5$$

$$r_2 = \frac{5}{2}cm.$$

Substituting r_2 value in (1)

$$\Rightarrow r_1 = 2cm$$

$\therefore$ Inner radius of pipe $(a) = 2cm$

Radius of outer cylinder $(r_2) = \frac{5}{2}cm.$

67.

Sol:

Given radius of cylindrical part $(r) = \frac{12}{2} = 6cm$

Height of cylinder $(h) = 110cm$

Length of cone $(l) = 9cm$

Volume of cylinder $= \pi r^2 h$

$V_1 = \pi (0)^2 110cm^3$ ………(1)

Volume of cone $= \frac{1}{3}\pi r^2 l$

$V_2 = \frac{1}{3}\pi (6)^2 9 = 108\pi cm^3$ ………(2)

Volume of pole = (1) + (2)

$V = V_1 + V_2$

$\Rightarrow V = \pi (6)^2 110 + 108\pi$

$\Rightarrow V = 12785 \cdot 14 cm^3$

Given mass of $1cm^3$ of iron $= 8gm$

Mass of $12785 \cdot 14 cm^3$ of iron $= 12785 \cdot 14 \times 8$

$= 102281 \cdot 12$

$= 102 \cdot 2kg$

$\therefore$ Mass of pole for $12785 \cdot 14 cm^3$ of iron is $102 \cdot 2kg$

68.

Sol:

Given radius of cone, cylinder and hemisphere $(r) = \frac{4}{2} = 2cm$

Height of cone $(l) = 2cm$

Height of cylinder $(h) = 4cm$

Volume of cylinder $= \pi r^2 h = \pi (2)^2 (4) cm^3$ ………(1)

Volume of cone $= \frac{1}{3}\pi r^2 l$

$= \frac{1}{3}\pi (2)^2 \times 2$

$= \frac{\pi}{3}(4) \times 2cm^3$ ……….(2)

Volume of hemisphere $=\frac{2}{3}\pi r^3$

$=\frac{2}{3}\times\pi(2)^3$

$=\frac{2}{3}\times\pi(8)cm^3$ ……….(3)

So remaining volume of cylinder when toy is inserted to it $=\pi r^2h-\left(\frac{1}{3}\pi r^2l+\frac{2}{3}\pi r^3\right)$

= (1) – ((2) + (3))

$=\pi(2)^2(4)-\left(\frac{\pi}{3}\times 8+\frac{2}{3}\times\pi\times 8\right)$

$=16\pi-\frac{2}{3}\pi(4+8)=16\pi-8\pi=8\pi cm^3$

$\therefore$ So remaining volume of cylinder when toy is inserted to it $=8\pi cm^3$

69.

Sol:

Given radius of circular cone $(a)=60cm$

Height of circular cone $(b)=120cm.$

Volume of a cone $=\frac{1}{3}\pi r^2 l$

$=\frac{1}{3}\pi(60)^2(120)cm^3$ …………(1)

Volume of hemisphere $=\frac{2}{3}\pi r^3$

Given radius of hemisphere $=60cm$

$=\frac{2}{3}\pi(60)^2\,cm^3$ ………….(2)

Given radius of cylinder $=60cm$

Height of cylinder $(h)=180cm.$

Volume of cylinder $=\pi r^2 h$

$=\pi(60)^2\times 180cm^3$ ……………(3)

Volume of water left in cylinder = (3) – ((1) + (2))

$\Rightarrow \frac{1}{3}\pi(60)^3(120) - \left(\frac{2}{3}\pi(60)^3 + \pi(60)^2 \times 180\right)$

$\Rightarrow 113\cdot 1cm^3 = 1\cdot 131m^3$

$\therefore$ Volume of water left in cylinder $= 1\cdot 131m^3$

70.

Sol:

Given internal radius $(r_1) = \frac{10}{2} = 5cm$

Height of cylindrical vessel $(h) = 10\cdot 5cm$

Outer radius of cylindrical vessel $(l_2) = \frac{7}{2} = 3\cdot 5cm$

Length of cone $(l) = 6cm.$

(i) Volume of water displaced = volume of cone

Volume of cone $= \frac{1}{3}\pi r_2^2 l$

$= \frac{1}{3}\pi \times 3\cdot 5^2 \times 6 = 76\cdot 9cm^3$

$= 77cm^3$

$\therefore$ Volume of water displaced $= 77cm^3$

Volume of cylinder $= \pi r_1^2 h = \pi(5)^2 10\cdot 5$

$= 824\cdot 6$

$= 825cm^2$

(ii) Volume of water left in cylinder = volume of
Cylinder – volume of cone

$= 825 - 77 = 748cm^3$

$\therefore$ Volume of water left in cylinder $= 748cm^3$

71.

Sol:

Given edge of wooden block $(a) = 21cm$

Given diameter of hemisphere = edge of cube

Radius $= \frac{21}{2} = 10\cdot 5cm$

Volume of remaining block = volume of box – volume of hemisphere

$= a^3 - \frac{2}{3}\pi r^3$

$=(2)^3-\frac{2}{3}\pi(10\cdot5)^3$

$=6835\cdot5cm^3$

Surface area of box $=6a^2$ ……..(1)

Curved surface area of hemisphere $=2\pi r^2$ ..……(2)

Area of base of hemisphere $=\pi r^2$ ……..(3)

So remaining surface area of box = (1) – (2) + (3)

$=6a^2-\pi r^2+2\pi r^2$

$=6(21)^2-\pi(10\cdot5)+2\pi(10\cdot5)^2$

$=2992\cdot5cm^2$

$\therefore$ Remaining surface area of box $=2992\cdot5cm^2$

Volume of remaining block $=6835\cdot5cm^3$

72.

Sol:

Given radius of cone = radius of hemisphere

Radius $(r)=21cm$

Given that volume of cone $=\frac{2}{3}$Volume of hemisphere

$\Rightarrow$ Volume of cone $=\frac{1}{3}\pi r^2h$

Volume of hemisphere $=\frac{2}{3}\pi r^3$

So $\frac{1}{3}\pi r^2 h=\frac{2}{3}\left(\frac{2}{3}\pi r^3\right)$

$\Rightarrow \frac{1}{3}\pi(21)^2 h=\frac{2}{3}\left(\frac{2}{3}\pi(21)^3\right)$

$\Rightarrow h=\frac{4(21)\pi\times 3}{4\pi(21)}$

$\Rightarrow h=\frac{4}{3}\times 21=28cm$

$\therefore$ Height of cone $(h)=28cm$

Curved surface area of cone $=\pi rl$

$S_1=\pi(21)(28)cm^2$ …………(1)

Curved surface area off hemisphere $=2\pi r^2$

$S_2=2\times\pi(21)^2\ cm^2$ …………..(2)

Total surface area $(s)=S_1+S_2=(1)+(2)$

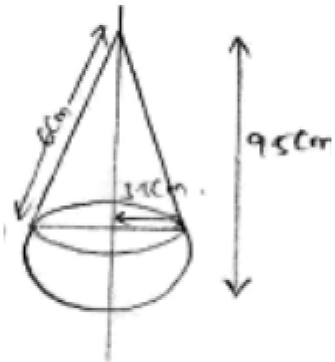
$S=\pi rl+2\pi r^2$

$S=5082cm^2$

$\therefore$ Curved surface area of toy $=5082cm^2$

73.

Sol:



Given radius of hemisphere and cone $= 3 \cdot 5 cm$

Given total height of solid $(h) = 9 \cdot 5 cm$

Length of cone $(l) = 9 \cdot 5 - 3 \cdot 5 = 6 cm$

Volume of a cone $= \frac{1}{3} \pi r^2 l$

$V_1 = \frac{1}{3} \pi (3 \cdot 5)^2 \times 6 \ cm^3$ ....(1)

Volume of hemisphere $= \frac{2}{3} \pi r^3$

$V_2 = \frac{2}{3} \pi (3 \cdot 5)^3 \ cm^3$ ....(2)

Volume of solid = (1) + (2)

$V = V_1 + V_2$

$V = \frac{1}{3} \pi (3 \cdot 5)^2 \times 6 + \frac{2}{3} \pi (3 \cdot 5)^3$

$V = 76 \cdot 96 + 89 \cdot 79 = 166 \cdot 75 cm^3$

$\therefore$ Volume of solid $(v) = 166 \cdot 75 cm^3$

Exercise 16.3

1.

Sol:

Given diameter to top of bucket = 40cm

Radius $(r_1) = \frac{40}{2} = 20 cm$

Depth of a bucket $(h) = 12 cm$

Volume of a bucket $= \frac{1}{3} \pi \left(r_1^2 + r_2^2 + r_1 r_2\right) h$

$= \frac{3}{1}\pi\left(20^2 + 10^2 + 20(10)\right)^{12}$

$= 8800 cm^3$.

Let ‘l’ be slant height of bucket

$\Rightarrow l = \sqrt{(r_1 - r_2)^2 + h_2}$

$\Rightarrow l = \sqrt{(20-10)^2 + 12^2}$

$\Rightarrow l = 2\sqrt{61} = 15\cdot 620 cm$

Total surface area of bucket $= \pi(r_1 + r_2)\times l + \pi r_2^2$

$= \pi(20+10)\times 15\cdot 620 + \pi(10)^2$

$= \frac{1320\sqrt{61} + 2200}{7} cm^2$

$= \frac{1320\sqrt{61} + 2200}{7\times 100} dm^2 = 17\cdot 87 dm^2$

Given that cost of tin sheet used for making bucket per $dm^2 = Rs 1.20$

So total cost for $17\cdot 87 dm^2 = 1\cdot 20 \times 17\cdot 87$

$= 21\cdot 40\ Rs.$

$\therefore$ Cost of tin sheet for $17\cdot 87 dm^2 = Rs 2140\, ps$

2.

Sol:

Given base diameter of cone $(d_1) = 20 cm$

Radius $(r_1) = \frac{20}{2} = 10 cm$

Top diameter of cone $(d_2) = 12 cm$

Radius $(r_2) = \frac{12}{2} = 6 cm$

Height of cone $(h) = 3 cm$

Volume of frustum right circular cone

$= \frac{1}{3}\pi\left(r_1^2 + r_2^2 + r_1 r_2\right)h$

$= \frac{1}{3}\pi\left(10^2 + 6^2 + (10)(6)\right)3$

$= 616 cm^3$

Let ‘l’ be slant height of cone

$\Rightarrow l=\sqrt{(r_1-r_2)^2+h_2}$

$\Rightarrow l=\sqrt{(10-6)^2+3^2}$

$\Rightarrow l=\sqrt{16+9}=\sqrt{25}cm=5cm$

$\therefore$ Slant height of cone $(l)=5cm$

Total surface area of cone $=\pi(r_1+r_2)l+\pi r_1^2+\pi r_2^2$

$=\pi(10+6)5+\pi(10)^2+\pi(6)^2$

$=\pi(80+100+36)$

$=\pi(216)=678\cdot 85cm^2$

$\therefore$ Total surface area of cone $=678\cdot 85cm^2$

3.

Sol:

Given slant height of cone $(r)=4cm$

Let radii of top and bottom circles be r_1 and r_2

Given perimeters of its ends as 18cm and 6cm

$\Rightarrow 2\pi r_1=18cm$

$\Rightarrow \pi r_1=9cm$ ……(1)

$\Rightarrow 2\pi r_2=6cm$

$\Rightarrow \pi r_2=3cm$ ……(2)

Curved surface area of frustum cone $=\pi(r_1+r_2)l$

$=\pi(r_1+r_2)l$

$=(\pi r_1+\pi r_2)l$

$=(9+3)4$

$=(12)4=48cm^2$

$\therefore$ Curved surface area of frustum cone $=48cm^2$

4.

Sol:

Given perimeters of ends of frustum right circular cone are 44cm an 33cm

Height of frustum cone $=16cm$

Perimeter $=2\pi r$

$2\pi r_1=44$

$r_1 = 7cm$

$2\pi r_2 = 33$

$r_2 = \frac{21}{4} = 5 \cdot 25cm$

Let slant height of frustum right circular cone be l

$l = \sqrt{(r_1 - r_2)^2 + h^2}$

$l = \sqrt{(7 - 5 \cdot 25)^2 + 16^2 cm}$

$l = 16 \cdot 1cm$

$\therefore$ Slant height of frustum cone $= 16 \cdot 1cm$

Curved surface area of frustum cone $= \pi(r_1 + r_2)l$

$= \pi(7 + 5 \cdot 25)16 \cdot 1$

C.S.A of cone $= 619 \cdot 65cm^2$

Volume of a cone $= \frac{1}{3}\pi(r_1^2 + r_2^2 + r_1 r_2) \times h$

$= \frac{1}{3}(7^2 + (5 \cdot 25)^2 + 7(5 \cdot 25) \times 16$

$= 1898 \cdot 56cm^3$

$\therefore$ Volume of a cone $= 1898 \cdot 56\ cm^3$

Total surface area of frustum cone $= \pi(r_1 + r_2)l + \pi r_1^2 + \pi r_2^2$

$= \pi(7 + 5 \cdot 25)16 \cdot 1 + \pi(7^2 + 5 \cdot 25^2)$

$= 860 \cdot 27cm^2$

$\therefore$ Total surface area of frustum cone $= 860 \cdot 27cm^2$

5.

Sol:

Given height of conical bucket $= 45cm$

Give radii of 2 circular ends of a conical bucket is 28cm and 7cm

$r_1 = 28cm$

$r_2 = 7cm$

Volume of a conical bucket $= \frac{1}{3}\pi(r_1^2 + r_2^2 + r_1 r_2)h$

$= \frac{1}{3}\pi\left(28^2 + 7^2 + 28(7)\right)45$

$= \frac{1}{3}\pi(1029)45$

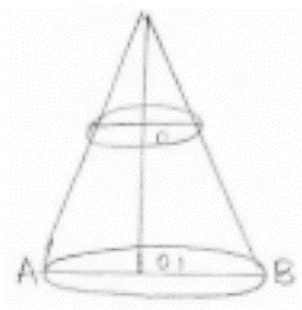
$= 15435$

$V = 48510cm^3$

Volume of a conical bucket $= 48510cm^3$

6.

Sol:



V AB be a cone of height $h_1 = VO_1 = 20cm$

Fronts triangles Vo_1A and VoA_1

$\frac{VO_1}{VO} = \frac{O_1A}{OA_1} \Rightarrow \frac{20}{VO} = \frac{O_1A}{OA_1}$

Volumes of cone $VA_1O = \frac{1}{125}$ times volumes of cone VAB

We have $\frac{1}{3}\pi \times OA_1^2 \times VO = \frac{1}{125} \times \frac{1}{3}\pi \times O_1A_1^2 \times 20$

$\Rightarrow \left(\frac{OA_1}{O_1A}\right)^2 \times VO = \frac{4}{25}$

$\Rightarrow \left(\frac{VO}{20}\right)^2 \times VO = \frac{4}{25}$

$\Rightarrow (VO)^3 = \frac{4 \times 400}{25}$

$\Rightarrow VO^3 = 64$

$\Rightarrow VO = 4$

Height at which section is made $= 20 - 4 = 16cm.$

7.

Sol:

Given height of a bucket $(R) = 24cm$

Radius of circular ends of bucket $5cm$ and $15cm$

$r_1 = 5cm$; $r_2 = 15cm$

Let $'l'$ be slant height of bucket

$l = \sqrt{(r_1 - r_2)^2 + h^2}$

$\Rightarrow l = \sqrt{(15-5)^2 + 24^2}$

$\Rightarrow l = \sqrt{100+576} = \sqrt{676}$

$l = 26cm$

Curved surface area of bucket $= \pi(r_1 + r_2)l + \pi r_2^2$

$= \pi(5+15)26 + \pi(15)^2$

$= \pi(20)26 + \pi(15)^2$

$= \pi(520+225)$

$= 745\pi cm^2$

$\therefore$ Curved surface area of bucket $= 745\pi cm^2$

8.

Sol:

Let slant height of frustum cone be ‘l’

Given height of frustum cone $12cm$

Radii of a frustum cone are 12cm and 23cm

$r_1 = 12cm \quad r_2 = 3cm$

$l = \sqrt{(r_1 - r_2)^2 + h^2}$

$l = \sqrt{(12-3)^2 + 12^2}$

$l = \sqrt{81+144} = 15cm$

$l = 15cm$

Total surface area of cone $= \pi(r_1 + r_2)l + \pi r_1^2 + \pi r_2^2$

$= \pi(12+3)15 + \pi(12)^2 + \pi(3)^2$

$\text{T.S.A} = 378\pi cm^2$

Volume of cone $= \frac{1}{3}\pi\left(r_1^2 + r_1 r_2 + r_2^2\right) \times h$

$= \frac{1}{3}\pi\left(12^2 + 3^2 + (12)(3)\right)12$

$= 756\pi cm^3$

Volume of frustum cone $= 756\pi cm^3$

9.

Sol:

Given height of frustum $(h) = 8m$

Radii of frustum cone are $13m$ and $7m$

$r_1 = 13m \quad r_2 = 7cm$

Let 'l' be slant height of frustum cone

$\Rightarrow l = \sqrt{(r_1 - r_2)^2 + h^2}$

$\Rightarrow l = \sqrt{(13-7)^2 + 8^2} = \sqrt{36+64}$

$\Rightarrow l = 10m$

Curved surface area of friction $(S_1) = \pi(r_1 + r_2) \times l$

$= \pi(13+7) \times 10$

$= 200\pi m^2$

C.S.A of frustum $(S_1) = 200\pi m^2$

Given slant height of conical cap = 12m

Base radius of upper cap cone $= 7m$

Curved surface area of upper cap cone $(S_2) = \pi rl$

$= \pi \times 7 \times 12 = 264m^2$

Total canvas required for tent $(S) = S_1 + S_2$

$S = 200\pi + 264 = 892 \cdot 57m^2$

$\therefore$ Total canvas $= 892 \cdot 57m^2$

10.

Sol:

Let depth of frustum cone be h

Volume of first cone $(V) = \frac{1}{3}\pi(r_1^2 + r_2^2 + r_1 r_2)h$

$r_1 = 50m \quad r_2 = 100m$

$V = \frac{1}{3} \times \frac{22}{7} \times (50^2 + 100^2 + 50(100))h$

$V = \frac{1}{3} \times \frac{22}{7} \times (2500 + 1000 + 5000)h \qquad \ldots(1)$

Volumes of reservoir $=44\times10^{7}$ liters ....(2)
Equating (1) and (2)

$$\frac{1}{3}\pi(2500)h=44\times10^{2}$$

$$h=24$$

Let 'l' be slant height of cone

$$l=\sqrt{(r_1-r_2)^2+h_2}$$

$$l=\sqrt{(50-100)^2+24^2}$$

$$l=55\cdot461m$$

Lateral surface area of reservoir

$$(S)=\pi(r_1+r_2)\times l$$

$$=\pi(50+100)55\cdot461$$

$$=1500(55\cdot461)\pi=26145\cdot225m^2$$

Lateral surface area of reservoir $=26145\cdot225m^2$

Volume of frustum cone $=\frac{1}{3}\pi\left(r_1^2+r_2^2+r_1r_2\right)h$

$$=\frac{1}{3}\pi\left(30^2+18^2+30(18)\right)9$$

$$=5292\pi cm^3$$

Volume $=5292\pi cm^3$

Total surface area of frustum cone =

$$=\pi(r_1+r_2)\times l+\pi r_1^2+\pi r_2^2$$

$$=(30+18)15+\pi(30)^2+(18)^2$$

$$=\pi\left(48(15)+(30)^2+(18)^2\right)$$

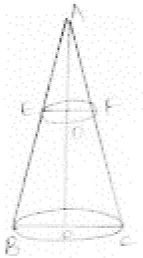
$$=\pi(720+900+324)$$

$$=1944\pi cm^2$$

$\therefore$ Total surface area $=1944\pi cm^2$

11.

Sol:



Let ABC be cone. Height of metallic cone $AO = 20cm$
Cone is cut into two parts at the middle point of its axis
Hence height of frustum cone $AD = 10cm$
Since angle A is right angled. So each angles B and $C = 45°$
Angles E and $F = 45°$
Let radii of top and bottom circles of frustum cone bee r_1 and $r_2 cm$

From $\Delta^{le} ADE \Rightarrow \frac{DE}{AD} = \cot 45°$

$\Rightarrow \frac{r_1}{10} = 1$

$\Rightarrow r_1 = 10cm.$

From $\Delta^{le} AOB$

$\Rightarrow \frac{OB}{OA} = \cot 45°$

$\Rightarrow \frac{r_2}{20} = 1$

$\Rightarrow r_2 = 20cm$

12.

Sol:

Given radii of top circular ends $(r_1) = 20cm$

Radii of bottom circular end of bucket $(r_2) = 12cm$

Let height of bucket be 'h'

Volume of frustum cone $= \frac{1}{3}\pi\left(r_1^2 + r_2^2 + r_1 r_2\right)h$

$= \frac{1}{3}\pi\left(20^2 + 12^2 + 20(12)\right)h$

$= \frac{784}{3}\pi h cm^3$ ……….(1)

Given capacity/volume of bucket $= 123308 \cdot 8cm^3$ ……….(2)

Equating (1) and (2)

$\Rightarrow \frac{784}{3}\pi h = 12308 \cdot 8$

$\Rightarrow h = \frac{12308 \cdot 8 \times 3}{784 \times \pi}$

$\Rightarrow h = 15cm$

$\therefore$ Height of bucket $(h) = 15cm$

Let 'l' be slant height of bucket

$\Rightarrow l^2 = (r_1 - r_2)^2 + h^2$

$\Rightarrow l = \sqrt{(r_1 - r_2)^2 + h^2}$

$\Rightarrow l = \sqrt{(20+2)^2 + 15^2} = \sqrt{64 + 225}$

$\Rightarrow l = 17cm$

Length of bucket/ slant height of

Bucket $(l) = 17cm$

Curved surface area of bucket $= \pi(r_1 + r_2)l + \pi r_2^2$

$= \pi(20+12)17 + \pi(12)^2$

$= \pi(32)17 + \pi(12)^2$

$= \pi(9248 + 144) = 2160 \cdot 32cm^2$

$\therefore$ Curved surface area $= 2160 \cdot 32cm^2$

13.

Sol:

Given height of bucket $(h) = 20cm$

Upper radius of bucket $(r_1) = 25cm$

Lower radius of bucket $(r_2) = 10cm$

Let 'l' be slant height of bucket

$l = \sqrt{(r_1 - r_2)^2 + h^2}$

$l = \sqrt{(25-10)^2 + 20^2} = \sqrt{225 + 400}$

$l = 25m$

$\therefore$ Slant height of bucket (l) $= 25cm$

Curved surface area of bucket $= \pi(r_1 + r_2)l + \pi r_2^2$

$= \pi(25+10)25 + \pi(10)^2$

$= \pi(35)25 + \pi(100) = 975\pi$

C.S.A $= 3061 \cdot 5cm^2$

Curved surface area $= 3061 \cdot 5cm^2$

Cost of making bucket per $100cm^2 = Rs70$

Cost of making bucket per $3061 \cdot 5cm^2 = \frac{3061 \cdot 5}{100} \times 70$

$= Rs\ 2143 \cdot 05$

$\therefore$ Total cost for $3061 \cdot 5cm^2 = Rs\ 2143 \cdot 05\ per$

14.

Sol:

Given slant height of frustum cone $= 10cm$

Radii of circular ends of frustum cone are 33 and 27cm

$r_1 = 33cm$; $r_2 = 27cm.$

Total surface area of a solid frustum of cone

$= \pi(r_1 + r_2) \times l + \pi r_1^2 + \pi r_2^2$

$= \pi(33 + 27) \times 10 + \pi(33)^2 + \pi(27)^2$

$= \pi(60) \times 10 + \pi(33)^2 + \pi(27)^2$

$= \pi(600 + 1089 + 729)$

$= 2418\pi cm^2$

$= 7599 \cdot 42cm^2$

$\therefore$ Total surface area of frustum cone $= 7599 \cdot 42cm^2$

15.

Sol:

Given height off frustum cone $= 16cm$

Diameter of lower end of bucket $(d_1) = 16cm$

Lower and radius $(r_1) = \frac{16}{2} = 8cm$

Upper and radius $(r_2) = \frac{40}{2} = 20cm$

Let 'l' be slant height of frustum of cone

$l = \sqrt{(r_1 - r_2)^2 + h^2}$

$l=\sqrt{(20-8)^2+16^2}$

$l=\sqrt{144+256}$

$l=20cm$

$\therefore$ Slant height of frustum cone $(l)=20cm.$

Volume of frustum cone $=\frac{1}{3}\pi\left(r_1^2+r_2^2+r_1r_2\right)h$

$=\frac{1}{3}\pi\left(8^2+20^2+8(20)\right)16$

$=\frac{1}{3}\pi(9984)$

Volume $=10449\cdot92cm^3$

Curved surface area of frustum cone

$=\pi(r_1+r_2)l+\pi r_2^2$

$=\pi(20+8)20+\pi(8)^2$

$=\pi(560+64)=624\pi cm^2$

Cost of metal sheet per $100cm^2=Rs20$

Cost of metal sheet for $624\pi cm^2=\frac{624\pi}{100}\times20$

$=Rs\ 391\cdot9$

$\therefore$ Total cost of bucket $=Rs\ 391\cdot9$

16.

Sol:

Given height of a frustum cone = 9cm

Lower end radius $(r_1)=\frac{60}{2}cm=30cm$

Upper end radius $(r_2)=\frac{36}{2}cm=18cm$

Let slant height of frustum cone be l

$l=\sqrt{(r_1-r_2)^2+h^2}$

$l=\sqrt{(8-30)^2+9^2}$

$l=\sqrt{144+81}$

$l=15cm$

Volume of frustum cone $=\frac{1}{3}\pi\left(r_1^2+r_2^2+r_1r_2\right)h$

$= \frac{1}{3}\pi\left(30^2 + 18^2 + 30(18)\right)9$

$= 5292\pi cm^3$

Volume $= 5292\pi cm^3$

Total surface area of frustum cone =

$= \pi(r_1 + r_2) \times l + \pi r_1^2 + \pi r_2^2$

$= \pi(30 + 18)15 + \pi(30)^2 + \pi(18)^2$

$= \pi\left(48(15) + (30)^2 + (18)^2\right)$

$= \pi(720 + 900 + 324)$

$= 1944\pi cm^2$

$\therefore$ Total surface area $= 1944\pi cm^2$

17.

Sol:

Given lower end radius of bucket $(r_1) = 8cm$

Upper end radius of bucket

Let height of bucket be 'h'

$V_1 = \frac{1}{3}\pi\left(8^2 + 20^2 + 8(20)\right)h\, cm^3$ ……….(1)

Volume of milk container $= 10459\frac{3}{4} cm^3$

$V_2 = \frac{73216}{7} cm^3$ ……….(2)

Equating (1) and (2)

$V_1 = V_2$

$\Rightarrow \frac{1}{3}\pi\left(8^2 + 20^2 + 8(20)\right)h = \frac{73216}{7}$

$\Rightarrow h = \frac{10459 \cdot 42}{653 \cdot 45}$

$\Rightarrow h = 16cm$

$\therefore$ Height of frustum cone $(h) = 16cm$

Let slant height of frustum cone be 'l'

$l = \sqrt{(r_1 - r_2)^2 + h^2}$

$= \sqrt{(20-8)^2 + 16^2} = \sqrt{144 + 256}$

$l = 20cm$

$\therefore$ Slant height of frustum cone $(l) = 20cm$

Total surface area of frustum cone

$= \pi(r_1 + r_2)l + \pi r_2^2 + \pi r_1^2$

$\Rightarrow \pi(20+8)20\pi(20)^2 + \pi(8)^2$

$= \pi(560 + 400 + 64)$

$= \pi(960 + 64) = 1024\pi = 3216 \cdot 99cm^2$

Total surface area $= 3216 \cdot 99cm^2$