RD Sharma Class 10 Solutions Chapter 15 Areas Related to Circles MCQS

Question 1.
If the circumference and the area of a circle are numerically equal, then diameter of the circle is
(a) π/2
(b) 2π
(c) 2
(d) 4
Solution:
Let r be the radius of the circle, then Circumference = 2πr
and area = πr^2
But $2\pi r = \pi r^2$
∴ $2r = r^2$
⇒ r = 2
Diameter = 2r = 2 x 2 = 4 **(d)**

Question 2.
If the difference between the circum-ference and radius of a circle is 37 cm., then using π = 227 the circumference (in cm) of the circle is
(a) 154
(b) 44
(c) 14
(d) 7 [CBSE 2013]
Solution:
Difference between circumference and radius of a circle = 37 cm
∴ 2πr-r = 37

$$(2\pi - 1)r = 37 \Rightarrow r = \frac{37}{2\pi - 1}$$

$$\Rightarrow r = \frac{37}{2 \times \frac{22}{7} - 1} = \frac{37}{\frac{44}{7} - 1}$$

$$= \frac{37 \times 7}{37} = 7 \text{ cm}$$

∴ Circumference = 37 + 7 = 44 cm (b)

Question 3.
A wire can be bent in the form of a circle of radius 56 cm. If it is bent in the form of a square, then its area will be
(a) 3520 cm²
(b) 6400 cm²
(c) 7744 cm²
(d) 8800 cm²

Solution:
Radius of a circular wire (r) = 56 cm
Circumference = 2πr = 2 x 227 x 56 cm = 352 cm
Now perimeter of square = 352 cm

$$\therefore \text{ Side of square} = \frac{352}{4} = 88 \text{ cm}$$

$$\text{and area of square} = (\text{side})^2 = (88)^2$$
$$= 7744 \text{ cm}^2 \quad (c)$$

Question 4.
If a wire is bent into the shape of a square, then the area of the square is 81 cm². When wire is bent into a semicircular shape, then the area of the semi-circle will be
(a) 22 cm²
(b) 44 cm²
(c) 77 cm²
(d) 154 cm²
Solution:
Area of a square wire = 81 cm²
∴ Side of square = Area−−−−√ = 81−−√ cm = 9 cm ard per in eret of square =4a = 4 x 9 = 36cm
Perimeter of semicircular wire whose bent = 36 cm
Let r be the radius, then

$$2r + \pi r = 36 \Rightarrow r\left(2 + \frac{22}{7}\right) = 36$$

$$\Rightarrow r\left(\frac{36}{7}\right) = 36 \Rightarrow r = \frac{36 \times 7}{36} = 7 \text{ cm}$$

$$\therefore \text{ Area of semicircle} = \frac{1}{2}\pi r^2$$

$$= \frac{1}{2} \times \frac{22}{7} \times (7)^2 \text{ cm}^2$$

$$= \frac{1}{2} \times \frac{22}{7} \times 7 \times 7 = 77 \text{ cm}^2 \quad (c)$$

Question 5.
A circular park has a path of uniform width around it. The difference between the outer and inner circumferences of the circular path is 132 m. Its width is
(a) 20 m
(b) 21 m
(c) 22 m

(d) 24 m

Solution:

Let R and r be the radii of the outer and inner circles of the park, then 2πR – 2πr = 132

$$\Rightarrow 2\pi (R - r) = 132 \Rightarrow \frac{2 \times 22}{7} (R - r) = 132$$

$$\Rightarrow R - r = \frac{132 \times 7}{2 \times 22} = 21$$

$$\therefore \text{ Width of path} = 21 \text{ m} \quad (b)$$

Question 6.

The radius of a wheel is 0.25 m. The number of revolutions it will make to travel a distance of 11 km will be

(a) 2800

(b) 4000

(c) 5500

(d) 7000

Solution:

Radius of the wheel (r) = 0.25 m = 25 cm

Circumference of the wheel

$$= 2\pi r = \frac{2 \times 22 \times 25}{7} \text{ cm}$$

$$= \frac{1100}{7} \text{ cm}$$

∴ To cover a distance of 11 km, the revolutions

$$\text{will be} = (11 \times 1000 \times 100) \div \frac{1100}{7}$$

$$= \frac{11 \times 1000 \times 100 \times 7}{1100} = 7000 \quad (d)$$

Question 7.

The ratio of the outer and inner perimeters of a circular path is 23 : 22. If the path is 5 metres wide, the diameter of the inner circle is

(a) 55 m

(b) 110 m

(c) 220 m

(d) 230 m

Solution:
Ratio in the outer and inner perimeter of a circular path = 23 : 22
Width of path = 5 m
Let R and r be the radii of outer and inner path then R- r = 5 m ....(i)

and $\frac{2\pi R}{2\pi r} = \frac{23}{22} \Rightarrow \frac{R}{r} = \frac{23}{22}$

$\Rightarrow 22R = 23r \Rightarrow R = \frac{23}{22}r$

$\therefore$ From (i)

$\frac{23}{22}r - r = 5 \Rightarrow r\left(\frac{23}{22} - 1\right) = 5$

$\Rightarrow r\left(\frac{1}{22}\right) = 5 \Rightarrow r = 5 \times 22 = 110$ m

$\therefore$ Diameter of inner circle $= 2r = 2 \times 110$
$= 220$ m (c)

Question 8.
The circumference of a circle is 100 cm. The side of a square inscribed in the circle is

(a) $50\sqrt{2}$ cm (b) $\frac{100}{\pi}$ cm

(c) $\frac{50\sqrt{2}}{\pi}$ cm (d) $\frac{100\sqrt{2}}{\pi}$ cm

Solution:
Circumference of a circle (c) = 100 cm

Diagonal of square which is inscribed in the circle

$$\therefore \text{ Radius } (r) = \frac{c}{2\pi} = \frac{100}{2\pi} = \frac{50}{\pi}$$

$\therefore$ Diagonal of square which is inscribed in the circle

$$= 2r = 2 \times \frac{50}{\pi} = \frac{100}{\pi}$$

$$\text{Side of square} = \frac{\text{Diagonal}}{\sqrt{2}} = \frac{100}{\sqrt{2} \times \pi}$$

$$= \frac{100 \times \sqrt{2}}{\sqrt{2} \times \sqrt{2} \times \pi} = \frac{100 \times \sqrt{2}}{2\pi} = \frac{50 \times \sqrt{2}}{\pi} \quad \text{(c)}$$

Question 9.
The area of the incircle of an equilateral triangle of side 42 cm is :
(a) 22 73 cm²
(b) 231 cm²
(c) 462 cm²
(d) 924 cm²
Solution:
Side of an equilateral triangle (a) = 42 cm
Radius of inscribed circle = 13 x altitude

$$= \frac{1}{3} \times \frac{\sqrt{3}}{2} \text{ side}$$

$$= \frac{\sqrt{3}}{6} \times 42 = 7\sqrt{3}$$

$$\therefore \text{ Area of the incircle} = \pi r^2$$

$$= \frac{22}{7} \times \left(7\sqrt{3}\right)^2 \text{ cm}^2$$

$$= \frac{22}{7} \times 7 \times 7 \times 3 \text{ cm}^2 = 462 \text{ cm}^2 \text{ (c)}$$

Question 10.
The area of incircle of an equilateral triangle is 154 cm2. The perimeter of the triangle is
(a) 71.5 cm
(b) 71.7 cm
(c) 72.3 cm
(d) 72.7 cm
Solution:
Area of incircle of an equilateral triangle = 154 cm^2

$$\therefore \text{ Radius} = \sqrt{\frac{154}{\pi}} = \sqrt{\frac{154 \times 7}{22}} = \sqrt{49} = 7 \text{ cm}$$

Let a be the side of the triangle, then

$$\text{altitude} = \frac{\sqrt{3}}{2}a$$

$$\therefore \frac{\sqrt{3}}{2}a = 3 \times r = 3 \times 7 \Rightarrow a = \frac{21 \times 2}{\sqrt{3}} = \frac{42}{\sqrt{3}}$$

$$a = \frac{42\sqrt{3}}{\sqrt{3} \times \sqrt{3}} = \frac{42\sqrt{3}}{3} = 14\sqrt{3}$$

$$\text{Perimeter} = 3 \times \text{side} = 3a = 3 \times 14\sqrt{3}$$

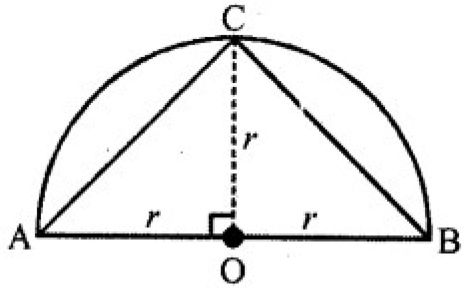
$$= 42\sqrt{3} \text{ cm}$$

$$= 42 \times (1.73) = 72.66 \text{ cm}$$

$$= 72.7 \text{ cm} \quad (d)$$

Question 11.
The area of the largest triangle that can be inscribed in a semi-circle of radius r. is
(a) r^2
(b) 2 r^2
(c) r^3
(d) $2r^3$
Solution:
The largest triangle inscribed in a semi-circle of radius r, can be ΔABC as shown in

the figure, whose base = AB = 2r



and altitude OC = r

Area of triangle = $\frac{1}{2}$ base × altitude

$= \frac{1}{2} \times 2r \times r = r^2$ (a)

Question 12.
The perimeter of a triangle is 30 cm and the circumference of its incircle is 88 cm. The area of the triangle is
(a) 70 cm²
(b) 140 cm²
(c) 210 cm²
(d) 420 cm²
Solution:
The perimeter of a triangle = 30 cm
and circumference of its incircle = 88 cm

$\therefore$ Radius of the incircle $= \frac{c}{2\pi} = \frac{88 \times 7}{2 \times 22} = 14$ cm

$\therefore$ Altitude of the triangle $= 14 \times \frac{3}{1} = 42$ cm

Base (side) of the triangle $= \frac{30}{3} = 10$ cm

$\therefore$ Area $= \frac{1}{2}$ base × altitude

$= \frac{1}{2} \times 10 \times 42 = 210 \text{ cm}^2$ (c)

Question 13.
The area of a circle is 220 cm², the area of a square inscribed in it is
(a) 49 cm²
(b) 70 cm²
(c) 140 cm²
(d) 150 cm²
Solution:
Area of a circle = 220 cm²

$$\therefore \text{ Radius } (r) = \sqrt{\frac{A}{\pi}} = \sqrt{\frac{220 \times 7}{22}}$$

$$= \sqrt{70} \text{ cm}$$

Diagonal of square = diameter of the circle

$$= 2 \times r = 2 \times \sqrt{70} \text{ cm}$$

$$\therefore \text{ Area of square} = \left(\frac{\text{Diagonal}}{\sqrt{2}}\right)^2 = \left(\frac{2\sqrt{70}}{\sqrt{2}}\right)^2$$

$$= \frac{4 \times 70}{2} = 140 \text{ cm}^2$$ (c)

Question 14.
If the circumference of a circle increases from 4π to 8π, then its area is
(a) halved
(b) doubled
(c) tripled
(d) quadrupled
Solution:

In first case circumference of a circle = 4π

$$\therefore \text{ Radius } (r) = \frac{c}{2\pi} = \frac{4\pi}{2\pi} = 2$$

$$\text{Area} = \pi r^2 = \pi \times (2)^2 = 4\pi$$

In second case, $c = 8\pi$

$$\therefore \text{ Radius (R)} = \frac{c}{2\pi} = \frac{8\pi}{2\pi} = 4$$

$$\text{Then area} = \pi R^2 = \pi \times (4)^2 = 16\pi$$

$\therefore$ 16π is fourtimes of 4π

$\therefore$ Area will be quadrupled

Question 15.
If the radius of a circle is diminished by 10%, then its area is diminished by
(a) 10%
(b) 19%
(c) 20%
(d) 36%
Solution:
Let in first case radius of a circle = r
Then area = πr^2

$$\text{In second case, radius} = \frac{r \times (100 - 10)}{100}$$

$$= \frac{r \times 90}{100} = \frac{9}{10} r$$

$$\text{Then area} = \pi \left(\frac{9}{10} r\right)^2 = \frac{81}{100} \pi r^2$$

$$\text{Difference} = \pi r^2 - \frac{81}{100} \pi r^2 = \frac{100 - 81}{100} \pi r^2$$

$$= \frac{19}{100} \pi r^2$$

$\therefore$ It is diminished by 19% (b)

Question 16.
If the area of a square is same as the area of a circle, then the ratio of their perimeter, in terms of 7t, is

(a) $\pi : \sqrt{3}$
(b) $2 : \sqrt{\pi}$
(c) $3 : \pi$
(d) $\pi : \sqrt{2}$

Solution:
Let side of square = a
Perimeter = 4 a
Then area = a^2
∴ Area of circle = a^2

$$\therefore \text{Radius} = \sqrt{\frac{\text{Area}}{\pi}} = \sqrt{\frac{a^2}{\pi}} = \frac{a}{\sqrt{\pi}}$$

and circumference $= 2\pi r$

$$= 2\pi \times \frac{a}{\sqrt{\pi}}$$

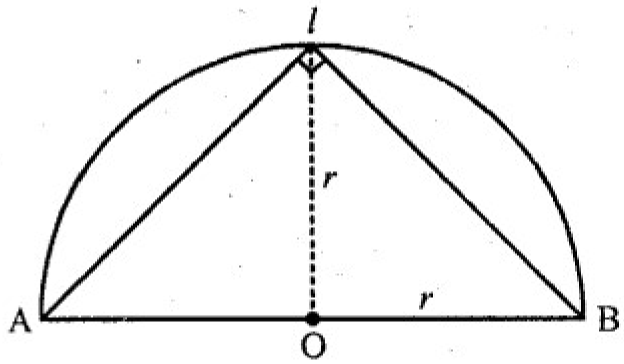
$$= 2a\sqrt{\pi}$$

$$\therefore \text{Ratio} = 4a : 2\sqrt{\pi}$$

$$= 2 : \sqrt{\pi}$$ (b)

Question 17.
The area of the largest triangle that can be inscribed in a semi-circle of radius r is
(a) 2r
(b) r^2
(c) r
(d) r√
Solution:

Radius of semicircule = r



The base of the largest triangle
$(b) = 2r$
and height $(h) = r$

$$\therefore \text{ Area} = \frac{1}{2} \text{ base} \times \text{height}$$

$$= \frac{1}{2} \times 2r \times r = r^2$$ (b)

Question 18.

The ratio of the areas of a circle and an equilateral triangle whose diameter and a side are respectively equal, is

(a) π : 2−√
(b) π : 3−√
(c) 3−√ :π
(d) 2−√ : π

Solution:

Let side of equilateral triangle = a
Then area = 3√4 a²

Diameter of circle = a

$$\therefore \text{ Radius} = \frac{a}{2}$$

$$\text{and area} = \pi r^2 = \pi\left(\frac{a}{2}\right)^2 = \frac{a^2\pi}{4}$$

Ratio of area of circle and triangle

$$= \frac{a^2\pi}{4} : \frac{\sqrt{3}}{4}a^2$$

$$= \pi : \sqrt{3} \qquad \text{(c)}$$

Question 19.

If the sum of the areas of two circles with radii r_1 and r_2 is equal to the area of a circle of radius r, then $r_1^2 + r_2^2$

(a) $>r^2$

(b) $= r^2$

(c) $< r^2$

(d) None of these

Solution:

Sum of area of two circles with radii r_1 and r_2

$$= \pi r_1^2 + \pi r_2^2 = \pi(r_1^2 + r_2^2)$$

and area of a circle with radius $r = \pi r^2$

$$\therefore \pi(r_1^2 + r_2^2) = \pi r^2$$

$$\Rightarrow r_1^2 + r_2^2 = r^2 \qquad \text{(b)}$$

Question 20.

If the perimeter of a semi-circular protractor is 36 cm, then its diameter is

(a) 10 cm

(b) 12 cm

(c) 14 cm

(d) 16 cm

Solution:

Perimeter of a semicircle = 36 cm

Let d be its diameter, then

$$\text{Perimeter} = \frac{\pi d}{2} + d$$

$$\therefore \frac{\pi d}{2} + d = 36$$

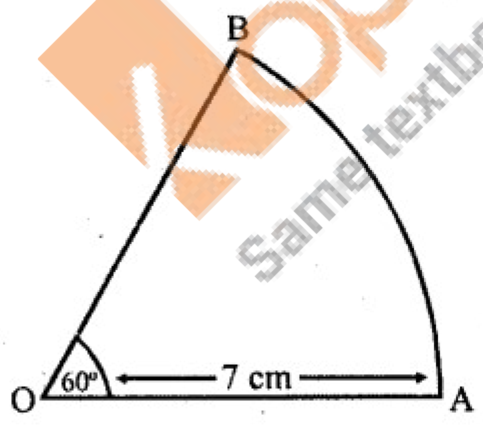
$$d\left(\frac{\pi}{2}+1\right) = 36 \Rightarrow d\left(\frac{22}{7\times 2}+1\right) = 36$$

$$\Rightarrow d\left(\frac{18}{7}\right) = 36 \Rightarrow d = \frac{36\times 7}{18} = 14 \text{ cm} \quad (c)$$

Question 21.
The perimeter of the sector OAB shown in the fiugre, is

(a) $\frac{64}{3}$ **cm** **(b) 26 cm**

(c) $\frac{64}{5}$ **cm** **(d) 19 cm**



Solution:
Radius of sector of 60° = 7 cm

∴ Perimeter = arc AB + 2 r

$$= 2\pi r \times \frac{60}{360} + 2 \times 7$$

$$= 2 \times \frac{22}{7} \times 7 \times \frac{1}{6} + 14$$

$$= \frac{22}{3} + 14 = \frac{64}{3} \text{ cm} \qquad \text{(a)}$$

Question 22.

If the perimeter of a sector of a circle of radius 6.5 cm is 29 cm, then its area is
(a) 58 cm^2
(b) 52 cm^2
(c) 25 cm^2
(d) 56 cm^2

Solution:

Radius of a sector = 6.5 cm

and perimeter = 29 cm

$$\therefore\ 2\pi r \times \frac{\theta}{360°} + 2r = 29$$

$$\Rightarrow 2 \times \pi\,(6.5) \times \frac{\theta}{360°} + 2 \times 6.5 = 29$$

$$13\pi \times \frac{\theta}{360°} + 13 = 29$$

$$\Rightarrow 13\pi \times \frac{\theta}{360°} = 29 - 13 = 16$$

$$\therefore\ \frac{\theta}{360°} = \frac{16}{13\pi} \quad \ldots.(i)$$

$$\text{Now area} = \pi r^2 \times \frac{\theta}{360°} = \pi\,(6.5)^2 \times \frac{16}{13\pi}$$

[From (*i*)]

$$= \frac{42.25 \times 16}{13} = 3.25 \times 16 = 52\ \text{cm}^2 \quad \text{(b)}$$

Question 23.

If the area of a sector of a circle bounded by an arc of length 5K cm is equal to 20K cm², then its radius is

(a) 12 cm
(b) 16 cm
(c) 8 cm
(d) 10 cm

Solution:

Let r be the radius, then

Length of the arc of sector of θ angle = 5π

$$\Rightarrow 2\pi r \frac{\theta}{360°} = 5\pi$$

$$\therefore r\frac{\theta}{360°} = \frac{5}{2} \quad ....(i)$$

and area of sector of θ angle = 20π cm²

$$\therefore \pi r^2 \frac{\theta}{360°} = 20\pi$$

$$r^2 \frac{\theta}{360°} = 20$$

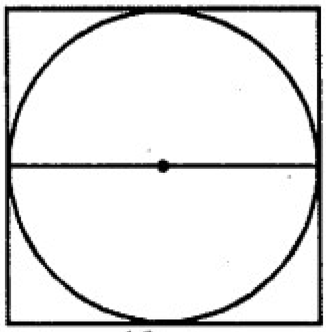
$$\Rightarrow r.r \frac{\theta}{360°} = 20 \Rightarrow r \times \frac{5}{2} = 20$$

$$\Rightarrow r = \frac{20 \times 2}{5} = 8$$

∴ Radius = 8 cm (c)

Question 24.
The area of the circle that can be inscribed in a square of side 10 cm is
(a) 40π cm²
(b) 30π cm²
(c) 100π cm²
(d) 25π cm²
Solution:
Side of square = 10 cm

∴ Diameter of the inscribed circle = 10 cm



and radius $(r) = \frac{10}{2} = 5$ cm

Area $= \pi r^2 = \pi \times (5)^2 = 25\pi\ cm^2$

Question 25.
If the difference between the circumference
(a) 154 cm²
(b) 160 cm²
(c) 200 cm²
(d) 150 cm²

Solution:
Let r be the radius of a circle then circum-ference = 2πr
∴ 2πr-r = 37

$$r\left(2\times\frac{22}{7}-1\right) = 37 \Rightarrow r\left(\frac{44}{7}-7\right) = 37$$

$$\Rightarrow r\left(\frac{37}{7}\right) = 37 \Rightarrow r = \frac{37\times 7}{37} = 7\text{ cm}$$

Now area of the circle $= \pi r^2$

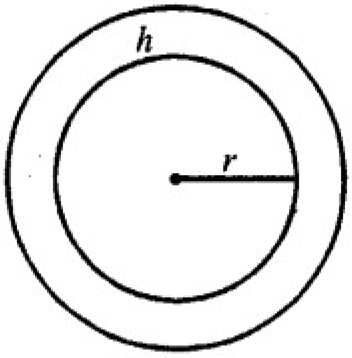
$$= \frac{22}{7}\times 7\times 7 = 154\text{ cm}^2 \qquad \text{(a)}$$

Question 26.
The area of a circular path of uniform width h surrounding a circular region of radius r is
(a) π (2r + h) r
(b) π (2r + h) h
(c) π (h + r)r

(d) π (h + r) A

Solution:

Let r be the radius of inner circle h is the width of circular path



$\therefore$ Outer radius $= r + h$

$\therefore$ Area of the path $= \pi [(r + h)^2 - r^2]$

$= \pi [r^2 + h^2 + 2rh - r^2]$

$= \pi (h^2 + 2rh) = \pi h (h + 2r)$ (b)

Question 27.

If AB is a chord of length 53–√ cm of a circle with centre O and radius 5 cm, then area of sector OAB is

(a) $\frac{3\pi}{8}$ cm² (b) $\frac{8\pi}{3}$ cm²

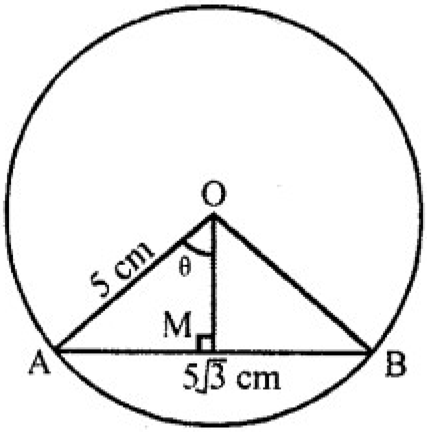
(c) 25π cm² (d) $\frac{25\pi}{3}$ cm²

Solution:

Radius of the circle (r) = 5 cm

AB is a chord = 53–√

Draw OM ⊥ AB which bisects the chord AB at M



$$AM = MB = \frac{5}{2}\sqrt{3}$$

Let $\angle AOM = \theta$

$$\sin\alpha = \frac{AM}{OA} = \frac{5\sqrt{3}}{2\times 5} = \frac{\sqrt{3}}{2}$$

$\therefore \alpha = 60°$

$\therefore \angle AOB = 2\alpha = 2 \times 60° = 120°$

$$\text{Now area of sector AOB} = \pi r^2 \times \frac{\theta}{360°}$$

$$= \pi (5)^2 \times \frac{120°}{360°}$$

$$= 25\pi \times \frac{1}{3} = \frac{25\pi}{3}\ \text{cm}^2$$ (d)

Question 28.

The area of a circle whose area and circumference are numerically equal, is

(a) 2π sq. units
(b) 4π sq. units
(c) 6π sq. units
(d) 8π sq. units

Solution:

Let radius of the circle = r

∴ Area = πr^2

and circumference = 2πr

$\therefore \pi r^2 = 2\pi r \Rightarrow r = 2$

$\therefore$ Area $= \pi r^2 = \pi (2)^2 = 4\pi$ sq. units (b)

Question 29.

If diameter of a circle is increased by 40%, then its area increases by
(a) 96%
(b) 40%
(c) 80%
(d) 48%

Solution:

Let the diameter of a circle in first case = 2r
Then radius = r
Area = πr^2

By increasing 40% of diameter or radius,

$$\text{new radius} = \frac{r \times 140}{100} = \frac{7r}{5}$$

$$\therefore \text{New area} = \pi \left(\frac{7r}{5}\right)^2 = \frac{49}{25}\pi r^2$$

$$\therefore \text{Difference of areas} = \frac{49}{25}\pi r^2 - \pi r^2$$

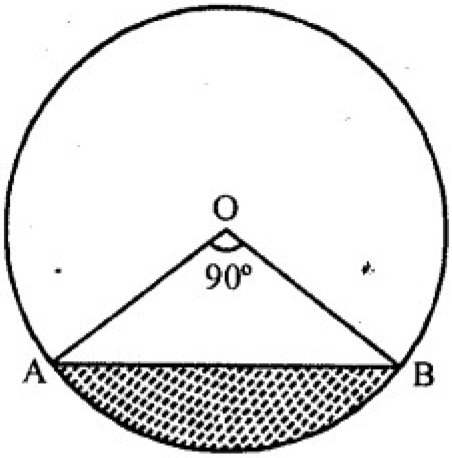
$$= \frac{24}{25}\pi r^2$$

$$\text{Percentage increase} = \frac{24\pi r^2}{25\pi r^2} \times 100$$

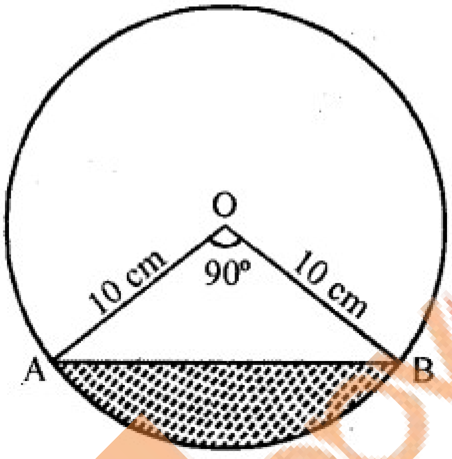
$= 96\%$ (a)

Question 30.

In the figure, the shaded area is
(a) 50 (π – 2) cm²
(b) 25 (π – 2) cm²
(c) 25 (π + 2) cm²
(d) 5 (π – 2) cm²



Solution:
In the figure, ∠AOB = 90°
and radius of the circle = 10 cm



∴ Area of shaded portion

= Area of quadrant AOB – area of ΔAOB

$= \frac{1}{4}\pi r^2 - \frac{1}{2}r^2$

$= \frac{1}{4}r^2(\pi - 2)\ cm^2$

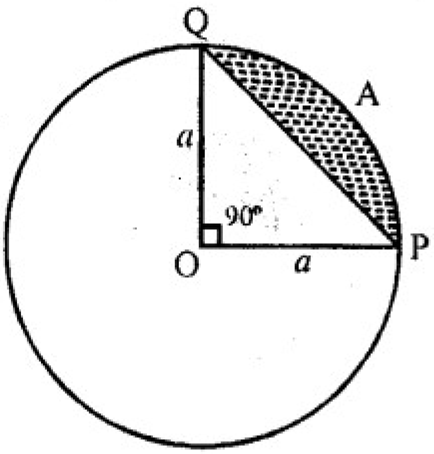
$= \frac{1}{4} \times 10 \times 10\ (\pi - 2)\ cm^2$

$= 25\ (\pi - 2)\ cm^2$ (b)

Question 31.
In the figure, the area of the segment PAQ is

(a) $\frac{a^2}{4}(\pi + 2)$ (b) $\frac{a^2}{4}(\pi - 2)$

(c) $\frac{a^2}{4}(\pi - 1)$ (d) $\frac{a^2}{4}(\pi + 1)$



Solution:
a is the radius of the circle arc PAQ subtends angle 90° at the centre
∴ Area of segment PAQ

= Area of quadrant – area of ΔOPQ

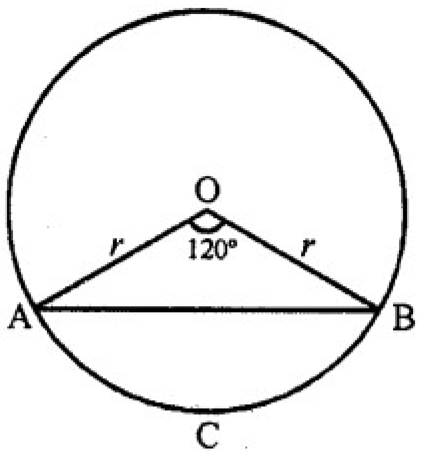
$= \frac{1}{4}\pi a^2 - \frac{1}{2} a \times a = \frac{1}{4}\pi a^2 - \frac{1}{2} a^2$

$= \frac{1}{4} a^2 (\pi - 2) = \frac{a^2}{4}(\pi - 2)$ (b)

Question 32.
In the figure, the area of segment ACB is

(a) $\left(\frac{\pi}{3}-\frac{\sqrt{3}}{2}\right) r^2$ (b) $\left(\frac{\pi}{3}+\frac{\sqrt{3}}{2}\right) r^2$

(c) $\left(\frac{\pi}{3}+\frac{2}{\sqrt{3}}\right) r^2$ (d) None of these



Solution:
r is the radius of the circle and arc ACB subtends angle of 120° at the centre
Area of segment ACB = r

$$= \left(\frac{\pi\theta}{360°} - \sin\frac{\theta}{2}\cos\frac{\theta}{2}\right) r^2$$

$$= \left(\pi \times \frac{120}{360} - \sin 60° \cos 60°\right) r^2$$

$$= \left(\frac{\pi}{3} - \frac{\sqrt{3}}{2} \times \frac{1}{2}\right) r^2 = \left(\frac{\pi}{3} - \frac{\sqrt{3}}{4}\right) r^2 \quad \text{(d)}$$

Question 33.
If the area of a sector of a circle bounded by an arc of length 5π cm is equal to 20rc cm², then the radius of the circle is
(a) 12 cm
(b) 16 cm
(c) 8 cm
(d) 10 cm
Solution:
Length of arc = 5π cm
area of sector = 20π cm²

Let the angle at the centre be θ

$$\text{then, } 2\pi r \times \frac{\theta}{360^\circ} = 5\pi$$

$$r \times \frac{\theta}{360^\circ} = \frac{5\pi}{2\pi} = \frac{5}{2} \quad \ldots.(i)$$

$$\text{Area} = \pi r \frac{\theta}{360^\circ}$$

$$\therefore \pi r^2 \frac{\theta}{360^\circ} = 20\pi$$

$$\pi r . r . \frac{\theta}{360^\circ} = 20\pi$$

$$\pi r \times \frac{5}{2} = 20\pi \Rightarrow r = \frac{20\pi \times 2}{5\pi}$$

$$\Rightarrow r = 8$$

$\therefore$ Radius of the circle = 8 cm (c)

Question 34.

In the figure, the ratio of the areas of two sectors S_1 and S_2 is

(a) 5 : 2

(b) 3 : 5

(c) 5 : 3

(d) 4 : 5

$$\therefore \text{Area of sector } S_1 = \pi r^2 \times \frac{120^\circ}{360^\circ}$$

$$\text{and area of sector } S_2 = \pi r^2 \times \frac{150^\circ}{360^\circ}$$

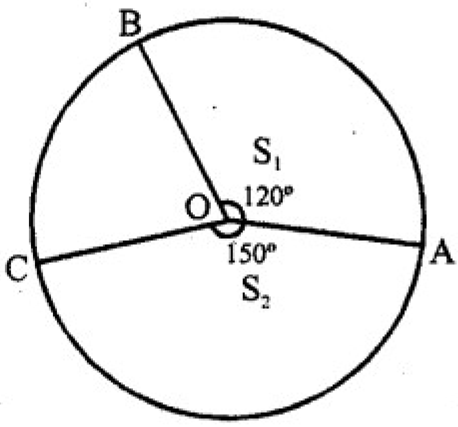
Solution:

Let r be the radius of the circle

$$\text{Ratio} = \pi r^2 \times \frac{120^\circ}{360^\circ} : \pi r^2 \times \frac{150^\circ}{360^\circ}$$

$= 120^\circ : 150^\circ$

$= 4 : 5$ (d)



Question 35.
If the area of a sector of a circle is 518 of the area of the circle, then the sector angle is equal to
(a) 60°
(b) 90°
(c) 100°
(d) 120°
Solution:
Area of sector of a circle = 518 x area of circle
Let θ be its angle at the centre and r be radius

$$\text{Then, } \pi r^2 \times \frac{\theta}{360^\circ} = \frac{5}{18} \pi r^2$$

$$\frac{\theta}{360^\circ} = \frac{5}{18} \Rightarrow \theta = \frac{5}{18} \times 360^\circ = 100^\circ \quad \text{(c)}$$

Question 36.
If the area of a sector of a circle is 720 of the area of the circle, then the sector angel is equal to
(a) 110°
(b) 130°
(c) 100°
(d) 126°
Solution:
Area of sector of a circle = 720 of the area of the circle

Let r be the radius and θ be its angle at the centre

$$\therefore \pi r^2 \times \frac{\theta}{360^\circ} = \frac{7}{20} \times \pi r^2$$

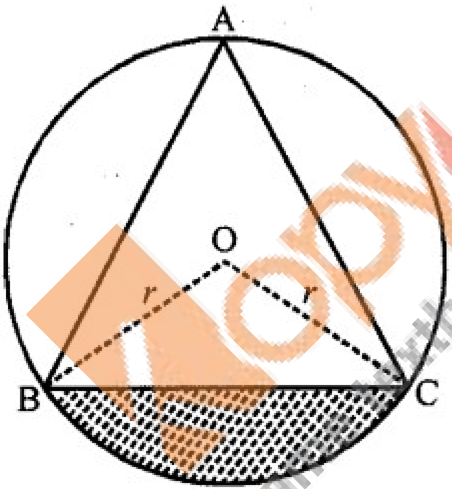
$$\Rightarrow \frac{\theta}{360^\circ} = \frac{7}{20} \Rightarrow \theta = \frac{7}{20} \times 360^\circ = 126^\circ \quad (d)$$

Question 37.

In the figure, if ABC is an equilateral triangle, then shaded area is equal to?

(a) $\left(\frac{\pi}{3} - \frac{\sqrt{3}}{4}\right) r^2$ (b) $\left(\frac{\pi}{3} - \frac{\sqrt{3}}{2}\right) r^2$

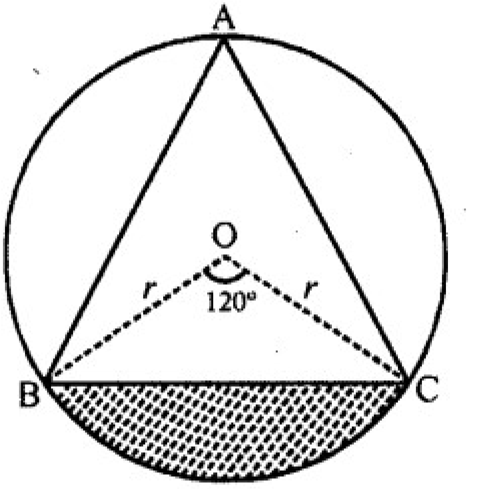
(c) $\left(\frac{\pi}{3} + \frac{\sqrt{3}}{4}\right) r^2$ (d) $\left(\frac{\pi}{3} + \sqrt{3}\right) r^2$



Solution:

ΔABC is an equilateral triangle inscribed in a circle with centre O and radius r

BO and CO are joined



$\therefore \angle BOC = 2\angle BAC = 2 \times 60° = 120°$

Area of shaded portion

$$= \left[\pi \frac{\theta}{360°} - \sin\frac{\theta}{2}\cos\frac{\theta}{2}\right] r^2$$

$$= \left[\frac{\pi \times 120°}{360°} - \sin 60° \cos 60°\right] r^2$$

$$= \left[\frac{\pi}{3} - \frac{\sqrt{3}}{2} \times \frac{1}{2}\right] r^2 = \left[\frac{\pi}{3} - \frac{\sqrt{3}}{4}\right] r^2 \quad (a)$$

Question 38.

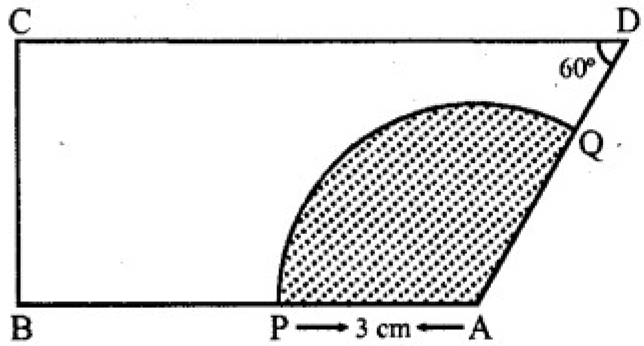
In the figure, the ara of the shaded region is

(a) 3π cm²

(b) 6π cm²

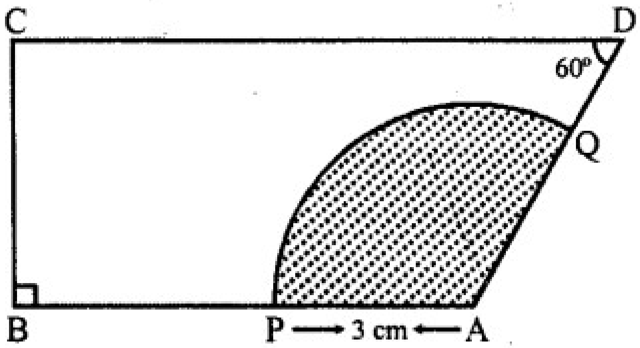
(c) 9π cm²

(d) 7π cm²



Solution:

In the figure, ∠B = ∠C = 90°, ∠D = 60?



∴ ∠A= 360° – (90° + 90° + 60°) = 360° – 240° = 120°
Radius of the sector = 3 cm
∴Area of shaded portion

$$= \pi r^2 \times \frac{\theta}{360°} = \pi \times 3 \times 3 \times \frac{120°}{360°}$$

$$= 9\pi \times \frac{1}{3} = 3\pi \text{ cm}^2 \qquad \text{(a)}$$

Question 39.
If the perimeter of a circle is equal to that of a square, then the ratio of their areas is
(a) 13 : 22
(b) 14 : 11
(c) 22 : 13
(d) 11 :14
Solution:
Let side of square = a units
∴ Area = a^2 sq. units
and perimeter = 4a units
Now perimeter of circle = 4a units

$$\therefore \text{ Radius} = \frac{\text{Perimeter}}{2\pi} = \frac{4a}{2\pi}$$

$$= \frac{2a}{\pi}$$

and area $= \pi r^2 = \pi\left(\frac{2a}{\pi}\right)^2$

$$= \frac{\pi \times 4a^2}{\pi^2} = \frac{4a^2}{\pi} \text{ sq. units}$$

$$= \frac{7 \times 4a^2}{22} = \frac{14}{11}a^2$$

$\therefore$ Ratio in the areas of circle and square

$\frac{14}{11}a^2 : a^2 = 14 : 11$ (b)

Question 40.

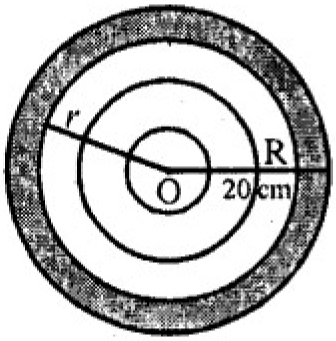
The radius of a circle is 20 cm. It is divided into four parts of equal area by drawing three concentric circles inside it. Then, the radius of the largest of three concentric circles drawn is

(a) $10\sqrt{5}$ **cm** **(b)** $10\sqrt{3}$ **cm**

(c) 10 cm **(d)** $10\sqrt{2}$ **cm**

Solution:

Radius of circle (R) = 20 cm



$\therefore$ Area $= \pi r^2 = \pi (20)^2$ cm^2

$= 400\pi$ cm^2

$\therefore$ Area of each part $= \frac{400\pi}{4}$

$= 100\pi$ cm^2

Let r be the radius of the larger circle

Then area $= \pi (R^2 - r^2)$

$\therefore \pi (20^2 - r^2) = 100\pi \Rightarrow 400 - r^2 = 100$

$\Rightarrow r^2 = 400 - 100 = 300$

$\Rightarrow r = \sqrt{300} = 10\sqrt{3}$ cm (b)

Question 41.

The area of a sector whose perimeter is four times its radius r units, is

(a) $\frac{r^2}{4}$ **sq. units** **(b)** $2r^2$ **sq. units**

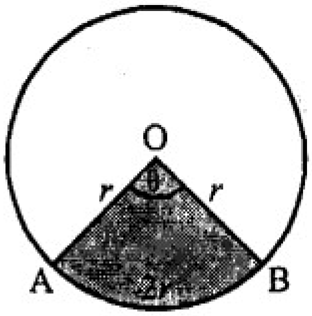
(c) r^2 **sq. units** **(d)** $\frac{r^2}{2}$ **sq. units**

Solution:

Radius of sector = r

Perimeter = 4r

and length of arc = 4r – 2r = 2r



∴ Let angle at the centre = θ

Then, $2\pi r = \frac{\theta}{360°} = 2r$

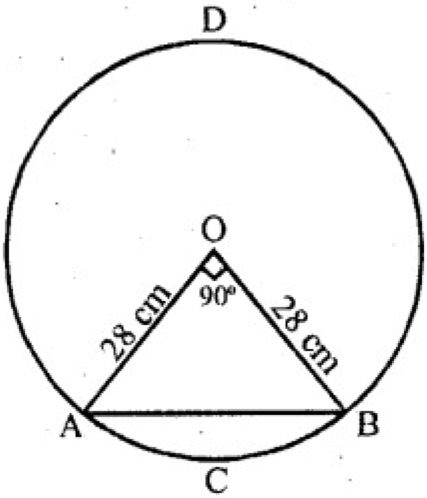
$\Rightarrow \pi \times \frac{\theta}{360°} = 1$ ....(*i*)

Now area $= \pi r^2 \times \frac{\theta}{360°} = r^2\left(\pi \times \frac{\theta}{360°}\right)$

$= r^2 \times 1$ [From (*i*)]

$= r^2$ (c)

Question 42.
If a chord of a circle of radius 28 cm makes an angle of 90° at the centre, then the area of the major segment is
(a) 392 cm²
(b) 1456 cm²
(c) 1848 cm²
(d) 2240 cm²
Solution:
A chord AB makes an angle of 90° at the centre
Radius of the circle = 28 cm



Area of minor segment ACB

$$= \pi r^2 \times \frac{\theta}{360^\circ} - \text{area of } \Delta AOB$$

$$= \pi r^2 \times \frac{90^\circ}{360^\circ} - \frac{1}{2} OA \times OB$$

$$= \frac{1}{4} \pi r^2 - \frac{1}{2} \times r^2$$

$$= \frac{1}{4} \times \frac{22}{7} \times 28 \times 28 - \frac{1}{2} \times 28 \times 28$$

$= 616 - 392$

$= 224$ cm^2

$\therefore$ Area of the major segment ADB

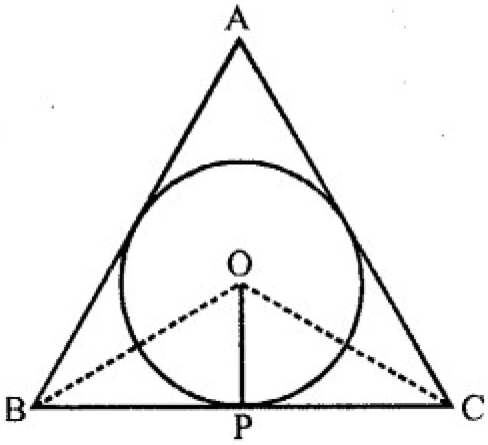
= Area of circle – area of minor segment

$$= \pi r^2 - 224 = \frac{22}{7} \times 28 \times 28$$

$= 2464 - 224$

$= 2240$ sq. cm^2 (d)

Question 43.
If the area of a circle inscribed in an equilateral triangle is 48π square units, then perimeter of the trianlge is
(a) 173–√ units
(b) 36 units
(c) 72 units
(d) 483–√ units
Solution:
Area of a circle inscribed in an equilateral triangle = 48π sq. units



$\therefore$ Radius of the circle $= \sqrt{\frac{\text{Area}}{\pi}} = \sqrt{\frac{48\pi}{\pi}}$

$= \sqrt{48}$ units $= 4\sqrt{3}$ units

$\because$ OP $\perp$ BC and $\angle B = 60^\circ$

$\therefore$ $\angle OBP = 30^\circ$

Now $\tan\theta = \frac{OP}{BP} \Rightarrow \tan 30^\circ = \frac{4\sqrt{3}}{BP}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{4\sqrt{3}}{BP} \Rightarrow BP\ 4\sqrt{3} \times \sqrt{3} = 12$ units

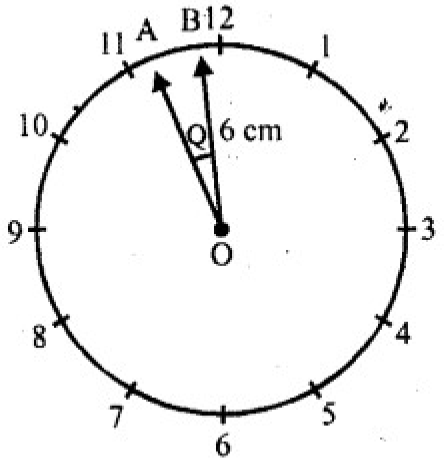
$\therefore$ BC $= 2 \times$ BP $= 2 \times 12 = 24$ units

$\therefore$ Perimeter of $\Delta ABC = 3 \times$ side

$= 3 \times 24 = 72$ units (c)

Question 44.
The hour hand of a clock is 6 cm long. The area swept by it between 11.20 am and 11.55 am is
(a) 2.75 cm²

(b) 5.5 cm²
(c) 11 cm²
(d) 10 cm²
Solution:
Length of hour hand of a clock (r) = 6 cm
Time 11.20 am to 11.55 am = 35 minutes



$$\therefore \text{ Centre angle} = \frac{35}{60} \times 5 = \frac{35}{12} \text{ min.}$$

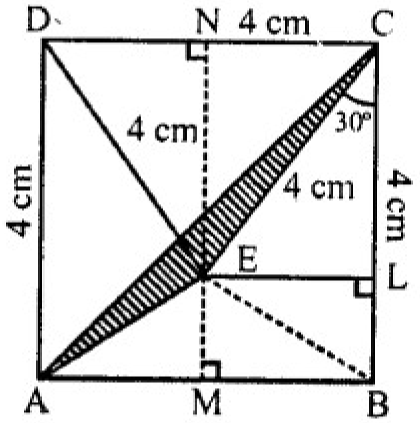
$$\therefore \text{ The hour hand will move } \frac{35^\circ}{12} \times 6^\circ = \frac{35^\circ}{2}$$

$$\therefore \text{ Area of section ACB} = \pi r^2 \times \frac{\theta}{360^\circ}$$

$$= \frac{22}{7} \times 6 \times 6 \times \frac{35}{360 \times 2} \text{ cm}$$

$$= \frac{11}{2} \text{ cm}^2 = 5.5 \text{ cm}^2 \qquad \text{(b)}$$

Question 45.
ABCD is a square of side 4 cm. If ? is a point in the interior of the square such that ΔCED is equilateral, then area of ΔACEis
(a) 2 (3–√ – 1) cm²
(b) 4 (3–√ -1) cm²
(c) 6(3–√-1)cm²
(d) 8(3–√-1)cm²
Solution:
Side of square ABCD = 4 cm
and side of equilateral ΔCED = 4 cm



Area of square = $(\text{side})^2 = 4 \times 4 = 16 \text{ cm}^2$

and area of $\Delta CED = \frac{\sqrt{3}}{4} (\text{side})^2$

$= \frac{\sqrt{3}}{4} \times 4 \times 4 = 4\sqrt{3} \text{ cm}^2$

Join AE, AB and AC and draw EL $\perp$ BC,
EM $\perp$ AB and EN $\perp$ CD
Now area of ΔABC

$= \frac{1}{2} AD \times BC = \frac{1}{2} \times 4 \times 4 = 8 \text{ cm}^2$

In ΔBEC, $EL = \frac{4}{2} = 2 \quad \left(\because \sin 30° = \frac{1}{2}\right)$

$\therefore$ area $\Delta BEC = \frac{1}{2} \times BC \times EL$

$= \frac{1}{2} \times 4 \times 2 = 4 \text{ cm}^2$

and in $\Delta AEB \perp EM = MN - EN$

$\left(4 - 2\sqrt{3}\right)$ cm

$\therefore$ area $\Delta AEB = \frac{1}{2} AB \times EM = \frac{1}{2} \times 4 \left(4 - 2\sqrt{3}\right)$

$= 4\left(2 - \sqrt{3}\right) = 8 - 4\sqrt{4} \text{ cm}^2$

$\therefore$ area ΔAEC = area ΔABC – (area ΔAEB + area ΔBEC)

$= 8 - \left(8 - 4\sqrt{3} + 4\right) = 8 - 8 - 4 + 4\sqrt{3}$

$= 4\sqrt{3} - 4 = 4\left(\sqrt{3} - 1\right) \text{ cm}^2$ (b)

Question 46.

If the area of a circle is equal to the sum of the areas of two circles of diameters 10 cm and 24 cm, then diameter of the larger circle (in cm) is
(a) 34
(b) 26
(c) 17
(d) 14

Solution:

Area of first circle of radius $= \frac{10}{2} = 5$ cm

$= \pi r^2 = \pi \times (5)^2 \text{ cm}^2 = 25\pi \text{ cm}^2$

and area of second circle of radius $= \frac{24}{2} =$

$12 \text{ cm} = \pi (12)^2 \text{ cm}^2 = 144\pi \text{ cm}^2$

$\therefore$ Total area $= (25\pi + 144\pi) \text{ cm}^2 = 169\pi \text{ cm}^2$

$\therefore$ Area of larger circle $= 169\pi \text{ cm}^2$

$\therefore$ Radius $= \sqrt{\frac{\text{Area}}{\pi}} = \sqrt{\frac{169\pi}{\pi}} = \sqrt{169}$

$= 13$ cm

$\therefore$ Diameter $= 2 \times \text{radius} = 2 \times 13 = 26 \text{ cm}^2$

(b)

Question 47.
If π is taken as 22/7, the-distance (in metres) covered by a wheel of diameter 35 cm, in one revolution, is (a) 2.2
(b) 1.1
(b) 9.625
(d) 96.25 [CBSE 2013]
Solution:
Diameter of a wheel = 35 cm = 35100 m
Circumference of the wheel = πd

$= \frac{35}{100} \times \frac{22}{7}$

$= \frac{110}{100} = 1.10 = 1.1$ m

$\therefore$ Distance in one revolution = 1.1 m (b)

Question 48.
ABCD is a rectangle whose three vertices are B (4, 0), C (4, 3) and D (0, 3). The length of one of its diagonals is
(a) 5
(b) 4
(c) 3

(d) 25 [CBSE 2014]

Solution:

Three vertices of a rectangle ABCD are B (4,0), C (4, 3) and D (0, 3) length of one of its diagonals

$$BD = \sqrt{(4-0)^2 + (0-3)^2} = \sqrt{4^2 + 3^2}$$

$$= \sqrt{16+9} = \sqrt{25} = 5 \quad \text{(a)}$$

Question 49.

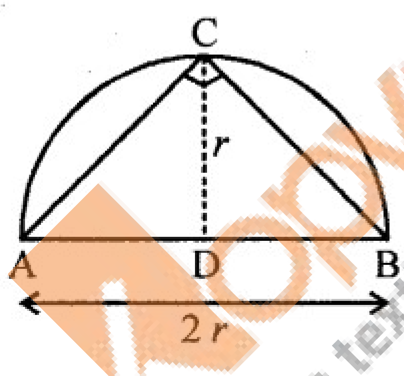
Area of the largest triangle that can be inscribed in a semi-circle of radius r units is

(a) r^2 sq. units (b) $\frac{1}{2}r^2$ sq. units

(c) $2r^2$ sq. units (d) $\sqrt{2}r^2$ sq. units

Solution:

Take a point C on the circumference of the semi-circle and join it by the end points of diameter A and B.



$\therefore \angle C = 90°$ [by property of circle]

[angle in a semi-circle are right angle]

So, ΔABC is right angled triangle.

$$\therefore \text{ Area of largest } \Delta ABC = \frac{1}{2} \times AB \times CD$$

$$= \frac{1}{2} \times 2r \times r$$

$$= r^2 \text{ sq units} \quad \text{(a)}$$

Question 50.

If the sum of the areas of two circles with radii r_1 and r_2 is equal to the area of a

circle of radius r, then

(a) $r = r_1 + r_2$ (b) $r_1^2 + r_2^2 = r^2$

(c) $r_1 + r_2 < r$ (d) $r_1^2 + r_2^2 < r^2$

Solution:

According to the given condition,

Area of circle = Area of first circle + Area of second circle.

$$\therefore \pi r^2 = \pi r_1^2 + \pi r_2^2$$

$$\Rightarrow r^2 = r_1^2 + r_2^2 \qquad \text{(b)}$$

Question 51.

If the sum of the circumference of two circles with radii r, and r2 is equal to the circumference of a circle of radius r, then

(a) $r = r_1 + r_2$ (b) $r_1 + r_2 > r$

(c) $r_1 + r_2 < r$ (d) None of these

Solution:

According to the given condition, Circumference of circle = Circumference of first circle + Circumference of second circle

$$\therefore 2\pi R = 2\pi R_1 + 2\pi R_2$$

$$\Rightarrow R = R_1 + R_2 \qquad \text{(a)}$$

Question 52.

If the circumference of a circle and the perimeter of a square are equal, then

(a) Area of the circle = Area of the square

(b) Area of the circle < Area of the square

(c) Area of the circle > Area of the square

(d) Nothing definite can be said

Solution:

According to the given condition, Circumference of a circle = Perimeter of square 2 πr = 4 a

[where, r and a are radius of circle and side of square respectively]

$$\Rightarrow \frac{22}{7}r = 2a \Rightarrow 11r = 7a$$

$$\Rightarrow a = \frac{11}{7}r \Rightarrow r = \frac{7a}{11} \quad ...(i)$$

Now, area of circle, $A_1 = \pi r^2$

$$= \pi\left(\frac{7a}{11}\right)^2 = \frac{22}{7} \times \frac{49a^2}{121} \quad \text{[from Eq. (i)]}$$

$$= \frac{14a^2}{11} \quad ...(ii)$$

and area of square, $A_2 = (a)^2$ ...(iii)

From Eqs. (ii) and (iii), $A_1 = \frac{14}{11}A_2$

$\therefore A_1 > A_2$

Hence, Area of the circle > Area of the square

(c)

Question 53.

If the perimeter of a circle is equal to that of a square, then the ratio of their areas is

(a) 22 : 7

(b) 14 : 11

(c) 7 : 22

(d) 11 : 14

Solution:

Let radius of circle be r and side of a square be a

According to the given condition,

Perimeter of a circle = Perimeter of a square

$$\therefore\ 2\pi r = 4a \Rightarrow a = \frac{\pi r}{2} \qquad \ldots(i)$$

$$\text{Now, } \frac{\text{Area of circle}}{\text{Area of square}} = \frac{\pi r^2}{(a)^2} = \frac{\pi r^2}{\left(\frac{\pi r}{2}\right)^2}$$

[from Eq. (*i*)]

$$= \frac{\pi r^2}{\frac{\pi^2 r^2}{4}} = \frac{4}{\pi} = \frac{4}{\frac{22}{7}} = \frac{28}{22} = \frac{14}{11} \qquad \text{(b)}$$