RD Sharma Class 10 Solutions Chapter 16 Surface Areas and Volumes MCQS

Question 1.

Mark the correct alternative in each of the following:

The diameter of a sphere is 6 cm. It is melted and drawn into a wire of diameter 2mm. The length of the wire is

(a) 12 m

(b) 18 m

(c) 36 m

(d) 66 m

Solution:

Diameter of sphere = 6 cm

$\therefore$ Radius $(r) = \frac{6}{2} = 3$ cm

$$\text{Volume} = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi \times (3)^3 \text{ cm}^3$$

$$= \frac{4}{3}\pi \times 3 \times 3 \times 3 = 36\pi \text{ cm}^3$$

Diameter of wire = 2 mm

$\therefore$ Radius $(r_2) = \frac{2}{2} = 1$ mm $= \frac{1}{10}$ cm

Let h be its length, then

$\pi r_2^2 h = 36\pi$

$$\Rightarrow \pi \times \left(\frac{1}{10}\right)^2 h = 36\pi \Rightarrow \frac{\pi}{100} h = 36\pi$$

$$\Rightarrow h = \frac{36\pi \times 100}{\pi} = 3600 \text{ cm}$$

$\therefore$ Height or length of wire = 3600 cm $= \frac{3600}{100}$

= 36 m (c)

Question 2.

A metallic sphere of radius 10.5 cm is melted and then recast into small cones, each of radius 3.5 cm and height 3 cm. The number of such cones is

(a) 63

(b) 126

(c) 21

(d) 130

Solution:

Radius of sphere (R) = 10.5 cm

$$\text{Volume} = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi\,(10.5)^3 \text{ cm}$$

$$= \frac{4}{3}\pi\left(\frac{21}{2}\right)^3 = \frac{4}{3}\pi \times \frac{21\times21\times21}{2\times2\times2} \text{ cm}^3$$

$$= \frac{3087}{2}\pi \text{ cm}^3$$

Radius of small cone (r) = 3.5 cm = $\frac{7}{2}$ cm

and height (h) = 3 cm

$$\therefore \text{ Volume of cone} = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\left(\frac{7}{2}\right)^2 \times 3 \text{ cm}^3$$

$$= \frac{1}{3}\pi \times \frac{49}{4} \times 3 = \frac{49}{4}\pi \text{ cm}^3$$

$$\therefore \text{ Number of cones} = \frac{3087\pi}{2} \div \frac{49\pi}{4}$$

$$= \frac{3087\pi\times4}{2\times49\pi} = 126 \qquad \text{(b)}$$

Question 3.

A solid is hemispherical at the bottom and conical above. If the surface areas of the two parts are equal, then the ratio of its radius and the height of its conical part is

(a) 1 : 3

(b) 1 : $3-\sqrt{}$

(c) 1 : 1

(d) $3-\sqrt{}$ = 1

Solution:

Surface area of hemispherical part = surface area of conical part

$$\Rightarrow 2\pi r^2 = \pi r l \Rightarrow 2r = l$$

$$\Rightarrow 2r = \sqrt{r^2 + h^2} \Rightarrow 4r^2 = r^2 + h^2$$

$$\Rightarrow 3r^2 = h^2 \Rightarrow \frac{r^2}{h^2} = \frac{1}{3}$$

$$\Rightarrow \frac{r}{h} = \frac{1}{\sqrt{3}}$$

$\therefore$ Roots = $1 : \sqrt{3}$ (b)

Question 4.

A solid sphere of radius r is melted and cast into the shape of a solid cone of height r, the radius of the base of the cone is

(a) 2r
(b) 3r
(c) r
(d) 4r

Solution:

Radius of solid sphere = r

Volume = $\left(\frac{4}{3}\right)\pi r^3$

Now height of the cone so formed = r and

let radius = r_1

$$\therefore \frac{1}{3}\pi r_1^2 h = \frac{4}{3}\pi r^3$$

$$\Rightarrow \frac{1}{3}\pi r_1^2 \times r = \frac{4}{3}\pi r^3$$

$$\Rightarrow r_1^2 = \frac{4}{3}\pi r^3 \times \frac{3}{1 \times \pi \times r}$$

$$= 4r^2 = (2r)^2$$

$$\therefore r_1 = 2r$$

$\therefore$ Radius of cone = 2r (a)

Question 5.

The material of a cone is converted into the shape of a cylinder of equal radius. If height of the cylinder is 5 cm, then height of the cone is

(a) 10 cm
(b) 15 cm
(c) 18 cm
(d) 24 cm

Solution:
Let height of cone = h
and let r be its radius
∴ Volume = $\left(\frac{1}{3}\right)$ πr²h
Now radius of cylinder so formed = r
and height = 5 cm

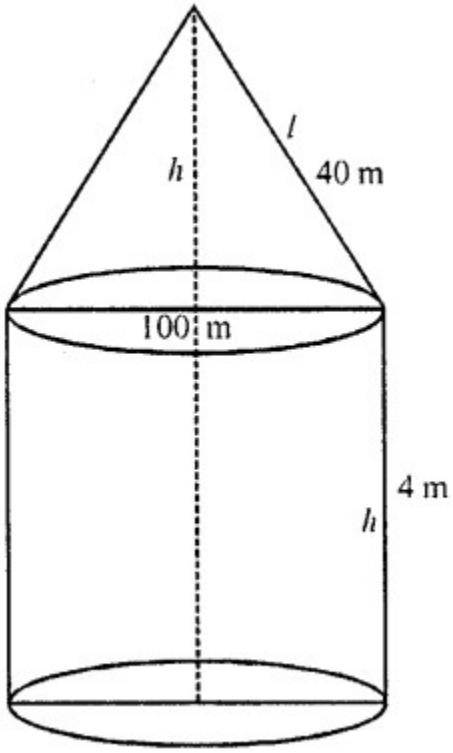
$$\therefore \text{ Volume} = \pi r^2 \times 5 = 5\pi r^2$$

$$\therefore \frac{1}{3}\pi r^2 h = 5\pi r^2$$

$$\Rightarrow \frac{1}{3}h = 5$$

$$h = 5 \times 3 = 15 \text{ cm} \quad \text{(b)}$$

Question 6.
A circus tent is cylindrical to a height of 4 m and conical above it. If its diameter is 105 m and its slant height is 40 m, the total area of the canvas required in m2 is
(a) 1760
(b) 2640
(c) 3960
(d) 7920
Solution:
Diameter of tent = 105 m
Height of the cylindrical part (h_1) = 4 m
Slant height of conical part (l) = 40 m
and radius (r) = $\left(\frac{105}{2}\right)$ m



surface area of the tent = curved surface area of conical part + curved surface area of cylindrical part =

$$= \pi rl + 2\pi rh$$

$$= \pi r\,(l + 2h) = \frac{22}{7} \times \frac{105}{2}\,(40 + 2 \times 4)\ \text{m}^2$$

$$= 165\,(40 + 8) = 165 \times 48\ \text{m}^2 = 7920\ \text{m}^2$$

(d)

Question 7.

The number of solid spheres, each of diameter 6 cm that could be moulded to form a solid metal cylinder of height 45 cm and diameter 4 cm, is

(a) 3
(b) 4
(c) 5
(d) 6

Solution:

Diameter of each sphere = 6 cm

$\therefore$ Radius $(r_1) = \frac{6}{2} = 3$ cm

Volume $= \frac{4}{3}\pi r_1^3 = \frac{4}{3}\pi \times (3)^3$ cm^3

$= 36\pi$ cm^3

Diameter of cylinder = 4 cm

$\therefore$ Radius $(r_2) = \frac{4}{2} = 2$ cm

and height $(h) = 45$ cm

$\therefore$ Volume $= \pi r^2 h = \pi \times (2)^2 \times 45$ cm^3

$= \pi \times 4 \times 45 = 180\pi$ cm^3

$\therefore$ Number of spheres required $= \frac{180\pi}{36\pi} = 5$ (c)

Question 8.

A sphere of radius 6 cm is dropped into a cylindrical vessel partly filled with water. The radius of the vessel is 8 cm. If the sphere is submerged completely, then the surface of the water rises by

(a) 4.5 cm
(b) 3 cm
(c) 4 cm
(d) 2 cm

Solution:

Radius of sphere (r) = 6 cm

Volume = (13) πr³ = (43) π (6)³ cm³

= (43) x216π = 4x 72π cm³ = 28871 cm³

Radius of vessel (r²) = 8 cm

Let height of water level = h

$\therefore$ Volume of water raised $= \pi r_2^2 h$

$\therefore$ $\pi r_2^2 h = 288\pi \Rightarrow (8)^2 h = 288$

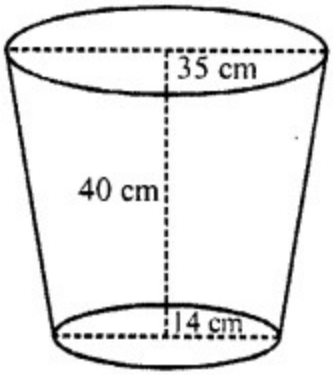
$\Rightarrow h = \frac{288}{8 \times 8} = \frac{36}{8} = \frac{9}{2}$ cm

$\therefore$ Height = 4.5 cm (a)

Question 9.

If the radii of the circular ends of a bucket of height 40 cm are of lengths 35 cm and

14 cm, then the volume of the bucket in cubic centimeters, is .
(a) 60060
(b) 80080
(c) 70040
(d) 80160
Solution:
Height of the bucket (h) = 40 cm
Upper radius ($r_{1)}$ = 35 cm
and lower radius (r_2) = 14 cm



$\therefore$ Volume of the bucket

$$= \frac{\pi}{3}[r_1^2 + r_1 r_2 + r_2^2] \times h$$

$$= \frac{22}{21}[(35)^2 + 35 \times 14 + (14)^2] \times 40 \text{ cm}^3$$

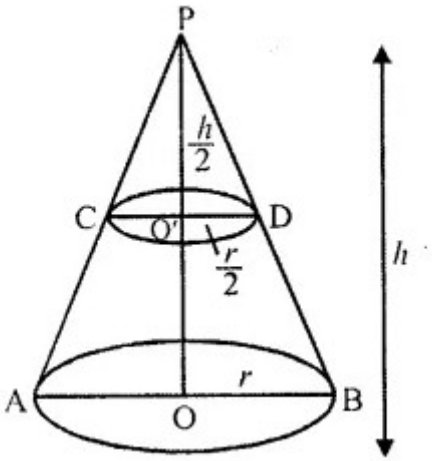
$$= \frac{22}{21}[1225 + 490 + 196] \times 40 \text{ cm}^3$$

$$= \frac{22}{21} \times 1911 \times 40 \text{ cm}^3$$

$$= \frac{22 \times 40 \times 1911}{21} \text{ cm}^3 = 80080 \text{ cm}^3 \qquad \text{(b)}$$

Question 10.
If a cone is cut into two parts by a horizontal plane passing through the mid¬point of its axis, the ratio of the volumes of the upper part and the cone is
(a) 1 : 2
(b) 1 : 4
(c) 1 : 6
(d) 1 : 8
Solution:
In the figure, C and D are the mid-points and CD || AB which divide the cone into two

parts



Height $OO' = \frac{1}{2}$ OP and diameter $CD = \frac{1}{2}$ AB

Let h be the height and r be the radius of the cone, then

$\frac{h}{2}$ will be the height of the smaller cone and

$\frac{r}{2}$ be its radius, then

Volume of bigger cone $= \frac{1}{3}\pi r^2 h$

and volume of smaller cone

$$= \frac{1}{3}\pi \left(\frac{r}{2}\right)^2 \left(\frac{h}{2}\right)$$

$$\therefore \frac{\frac{1}{3}\pi\left(\frac{r}{2}\right)^2\left(\frac{h}{2}\right)}{\frac{1}{3}\pi r^2 h} = \frac{\frac{1}{3}\pi\frac{r^2}{4}\times\frac{h}{2}}{\frac{1}{3}\pi r^2 h}$$

$$= \frac{\frac{1}{8}\left(\frac{1}{3}\pi r^2 h\right)}{\frac{1}{3}\pi r^2 h} = \frac{1}{8}$$

$\therefore$ Ratio = 1 : 8 (d)

Question 11.

The height of a cone is 30 cm. A small cone is cut off at the top by a plane parallel to the base. If its volume be (127) of the volume of the given cone, then the height above the base at which the section has been made, is

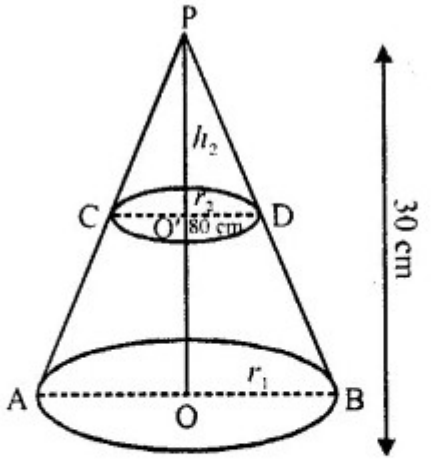
(a) 10 cm
(b) 15 cm
(c) 20 cm
(d) 25 cm

Solution:

Height of given cone (h_1) = 30 cm

Let r_1 be its radius

Then volume of the larger cone = $(13)\ \pi r_1^2 h_1$



A cone is cut off from the top of the larger cone, such that volume of smaller cone

$= \frac{1}{27}$ of that of larger cone

$$\therefore \frac{\text{Volume of smaller cone}}{\text{Volume of bigger cone}} = \frac{1}{27}$$

$$= \frac{\frac{1}{3}\pi r_2^2 h_2}{\frac{1}{3}\pi r_1^2 h_1} = \frac{1}{27} \Rightarrow \frac{r_2^2 h_2}{r_1^2 h_1} = \frac{1}{27} = \left(\frac{1}{3}\right)^3$$

$$\Rightarrow \frac{h_2}{h_1} = \frac{1}{3} \quad \frac{h_2}{30} = \frac{1}{3} \Rightarrow h_2 = \frac{30}{3} = 10$$

Height of smaller cone = 10 cm

Height from the base of bigger cone will be

= 30 – 10 = 20 cm (c)

Question 12.

A solid consists of a circular cylinder with an exact fitting right circular cone placed at the top. The height of the cone is h. If the total volume of the solid is 3 times the volume of the cone, then the height of the circular cylinder is

(a) $2h$ (b) $\frac{2h}{3}$

(c) $\frac{3h}{2}$ (d) $4h$

Solution:

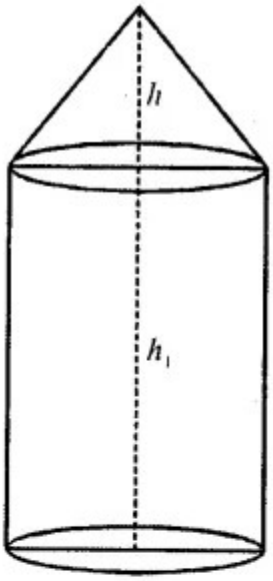
Height of cone = h

Volume of solid = 3 x volume of cone

Let h be the height of the cylinder and r be its radius, then

Volume of cylinder = $\pi r^2 h_1$

and volume of cone = $(13)\ \pi r^2 h_1$



Then volume of solid $= \pi r^2 h_1 + \frac{1}{3}\pi r^2 h$

$= \pi r^2 \left(h_1 + \frac{1}{3}h\right)$

Now $\pi r^2 \left(h_1 + \frac{1}{3}h\right) = 3 \times \frac{1}{3}\pi r^2 h = \pi r^2 h$

$\Rightarrow h_1 + \frac{1}{3}h = h$ (comparing)

$h_1 = h - \frac{1}{3}h = \frac{2}{3}h$

Hence height of cylinder $= \frac{2h}{3}$ (b)

Question 13.

A reservoir is in the shape of a frustum of a right circular cone. It is 8 m across at the top and 4 m across at the bottom. If it is 6 m deep, then its capacity is

(a) 176 m^3

(b) 196 m^3

(c) 200 m^3

(d) 110 m^3

Solution:

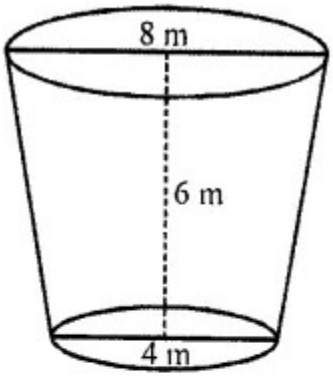
A reservoir is a frustum in shape which Upper diameter = 8 m

and lower diameter = 4 m

$\therefore$ Upper radius = $\left(\frac{8}{2}\right)$ = 4 m

and lower radius = $\left(\frac{4}{2}\right)$ = 2 m

Height (h) = 6m



$$\therefore \text{ Volume} = \frac{\pi}{3}\left[r_1^2 + r_1 r_2 + r_2^2\right] \times h$$

$$= \frac{\pi}{3}\left[(4)^2 + 4 \times 2 + (2)^2\right] \times 6 \text{ m}^3$$

$$= \frac{22}{7 \times 3}\left[16 + 8 + 4\right] \times 6 \text{ m}^3$$

$$= \frac{22}{21}(28 \times 6) \text{ m}^3 = 176 \text{ m}^3$$ (a)

Question 14.

Water flows at the rate of 10 metre per minute from a cylindrical pipe 5 mm in diameter. How long will it take to fill up a conical vessel whose diameter at the base is 40 cm and depth 24 cm ?

(a) 48 minutes 15 sec

(b) 51 minutes 12 sec

(c) 52 minutes 1 sec

(d) 55 minutes

Solution:

Flow of water = 10 m per minute

Diameter of pipe = 5 mm

$\therefore$ Radius $(r_1) = \frac{5}{2}$ mm $= \frac{5}{10 \times 2} = \frac{1}{4}$ cm

Diameter of conical vessel = 40 cm

$\therefore$ Radius $(r_2) = \frac{40}{2} = 20$ cm

and depth $(h_2) = 24$ cm

$\therefore$ Volume of water filled in it $= \frac{1}{3}\pi r_2^2 h_2$

$= \frac{1}{3} \times \pi \times (20)^2 \times 24 \text{ cm}^3$

$= \frac{1}{3}\pi \times 400 \times 24 = 3200\pi \text{ cm}^3$

$\therefore$ Volume of water in the pipe $= 3200\pi$ cm^2

Let h_1 be its length, then

$$\pi r_1^2 h_1 = 3200\pi \Rightarrow \pi\left(\frac{1}{4}\right)^2 h_1 = 3200\pi$$

$$\Rightarrow \frac{1}{16}\pi h_1 = 3200\pi \Rightarrow h_1 = \frac{3200\pi \times 16}{\pi}$$

$$\Rightarrow h_1 = 51200 \text{ cm} = \frac{51200}{100} = 512 \text{ m}$$

$\therefore$ Time taken $= \frac{512}{10}$ minutes $= 51\frac{1}{5}$ minutes

= 51 minutes 12 seconds (b)

Question 15.

A cylindrical vessel 32 cm high and 18 cm as the radius of the base, is filled with sand. This bucket is emptied on the ground and a conical heap of sand is formed. If the height of the conical heap is 24 cm, the radius of its base is

(a) 12 cm
(b) 24 cm
(c) 36 cm
(d) 48 cm

Solution:

Radius of a cylindrical vessel (r_1) = 18 cm
and height (h_1) = 32 cm
∴ Volume of sand filled in it = $\pi r_1^2 h_1$

$= \pi (18)^2 \times 32 = \pi \times 324 \times 32 \text{ cm}^3$

$= 10368\pi \text{ cm}^3$

Now height of the conical heap (h_2) = 24 cm

Let r_2 be its radius, then

$$\frac{1}{3}\pi r_2^2 h_2 = 10368\pi$$

$$\Rightarrow \frac{1}{3}\pi r_2^2 \times 24 = 10368\pi$$

$$\Rightarrow 8\pi r_2^2 = 10368\pi$$

$$r_2^2 = \frac{10368\pi}{8\pi} = 1296$$

$$\therefore r_2 = \sqrt{1296} = 36$$

Hence radius of the base of the heap = 36 cm (c)

Question 16.

The curved surface area of a right circular cone of height 15 cm and base diameter 16 cm is
(a) 607t cm²
(b) 6871 cm²
(c) 12071 cm²
(d) 136TI cmᶜ

Solution:

Diameter of base of a right circular cone = 16 cm
Radius (r) = $\left(\frac{16}{2}\right)$ = 8 cm

and height (h) = 15 cm

$\therefore$ Slant height $(l) = \sqrt{r^2 + h^2}$

$= \sqrt{(8)^2 + (15)^2}$

$= \sqrt{64 + 225} = \sqrt{289} = 17$ cm

$\therefore$ Curved surface area $= \pi rl$

$= \pi \times 8 \times 17 = 136\pi$ cm^2 (d)

Question 17.

A right triangle with sides 3 cm, 4 cm and 5 cm is rotated about the side of 3 cm to form a cone. The volume of the cone so formed is

(a) 12π cm^3

(b) 15π cm^3

(c) 16π cm^3

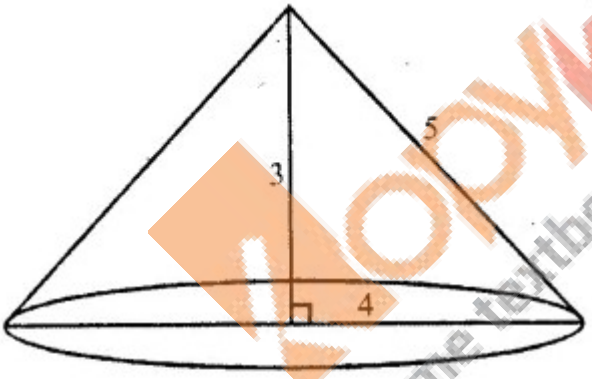
(d) 20π cm^3

Solution:

A cone is formed be rotating the right angled triangle above the side 3 cm

Height of cone (h) = 3 cm

and radius (r) = 4 cm



$\therefore$ Volume $= \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi \times (4)^2 \times 3$ cm^3

$= \frac{1}{3}\pi \times 16 \times 3 = 16\pi$ cm^3 (c)

Question 18.

The curved surface area of a cylinder is 264 m^2 and its volume is 924 m^3 The ratio of its diameter to its height is

(a) 3 : 7

(b) 7 : 3

(c) 6 : 7

(d) 7 : 6

Solution:

Curved surface area of a cylinder = 264 m^2

and its volume = 924 m^3

Let r be its radius and h be its height, then $2\pi rh = 264$

$$\Rightarrow 2 \times \frac{22}{7} rh = 264 \Rightarrow rh = \frac{264 \times 7}{2 \times 22} = 42 \quad \ldots(i)$$

and $\pi r^2 h = 924$

$$\Rightarrow \frac{22}{7} r^2 h = 924$$

$$\Rightarrow r^2 h = \frac{924 \times 7}{22} = 42 \times 7 = 294 \quad \ldots(ii)$$

Dividing (ii) by (i)

$$\frac{r^2 h}{rh} = \frac{294}{42} \Rightarrow r = 7$$

$$\therefore h = \frac{42}{r} = \frac{42}{7} = 6$$

Now ratio between diameter and height

$= 2 \times 7 : 6 = 14 : 6 \Rightarrow 7 : 3$ (b)

Question 19.

A cylinder with base radius of 8 cm and height of 2 cm is melted to form a cone of height 6 cm. The radius of the cone is

(a) 4 cm

(b) 5 cm

(c) 6 cm

(d) 8 cm

Solution:

Radius of cylinder $(r_1) = 8$ cm

and height $(h_1) = 2$ cm

$\therefore$ Volume $= \pi r_1^2 h_1 = \pi \times (8)^2 \times 2$ cm^3

$= 128\pi$ cm^3

Now volume of cone $= 128\pi$ cm^3

Height of cone $(h_2) = 6$ cm

Let r_2 be its radius, then

$$\frac{1}{3}\pi r_2^2 h_2 = 128\pi \Rightarrow \frac{1}{3}\pi r_2^2 \times 6 = 128\pi$$

$$\Rightarrow r_2^2 = \frac{128\pi \times 3}{\pi \times 6} = 64 = (8)^2$$

$$\Rightarrow r_1 = 8$$

$\therefore$ Radius of cone $= 8$ cm (d)

Question 20.

The volumes of two spheres are in the ratio 64 : 27. The ratio of their surface areas is

(a) 1 : 2

(b) 2 : 3

(c) 9 : 16

(d) 16 : 9

Solution:

Ratio in volumes of two spheres = 64 : 27

$= (4)^3 : (3)^3$

$\because$ Volume is in cubic units

$\therefore$ Length will be units while areas are in square units

$\therefore$ Areas will be in the ratio $= (4)^2 : (3)^2 = 16:9$ **(d)**

Question 21.

If three metallic spheres of radii 6 cm, 8 cm and 10 cm are melted to form a single sphere, the diameter of the sphere is

(a) 12 cm

(b) 24 cm

(c) 30 cm

(d) 36 cm

Solution:

Let radii of 3 metallic spheres are

$r_1 = 6$ cm

$r_2 = 8$ cm

$r_3 = 10$ cm

Volume of first sphere $= \frac{4}{3}\pi r_1^3 = \frac{4}{3}\pi\,(6)^3$

$= \frac{4}{3} \times 216\pi \text{ cm}^3$

$= \frac{864}{3}\pi \text{ cm}^3$

Volume of second sphere

$= \frac{4}{3}\pi r_2^3 = \frac{4}{3}\pi \times (8)^3 \text{ cm}^3$

$= \frac{4}{3} \times 512\pi = \frac{2048}{3}\pi \text{ cm}^3$

and volume of third sphere

$= \frac{4}{3}\pi r_3^3 = \frac{4}{3}\pi\,(10)^3 \text{ cm}^3$

$= \frac{4}{3}\pi \times 1000 = \frac{4000}{3}\pi \text{ cm}^3$

$\therefore$ Sum of volumes of the 3 spheres

$= \frac{864}{3}\pi + \frac{2048}{3}\pi + \frac{4000}{3}\pi$

$= \frac{864 + 2048 + 4000}{3}\pi = \frac{6912}{3}\pi \text{ cm}^3$

$\therefore$ Volume of new sphere $= \frac{6912}{3}\pi$ cm^3

Let R be its radius, then

$$\frac{4}{3}\pi R^3 = \frac{6912\pi}{3}$$

$$R^3 = \frac{6912\pi}{3} \times \frac{3}{4\pi} = 1728 = (12)^3$$

$\therefore$ R = 12 cm

$\therefore$ Diameter of the new sphere = 2R = 2 × 12

= 24 cm (b)

Question 22.

The surface area of a sphere is same as the curved surface area of a right circular cylinder whose height and diameter are 12 cm each. The radius of the sphere is

(a) 3 cm

(b) 4 cm

(c) 6 cm

(d) 12 cm

Solution:

Diameter of cylinder = 12 cm

$\therefore$ Radius (r_1) = $\left(\frac{12}{2}\right)$ = 6 cm

and height (h) = 12 cm

$\therefore$ Surface area = $2\pi rh = 2\pi \times 6 \times 12$ cm^2

$= 144\pi$ cm^2

Now surface area of sphere = 1447c cm^2

Let r_2 be its radius, then

Question 23.

The volume of the greatest sphere that can be cut off from a cylindrical log of wood of base radius 1 cm and height 5 cm is

(a) $\frac{4}{3}\pi$ **(b)** $\frac{10}{3}\pi$

(c) 5π **(d)** $\frac{20}{3}\pi$

Solution:

Radius of cylindrical log (r) = 1 cm

and height (h) = 5 cm

The radius of the greatest sphere cut off from the cylindrical log will be = radius of

the log = 1 cm

$$\therefore \text{ Volume } = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi\,(1)^3 = \frac{4}{3}\pi \text{ cm}^3 \quad \text{(a)}$$

Question 24.

A cylindrical vessel of radius 4 cm contains water. A solid sphere of radius 3 cm is lowered into the water until it is completely immersed. The water level in the vessel will rise by

(a) $\frac{2}{9}$ **cm** **(b)** $\frac{4}{9}$ **cm**

(c) $\frac{9}{4}$ **cm** **(d)** $\frac{9}{2}$ **cm**

Solution:

Radius of sphere $(r_1) = 3$ cm

$\therefore$ Volume $= \frac{4}{3}\pi r^3 = \frac{4}{3}\pi\,(3)^3$ cm^3

$= 36\pi$ cm^3

$\therefore$ Volume of water in the cylinder $= 36\pi$ cm^3

Radius of cylindrical vessel $(r_2) = 4$ cm

Let h be its height, then

$\pi r_2^2 h = 36\pi \Rightarrow \pi\,(4)^2\, h = 36\pi$

$\Rightarrow 16\pi h = 36\pi \Rightarrow h = \frac{36\pi}{16\pi} = \frac{9}{4}$ cm (c)

Question 25.

12 spheres of the same size are made from melting a solid cylinder of 16 cm diameter and 2 cm height. The diameter of each sphere is

(a) $3-\sqrt{}$ **cm**

(b) 2 cm

(c) 3 cm

(d) 4 cm

Solution:

Diameter of solid cylinder = 16 cm

$\therefore$ Radius $(r_1) = \frac{16}{2} = 8$ cm

Height $(h_1) = 2$ cm

$\therefore$ Volume $= \pi r^2 h = \pi \times 8 \times 8 \times 2$ cm^3

$= 128\pi$ cm^3

Now volume 12 spheres $= 128\pi$ cm^3

$\therefore$ Volume of 1 sphere $= \frac{128\pi}{12} = \frac{32}{3}\pi$ cm^3

Let r_2 be its radius, then

$$\frac{4}{3}\pi r_2^3 = \frac{32}{3}\pi \Rightarrow r_2^3 = \frac{32\pi}{3} \times \frac{3}{4\pi}$$

$\Rightarrow r^3 = 8 = (2)^3 \Rightarrow r = 2$

$\therefore$ Radius of each sphere $= 2$ cm

$\therefore$ Diameter $= 2r_2 = 2 \times 2 = 4$ cm (d)

Question 26.

A solid metallic spherical ball of diameter 6 cm is melted and recast into a cone with diameter of the base as 12 cm. The height of the cone is

(a) 2 cm
(b) 3 cm
(c) 4 cm
(d) 6 cm

Solution:

Diameter of a metallic sphere = 6 cm

$\therefore$ Radius = (62) = 3 cm

$\therefore$ Volume = (43) πr_{1^3} = (43) π $(3)^3$ cm^3 = 36π cm^3

∴ Volume of cone = 36π cm³

Diameter of cone = 12 cm

$\therefore$ Radius $(r_2) = \frac{12}{2} = 6$ cm

Let h_1 be its height, then

$$\frac{1}{3}\pi r_2^2 h_1 = 36\pi$$

$$\frac{1}{3}\pi (6)^2 h_1 = 36\pi \Rightarrow 12\pi h_1 = 36\pi$$

$$\Rightarrow h_1 = \frac{36\pi}{12\pi} = 3$$

$\therefore$ Height = 3 cm (b)

Question 27.

A hollow sphere of internal and external diameters 4 cm and 8 cm respectively is melted into a cone of base diameter 8 cm. The height of the cone is

(a) 12 cm

(b) 14 cm

(c) 15 cm

(d) 18 cm

Solution:

Internal diameter of a hollow sphere = 4 cm

and external diameter = 8 cm

∴ Internal radius (r) = $\left(\frac{4}{2}\right)$ = 2 cm

and external radius (R) = $\left(\frac{8}{2}\right)$ = 4 cm

∴ Volume of metal used = $(43)\ \pi (R^3 - r^3)$

$$= \frac{4}{3}\pi (4^3 - 2^3) = \frac{4}{3}\pi (64 - 8)\ cm^3$$

$$= 56 \times \frac{4}{3}\pi = \frac{224}{3}\pi\ cm^3$$

$$\therefore \text{ Volume of cone } = \frac{224}{3}\pi\ cm^3$$

Diameter of cone = 8 cm

$$\therefore \text{ Radius } (r_1) = \frac{8}{2} = 4\ cm$$

$$\therefore \frac{1}{3}\pi r_1^2 h_1 = \frac{224}{3}\pi$$

$$\Rightarrow \frac{1}{3}\pi (4)^2 h_1 = \frac{224}{3}\pi \Rightarrow \frac{16}{3}\pi h_1 = \frac{224}{3}\pi$$

$$\Rightarrow h_1 = \frac{224\pi}{3} \times \frac{3}{16\pi} = 14$$

∴ Height of cone = 14 cm (b)

Question 28.

A solid piece of iron of dimensions 49 x 33 x 24 cm is moulded into a sphere. The radius of the sphere is (a) 21 cm

(b) 28 cm

(c) 35 cm

(d) None of these

Solution:

Dimension of a solid piece = 49 x 33 x 24 cm

Volume = 49 x 33 x 24 cm^3 = 38808 cm^3

Volume of a sphere = 38808 cm^3

Let r be its radius, their

$$\frac{4}{3}\pi r^3 = 38808$$

$$\Rightarrow \frac{4}{3} \times \frac{22}{7} r^3 = 38808 \Rightarrow r^3 = \frac{38808 \times 3 \times 7}{4 \times 22}$$

$$\Rightarrow r^3 = 9261 = (21)^3$$

$$\therefore r = 21$$

$$\therefore \text{Radius of sphere} = 21 \text{ cm} \qquad \text{(a)}$$

Question 29.

The ratio of lateral surface area to the total surface area of a cylinder with base diameter 1.6 m and height 20 cm is

(a) 1 : 7

(b) 1 : 5

(c) 7 : 1

(d) 5 : 1

Solution:

Ratio in lateral surface area and total surface area

Base diameter = 1.6 m = 160 cm

Height (h) = 20 cm

∴ Radius = 80 cm

Now, lateral surface = 2 πrh = 2 π x 80 x 20 = 3200 π

and 2 πrh x 2 πr2 = 3200 π + 2 π $(80)^2$

= 3200 π + 2 π x 6400

= (3200 + 12800) π = 16000 π

Ratio = 3200 π : 6000 π = 1.5 **(b)**

Question 30.

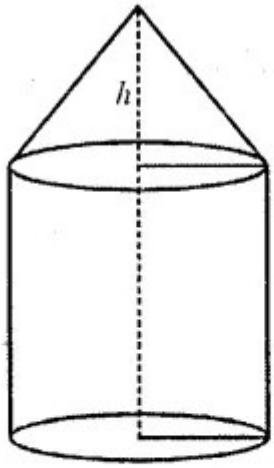
A solid consists of a circular cylinder surmounted by a right circular cone. The height of the cone is h. If the total height of the solid is 3 times the volume of the cone, then the height of the cylinder is

(a) $2h$ **(b)** $\frac{3h}{2}$

(c) $\frac{h}{2}$ **(d)** $\frac{2h}{3}$

Solution:

Let r be the radius of the solid = height of the conical part = h



Volume of cone $= \frac{1}{3}\pi r^2 h$

$\therefore$ Total volume of the solid = 3 × volume of cone

$= 3 \times \frac{1}{3}\pi r^2 h$

$= \pi r^2 h$

and volume of cylinder $= \pi r^2 h = \frac{1}{3}\pi r^2 h$

$= \frac{2}{3}\pi r^2 h$

$\therefore$ Height $= \frac{\text{Volume}}{\pi r^2} = \frac{2\pi r^2 h}{3\pi r^2} = \frac{2}{3}h$ (d)

Question 31.

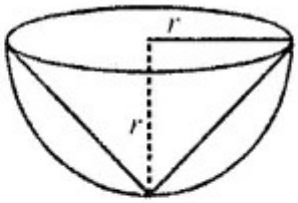
The maximum volume of a cone that can be carved out of a solid hemisphere of radius r is

(a) $3\pi r^2$ **(b)** $\frac{\pi r^3}{3}$

(c) $\frac{\pi r^2}{3}$ **(d)** $3\pi r^3$

Solution:

Radius of cone = r
and height = r



$$\therefore \text{ Volume} = \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3}\pi r^2 \times r = \frac{1}{3}\pi r^3 \qquad \text{(b)}$$

Question 32.

The radii of two cylinders are in the ratio 3 : 5. If their heights are in the ratio 2 : 3, then the ratio of their curved surface areas is

(a) 2 : 5
(b) S : 2
(c) 2 : 3
(d) 3 : 5

Solution:

Ratio in radii of two cylinders = 3 : 5
and in their heights = 2 : 3
Let $r_1 = 3x$, $r_2 = 5x$
$h_1 = 2y$, $h_2 = 3y$
∴ Curved surface area of first cylinder = $2\pi r_1 h_1$
$= 2\pi \times 3x \times 2y = 12\pi xy$
and curved surface area of second cylinder
$= 2\pi r_2 h_2 = 2\pi \times 5x \times 3y = 30\pi xy$
∴ Ratio = $12\pi xy : 30\pi xy = 2 : 5$ **(a)**

Question 33.

A right circular cylinder of radius r and height It (h = 2r) just enclose a spehre of diameter

(a) h
(b) r
(c) 2r
(d) 2h

Solution:

Radius of right cylinder = r
Height = h or 2r(∵ h = 2r)
Diameter of sphere encloses by the cylinder = 2r **(c)**

Question 34.

The radii of the circular ends of a frustum are 6 cm and 14 cm. If its slant height is 10 cm, then its vertical height is

(a) 6 cm

(b) 8 cm

(c) 4 cm

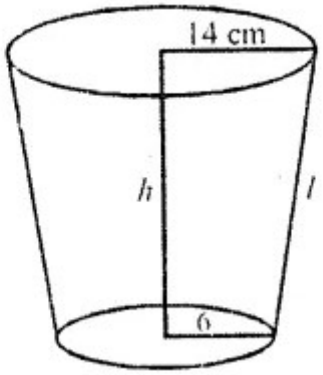
(d) 7 cm

Solution:

Radii of circular ends of frustum an 6 cm and then

$\therefore r_1 = 14, r_2 = 6$

and slant height (l) = 10 cm



$\therefore$ Vertical height (h)

$$= \sqrt{l^2 - (r_1 - r_2)^2} = \sqrt{(10)^2 - (14-6)^2}$$

$$= \sqrt{100 - (8)^2} = \sqrt{100 - 64}$$

$$= \sqrt{36} = \sqrt{(36)^2} = 6 \text{ cm} \qquad \text{(a)}$$

Question 35.

The height and radius of the cone of which the frustum is a part are h_1, and r_1 respectively. If h_2 and r_2 are the heights and radius of the smaller base of the frustum respectively and $h_2 : h_1 = 1 : 2$, then $r_2 : r_1$ is equal to

(a) 1 : 3

(b) 1 : 2

(c) 2 : 1

(d) 3 : 1

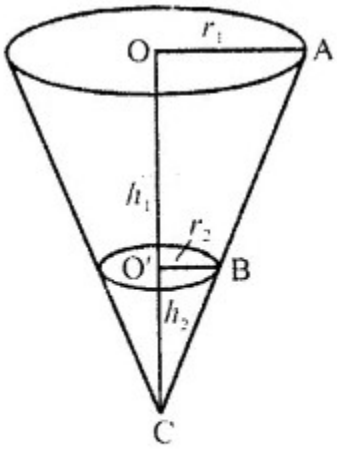
Solution:

Height of cone = h_1

and radius = r_1

Height of frustum = h_2

and radius = r_2



In ΔOAC and $\Delta O'BC$

$\angle OCA = \angle O'CB$ (common)

$\angle COA = \angle CO'B$ (each 90°)

$\therefore \Delta OAC \sim \angle O'BC$

$$\therefore \frac{O'C}{OC} = \frac{O'B}{OA}$$

$$\Rightarrow \frac{h_2}{h_1} = \frac{r_2}{r_1} = \frac{1}{2} \quad (\because h_2 : h_2 = 1 : 2)$$

$$\therefore \frac{r_2}{r_1} = \frac{1}{2}$$

$\therefore r_2 : r_1 = 1 : 2$ (b)

Question 36.

The diameters of the ends of a frustum of a cone are 32 cm and 20 cm. If its slant height is 10 cm, then its lateral surface area is

(a) 321π cm²

(b) 300π1 cm²

(c) 260π cm²

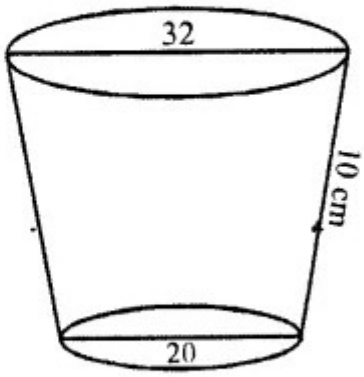
(d) 250π cm²

Solution:

In the frustum

$$r_1 = \frac{32}{2} = 16 \text{ cm}, r_2 = \frac{20}{2} \text{ cm} = 10 \text{ cm}$$

and $l = 10$ cm



Lateral surface area $= \pi (r_1 + r_2) l$

$= \pi (16 + 10) \times 10 \text{ cm}^2$

$= 26\pi \times 10 \text{ cm}^2 = 260 \text{ cm}^2$ (c)

Question 37.

A solid frustum is of height 8 cm. If the radii of its lower and upper ends are 3 cm and 9 cm respectively, then its slant height is

(a) 15 cm

(b) 12 cm

(c) 10 cm

(d) 17 cm

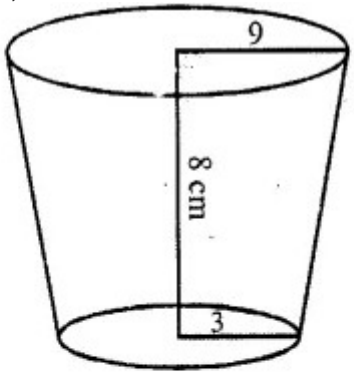
Solution:

In the frustum,

Upper radius (r_1) = 9 cm

and lower radius (r_2) = 3 cm

and height (h) = 8 cm



$\therefore$ Slant height $(l) = \sqrt{h^2 + (r_1 - r_2)^2}$

$= \sqrt{(8)^2 + (9-3)^2} = \sqrt{64+36}$

$= \sqrt{100} = 10$ cm (c)

Question 38.

The radii of the ends of a bucket 16 cm high are 20 cm and 8 cm. The curved surface area of bucket is

(a) 1760 cm²

(b) 2240 cm²

(c) 880 cm²

(d) 3120 cm²

Solution:

Height of bucket (h) = 16 cm

Upper radius (r_1) = 20 cm

and lower radius (r_2) = 8 cm

$\therefore$ Slant height $= \sqrt{h^2 + (r_1 - r_2)^2}$

$= \sqrt{(16)^2 + (20-8)^2} = \sqrt{256+144}$

$= \sqrt{400} = 20$ cm

$\therefore$ Curved surface area $= \pi (r_1 + r_2)\, l$

$= \frac{22}{7} (20 + 8) \times 20$ cm²

$= \frac{22}{7} \times 28 \times 20 = 1760$ cm² (a)

Question 39.

The diameters of the top and the bottom portions of a bucket are 42 cm and 28 cm respectively. If the height of the bucket is 24 cm, then the cost of painting its outer surface at the rate of 50 paise/ cm² is

(a) Rs. 1582.50

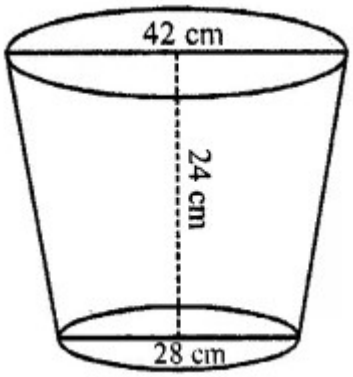
(b) Rs. 1724.50

(c) Rs. 1683

(d) Rs. 1642

Solution:

Diameter of upper and lower portions of a bucket are 42 cm and 28 cm
and height (h) = 24 cm



$\therefore\ r_1 = \dfrac{42}{2} = 21$ cm

$r_2 = \dfrac{28}{2} = 14$ cm

$h = 24$ cm

$\therefore$ Slant height (l)

$= \sqrt{h^2 + (r_1 - r_2)^2}$

$= \sqrt{(24)^2 + (21-14)^2}$

$= \sqrt{(24)^2 + (7)^2} = \sqrt{576+49}$

$= \sqrt{625} = 25$ cm

Now area of outer surface

$= \pi (r_1 + r_2)\, l + \pi r_2^2$

$= \dfrac{22}{7} (21 + 14) \times 25 + \dfrac{22}{7} \times (14)^2$ cm²

$= \dfrac{22}{7} \times 35 \times 25 + \dfrac{22}{7} \times 14 \times 14$ cm²

$= 2750 + 616 = 3366$ cm²

Rate of polishing = 50 p. per cm²

$\therefore$ Total cost = $3366 \times \dfrac{50}{100}$ = Rs. 1683 (c)

Question 40.

If four times the sum of the areas of two circular faces of a cylinder of height 8 cm is equal to twice the curve surface area, then diameter of the cylinder is

(a) 4 cm

(b) 8 cm

(c) 2 cm

(d) 6 cm

Solution:

Let r be the radius of the cylinder

Height of = 8 cm

Sum of areas of two circular faces = $2\pi r^2$

Curved surface area = $2\pi rh = 2\pi r \times 8$

$$\therefore \quad 4 \times 2\pi r^2 = 2 \times 2\pi r \times 8$$

$$8\pi r^2 = 32\pi r$$

$$r = \frac{32}{8} = 4 \text{ cm}$$

$$\therefore \quad \text{Diameter} = 2r = 2 \times 4 \text{ cm} = 8 \text{ cm} \qquad \text{(b)}$$

Question 41.

If the radius of the base of a right circular cylinder is halved, keeping the height the same, then the ratio of the volume of the cylinder thus obtained to the volume of orginal cylinder is

(a) 1 : 2

(b) 2 : 1

(c) 1 : 4

(d) 4 : 1 (CBSE 2012)

Solution:

Let h be height in each case of the cylinder

Let r be radius in first case, then

volume $= \pi r^2 h$

and if radius is halved *i.e.* radius $= \frac{r}{2}$, then

$$\text{volume} = \pi \frac{r^2}{4} h$$

$$\therefore \text{ Ratio } \pi r^2 h : \frac{\pi r^2 h}{4}$$

$$= 1 : \frac{1}{4} = 4 : 1$$ (d)

Question 42.

A metalic solid cone is melted to form a solid cylinder of equal radius. If the height of the cylinder is 6 cm, then the height of the cone was

(a) 10 cm

(b) 12 cm

(c) 18 cm

(d) 24 cm [CBSE 2014]

Solution:

Let r be the radius in each case = r

Height of cylinder = 6 cm

Volume of cylinder = Volume of cone

$$\pi r^2 h_1 = \frac{1}{3} \pi r^2 h_2$$

$$h_1 = \frac{1}{3} h_2$$

$$\therefore h_2 = h_1 \times 3 = 6 \times 3 = 18 \text{ cm}$$

$$\therefore \text{ Height of cone} = 18 \text{ cm}$$ (c)

Question 43.

A rectangular sheet of paper 40 cm x 22 cm, is rolled to form a hollow cylinder of height 40 cm. The radius of the cylinder (in cm) is

(a) 3.5

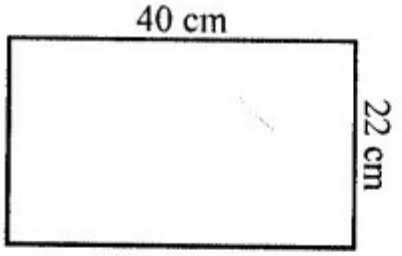
(b) 7

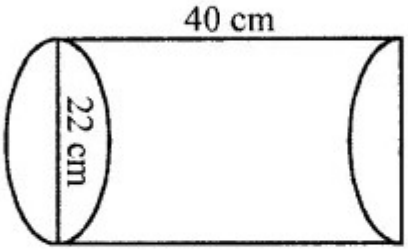
(c) 80/7

(d) 5

Solution:

Length of rectangular sheet(l) = 40 cm
and width (b) = 22 cm





By rolling it a cylinder is formed

$\therefore$ Circumference of cylinder = b = 22 cm

Let r be the radius

$$\therefore 22 = 2 \times \frac{22}{7} r \Rightarrow r = \frac{22 \times 7}{2 \times 22} = \frac{7}{2}$$

$$\therefore \text{Radius} = \frac{7}{2} = 3.5 \text{ cm}$$ (a)

Question 44.

The number of solid spheres, each of diameter 6 cm that can be made by melting a solid metal cylinder of height 45 cm and diameter 4 cm is

(a) 3

(b) 5

(c) 4

(d) 6 [CBSE 2014]

Solution:

Diameter of solid sphere = 6 cm

∴ Radius = (62) = 3 cm

$\therefore$ Volume = $\frac{4}{3}\pi r^3 = \frac{4}{3}\pi \times 3 \times 3 \times 3\ cm^3$

$= 36\pi\ cm^3$

Now volume of one sphere = $36\pi\ cm^3$

Height (h) = 45 cm

Radius of cylinder (R) = $\frac{4}{2}$ = 2 cm

$\therefore$ Volume of cylinder = $\pi r^2 h$

$= \pi \times 2 \times 2 \times 45\ cm^3 = 180\pi\ cm^3$

$\therefore$ Number of spheres = $\frac{180\pi}{36\pi} = 5$ (b)

Question 45.

Volumes of two spheres are in the ratio 64 : 27. The ratio of their surface areas is

(a) 3 : 4

(b) 4 : 3

(c) 9 : 16

(d) 16 : 9

Solution:

Let the radii of the two spheres are r_1 and r_2, respectively.

$\therefore$ Volume of the sphere of radius, $r_1 = V_1$

$$= \frac{4}{3}\pi r_1^3 \qquad ...(i)$$

[$\because$ volume of sphere = $\frac{4}{3}\pi$ (radius)^3]

and volume of the sphere of radius,

$$r_2 = V_2 = \frac{4}{3}\pi r_2^3 \qquad ...(ii)$$

Given, ratio of volume = $V_1 : V_2$

$$= 64 : 27 \Rightarrow \frac{\frac{4}{3}\pi_1^3}{\frac{4}{3}\pi_2^3} = \frac{64}{27}$$

[using Eqs. (*i*) and (*ii*)]

$$\Rightarrow \frac{r_1^3}{r_2^3} = \frac{64}{27} \Rightarrow \frac{r_1}{r_2} = \frac{4}{3} \qquad ...(iii)$$

Now, ratio of surface area = $\frac{4\pi r_1^2}{4\pi r_2^2}$

[$\because$ surface area of a sphere = 4π (radius)^2]

$$= \frac{r_1^2}{r_2^2}$$

$$= \left(\frac{r_1}{r_2}\right)^2 = \left(\frac{4}{3}\right)^2 \qquad \text{[using Eq. (iii)]}$$

$= 16 : 9$

Hence, the required ratio of their surface area is 16 : 9. (d)

Question 46.
A right circular cylinder of radius r and height h (h > 2r) just encloses a sphere of diameter
(a) r
(b) 2r
(c) h
(d) 2h
Solution:
Because the sphere enclose in the cylinder, therefore the diameter of sphere is equal to diameter of cylinder which is 2r. **(b)**

Question 47.
In a right circular cone, the cross-section made by a plane parallel to the base is a
(a) circle
(b) frustum of a cone
(c) sphere
(d) hemisphere
Solution:
We know that, if a cone is cut by a plane parallel to the base of the cone, then the portion between the plane and base is called the frustum of the cone. **(b)**

Question 48.
If two solid-hemispheres of same base radius r are joined together along their bases, then curved surface area of this new solid is
(a) $4\pi r^2$
(b) $6\pi r^2$
(c) $3\pi r^2$
(d) $8\pi r^2$
Solution:
Because curved surface area of a hemisphere is $2\pi r^2$ and here, we join two solid hemispheres along their bases of radius r, from which we get a solid sphere.
Hence, the curved surface area of new solid = $2\pi r^2 + 2\ \pi r^2 = 4\pi r^2$. **(a)**

Question 49.
The diameters of two circular ends of the bucket are 44 cm and 24 cm. The height of the bucket is 35 cm. The capacity of the bucket is
(a) 32.7 litres
(b) 33.7 litres
(c) 34.7 litres
(d) 31.7 litres
Solution:
Given, diameter of one end of the bucket, $2R = 44 \Rightarrow R = 22$ cm [∵ diameter, r = 2 x radius]
and diameter of the other end,
$2r = 24 \Rightarrow r = 12$ cm [∵ diameter, r = 2 x radius]
Height of the bucket, h = 35 cm
Since, the shape of bucket is look like as frustum of a cone.

∴ Capacity of the bucket = Volume of the frustum of the cone

$$= \frac{1}{3}\pi h\,[R^2 + r^2 + Rr]$$

$$= \frac{1}{3} \times \pi \times 35[(22)^2 + (12)^2 + 22 \times 12]$$

$$= \frac{35\pi}{3}[484 + 144 + 264]$$

$$= \frac{35\pi \times 892}{3} = \frac{35 \times 22 \times 892}{3 \times 7}$$

$$= 32706.6 \text{ cm}^3 = 32.7 \text{ L}$$

$$[\because 1000 \text{ cm}^3 = 1 \text{ L}]$$

Question 50.

A spherical ball of radius r is melted to make 8 new identical balls each of radius r,. Then r:rl =

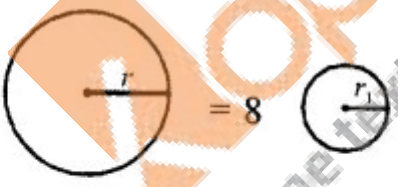
(a) 2 : 1

(b) 1 : 2

(c) 4 : 1

(d) 1 : 4

Solution:



Volume of bigger ball = 8 × volume of smaller ball

$$\frac{4}{3}\pi r^3 = 8 \times \frac{4}{3}\pi r_1^3$$

$$r^3 = (2r_1)^3$$

$$r = 2r_1$$

$$\frac{r}{r_1} = \frac{2}{1}$$

$$\therefore\ r : r_1 = 2 : 1 \qquad \text{(a)}$$