Exercise 15.1

1.

Sol:

Radius (r) = 4.2 cm

Circumference = 2 × r

$= 2 \times \frac{22}{7} \times 4.2$

$= \left(\frac{44}{10} \times 6\right) = \frac{264}{10}$

$= 26.4\ cm$

Area = $\pi r^2 = \frac{22}{7} \times 4.2 \times 4.2$

$= \frac{22 \times 6 \times 42}{10 \times 10} = \frac{5544}{100} = 55.44 cm^2$

2.

Sol:

Area of circle = 301.84 cm^2.

Let radius = r cm

Area of circle = πr^2

$\pi r^2 = 301.84$

$\frac{22}{7} \times r^2 = 301.84$

$r^2 = \frac{301.84 \times 7}{22} = \left(\sqrt{7 \times 7}\right)^{\frac{1}{2}} \times 13.75$

$r = \sqrt{13.72 \times 7} = \sqrt{7 \times 7 \times 1.96} = 7 \times 1.4 = 9.8\ cm$

Radius = r = 9.8 cm

Circumference = 2 × r = $2 \times \frac{22}{7} \times 9.8$

$= 44 \times 1.4$

$= 61.6\ cm$

3.

Sol:

Circumference = 44 cm

Let radius = r cm

Circumference = 2 × r = 44 cm

$2 \times \frac{22}{7} \times r = 44$

r $= \frac{44 \times 7}{2 \times 22} = 7cm$

radius = 7 cm

Area of circle = πr^2

$= \frac{22}{7} \times 7 \times 7 = (22 \times 7) = 154cm^2$

4.

Sol:

Let radius of circle = r cms

Diameter(d) = $2 \times radius = 2r$

Circumference (c) = $2\pi r$

Given circumference exceeds diameter by 16.8cm

C = d + 16.8

$\Rightarrow 2\pi r = 2r + 16.8$

$\Rightarrow 2r(\pi - 1) = 16.8$

$\Rightarrow 2r \times \left(\frac{22}{7} - 1\right) = 16.8$

$\Rightarrow 2r \times \frac{15}{7} = 16.8$

$\Rightarrow r = \frac{16.8 \times 7}{30} = 5.6 \times 0.7$

$\Rightarrow r = 3.92$ cms

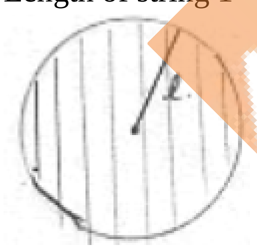
Circumference $= 2\pi r = 2 \times \frac{22}{7} \times 3.92$

$= \frac{2464}{100} = 24.64\ cms$

5.

Sol:

Length of string 1 = 28m



Area it can graze is area of circle with radius equal to length of string

Area = πl^2

$= \frac{22}{7} \times 28 \times 28$

$= 88 \times 28$

$= 2464\ cm^2$

$\therefore$ area grazed by horse $= 2464cm^2$.

6.

Sol:

Let side of square = s and length of wire be l. As wire is bent into square

l = perimeter of square = 4s.

Area of square = $121cm^2 = s^2$.

S = $\sqrt{121} = 11cm$

∴ length of wire l = 4(11) = 44cm

As wire is bent into circle (let radius be r)

Length of wire = circumference

$44 = 2\pi r$

$\frac{22}{7} \times 2 \times r = 44 \Rightarrow r = \frac{44\times7}{2\times22} = 7cm$

$Area\ of\ circle = \pi r^2$

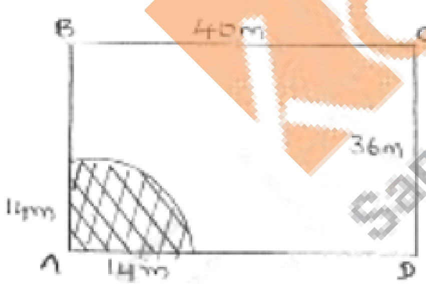
$= \frac{22}{7} \times 7 \times 7$

$= 22 \times 7$

$= 154cm^2$

7.

Sol:



The fig shows rectangular field ABCD at corner A, a horse is tied with rope length = 14m. The area it can graze is represented A as shaded region= area of quadrant with (radius = length) of string

Area = $\frac{1}{4} \times (area\ of\ circle) = \pi r^2$

= $\frac{1}{4} \times \frac{22}{7} \times 14 \times 14$

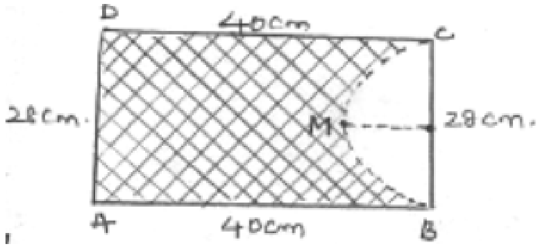
= (22×7)

= $154\ m^2$.

Area it can graze = $154m^2$.

8.

Sol:



Given sheet of paper ABCD

AB = 40 cm, AD = 28 cm

⇒ CD = 40 cm, BC = 28 cm [since ABCD is rectangle]

Semicircle be represented as BMC with BC as diameter

Radius = $\frac{1}{2} \times BC = \frac{1}{2} \times 28 = 14cms$

Area of remaining (shaded region) = (area of rectangle) – (area of semicircle)

= $(AB \times BC) - \left(\frac{1}{2}\pi r^2\right)$

= $(40 \times 28) - \left(\frac{1}{7} \times \frac{22}{7} \times 14 \times 14\right)$

= $1120 - 308$

= 812 cm^2.

9.

Sol:

Let radius of two circles be r_1 and r_2 then their circumferences will be $2\pi r_1$: $2\pi r_2$

= $r_1 : r_2$

But circumference ratio is given as 2 : 3

$r_1 : r_2 = 2 : 3$

Ratio of areas = $\pi {r_2}^2 : \pi {r_2}^2$

$= \left(\frac{r_1}{r_2}\right)^2$

$= \left(\frac{2}{3}\right)^2$

$= \frac{4}{9}$

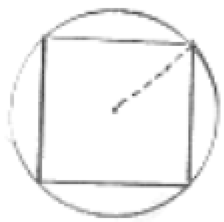
$= 4 : 9$

$\therefore ratio\ of\ areas = 4 : 9$

10.

Sol:

Circumscribed circle



Radius = $\frac{1}{2}(diagonal\ of\ square)$

= $\frac{1}{2} \times \sqrt{2}$ side

= $\frac{1}{2} \times \sqrt{2} \times 10$

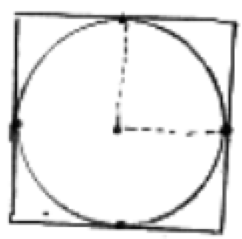
= $5\sqrt{2}$ cm

Area = πr^2

= $\frac{22}{7} \times 25 \times 2$

= $\frac{1100}{7} cm^2$

Inscribed circle



Radius = $\frac{1}{2}$ (sides)

= $\frac{1}{2} \times 10$

= 5 cm

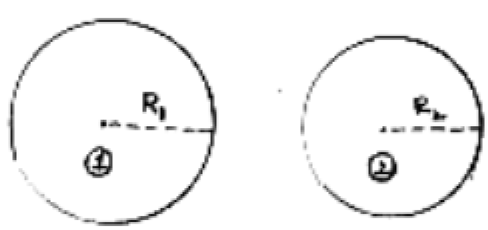
Area = πr^2

= $\frac{22}{7} \times 5 \times 5$

= $\frac{550}{7} cm^2$

11.

Sol:



Let radius of circles be $r_1 and\ r_2$

Given sum of radius = 140cm

$r_1 + r_2 = 140$ …..(i)

$difference\ in\ circumgerences = 88\ cm$

$2 \times r_1 - 2\pi r_2 = 88$

$2 \times \frac{22}{7}(r_1 - r_2)\ 88$

$r_1 - r_2 = \frac{88\times7}{2\times22} = 14$

$r_1 = r_2 + 14$ ….(ii)

$(ii) in\ (i) \Rightarrow\ r_2 + r_2 + 14 = 140$

$\Rightarrow 2r_2 = 126$

$\Rightarrow r_2 = \frac{126}{2} = 63cms$

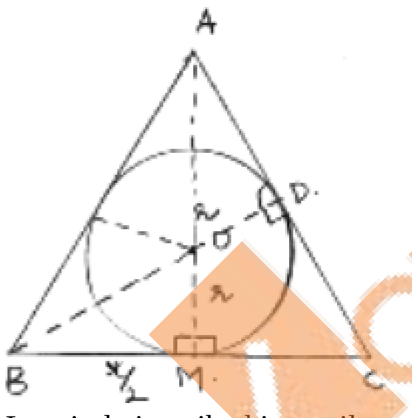
$r_2 = 63cms\ in\ (ii)\ r_1 = 63 + 14 = 77\ cms$

Diameter of circle (i) = $2r_1 = 2 \times 77 = 154\ cms$

Diameter of circle (ii) = $2r_2 = 2 \times 63 = 126\ cms$

12.

Sol:



Let circle inscribed in equilateral triangle

Be with centre O and radius 'r'

Area of circle = πr^2

But given that area = 154 cm2.

$\pi r^2 = 154$

$\frac{22}{7} \times r^2 = 154$

$r^2 = 7 \times 7$

r = 7cms

Radius of circle = 7cms

From fig. at point M, BC side is tangent at point M, BM ⊥ OM. In equilateral triangle, the perpendicular from vertex divides the side into two halves

BM = $\frac{1}{2}BC = \frac{1}{2}(side = x) = \frac{x}{2}$

ΔBMO is right triangle, by Pythagoras theorem

$OB^2 = BM^2 + MO^2$

$OB = \sqrt{r^2 + \frac{x^2}{4}} = \sqrt{49 + \frac{x^2}{4}} \qquad OD = r$

Altitude BD = $\frac{\sqrt{3}}{2}\ (side) = \frac{\sqrt{3}}{2}x = OB + OD$

BD – OD = OB

$\Rightarrow \frac{\sqrt{3}}{2}x - r = \sqrt{49 + \frac{x^2}{4}}$

$\Rightarrow \frac{\sqrt{3}}{2}x - 7 = \sqrt{49 + \frac{x^2}{4}}$

$\Rightarrow \left(\frac{\sqrt{3}}{2}x - 7\right)^2 = \left(\sqrt{\frac{x^2}{4} + 49}\right)^2$

$\Rightarrow \frac{3}{4}x^2 - 7\sqrt{3}x + 49 = \frac{x^2}{4} + 49$

$\Rightarrow \frac{x}{2} = 7\sqrt{3} \Rightarrow x = 14\sqrt{3}\ cm$

Perimeter = 3x = $3 \times 14\sqrt{3}$

= $42\sqrt{3}$ cms

13.

Sol:

Given

Total cost of fencing the circular field = Rs. 2640

Cost per metre fencing = Rs 12

Total cost of fencing = circumference × cost per fencing

⇒ 2640 = circumference × 12

⇒ circumference $= \frac{2640}{12} = 220m$

Let radius of field be r m

Circumference = 2 πr m

$2\pi r = 220$

$2 \times \frac{22}{7} \times r = 220$

$r = \frac{70}{2} = 35m$

Area of field = πr^2

$= \frac{22}{7} \times 35 \times 35$

= 3850 m^2.

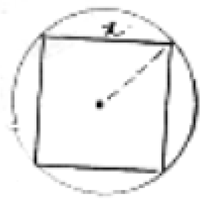
Cost of ploughing per m^2 land = Rs. 0.50

Cost of ploughing 3850 m^2 land = $\frac{1}{2} \times 3850$

= Rs. 1925.

14.

Sol:



Let side of square be x cms inscribed in a circle.

Radius of circle (r) = $\frac{1}{2}(diagonal\ of\ square)$

$= \frac{1}{2}(\sqrt{2}x)$

$= \frac{x}{\sqrt{2}}$

Area of square = $(side)^2 = x^2$

Area of circle = πr^2

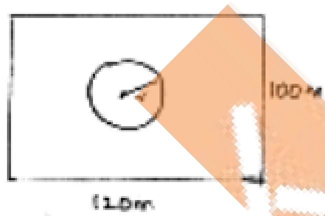
$= \pi\left(\frac{x}{\sqrt{2}}\right)^2$

$= \frac{\pi x^2}{2}$

$\frac{area\ of\ circle}{area\ of\ square} = \frac{\frac{\pi}{2}x^2}{x^2} = \frac{\pi}{2} = \pi : 2$

15.

Sol:



Dimensions of rectangular park length = 120m

Breadth = 100m

Area of park = l × b

$= 120 \times 100 = 12000m^2.$

$Let\ radius\ of\ circular\ lawn\ be\ r$

Area of circular lawn = πr^2

Area of remaining park excluding lawn = (area of park) – (area of circular lawn)

$\Rightarrow 8700 = 12000 - \pi r^2$

$\Rightarrow \pi r^2 = 12000 - 8700 = 3300$

$\Rightarrow \frac{22}{7} \times r^2 = 3300$

$\Rightarrow r^2 = 150 \times 7 = 1050$

$\Rightarrow r = \sqrt{1050} = 5\sqrt{42}\ metres$

∴ radius of circular lawn = $5\sqrt{42}$ metres.

16.

Sol:

Radius of circles are 8cm and 6 cm

Area of circle with radius 8 cm = $\pi(8)^2 = 64\pi cm^2$

Area of circle with radius 6cm = $\pi(6)^2 = 36\pi\ cm^2$

Areas sum = $64\pi + 36\pi = 100\pi$ cm^2

Radius of circle be x cm

Area = πx^2

$\pi x^2 = 100\pi$

$x^2 = 100 \Rightarrow x = \sqrt{100} = 10cm$

17.

Sol:

Radius of 1st circle = 19cm

Radius of 2nd circle = 9 cm

Circumference of 1st circle = 2(19) = 38π cm

Circumference of 2nd circle = 2π (9) = 18π cm

Let radius of required circle = R cm

Circumference of required circle = $2\pi R = c_1 + c_2$

$2\pi R = 38\pi + 18\pi$

$2\pi R = 56\pi$

R = 28 cms

Area of required circle = πr^2

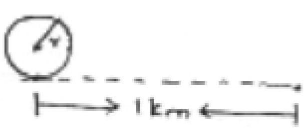
$= \frac{22}{7} \times 28 \times 28$

$= 2464\ cm^2$

18.

Sol:

Let radius of wheel = ‘r’ m



Circumference of wheel = ($2\pi r$)m.

No. of revolutions = 450

Distance for 450 revolutions = $450 \times 2\pi r = 900\pi r\ m$

But distance travelled = 1000 m.

$900\pi r = 1000$

r = 10000 $9\pi \times 100$

$= \frac{10}{9\pi} m$

$= \frac{1000}{9\pi} cms$

$radius\ (r) = \frac{1000}{9\pi}\ cms$

19.

Sol:

Radius of outer circle = $21cm$

Radius of inner circle = R_2

Area between concentric circles = area of outer circle – area of inner circle

$\Rightarrow 770 = \frac{22}{7}(21^2 - {R_2}^2)$

$\Rightarrow 21^2 - {R_2}^2 = 35 \times 7 = 245$

$\Rightarrow 441 - 245 = {R_2}^2$

$\Rightarrow R_2 = \sqrt{196} = 14\ cm$

Radius of inner circle = 14cm.

Exercise 15.2

1.

Sol:

Length of arc = $\frac{\theta}{360°} \times 2\pi r$

Radius = r = 4 cm

θ = angle subtended at centre = 30°

Arc length = $\frac{30°}{360°} \times 2 \times (7)$

= $\frac{2\pi}{3}$ cm

2.

Sol:

Radius (r) = 5 cm

θ = angle subtended at centre (degrees)

Length of Arc = $\frac{\theta}{360°} \times 2\pi$r cm

But arc length = $\frac{5\pi}{3} cm$

$\frac{\theta}{360°} \times 2\pi \times 5 = \frac{5\pi}{3}$

$\theta = \frac{360° \times \pi}{3 \times 2\pi} = 60°$

∴ Angle subtended at centre = 60°

3.

Sol:

Length of arc = 20πcm

Let radius = 'r' cm

O = angle subtended at centre = 144°

Length of arc = $\frac{\theta}{360°} \times 2\pi r$

= $\frac{144}{360} \times 2\pi r = \frac{4\pi}{5} r$

$= \frac{4\pi}{5} r = 20\pi$

$r = \frac{20\pi \times 5}{4\pi} = 25\ cms$

4.

Sol:

Length of arc = 15cm

θ = angle subtended at centre = 45°

Let radius = r cm

arc length = $\frac{\theta}{360°} \times 2\pi r$

= $\frac{45°}{360°} \times 2\pi r$

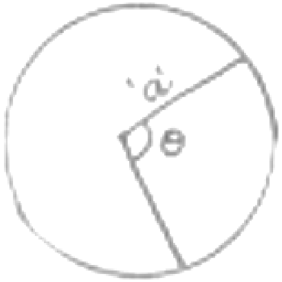
$\frac{45}{360} \times 2\pi r = 15$

$r = \frac{15 \times 360}{45 \times 2\pi} = \frac{60}{\pi}$ cms

$Radius = \frac{60}{\pi} cms$

5.

Sol:



Length of arc = $\frac{a\pi}{4}\ cm$

Radius r = 'a' cm

θ = angle subtended at centre

arc length = $\frac{\theta}{360°} \times 2\pi r$

= $\frac{\theta}{360°} \times 2\pi a$

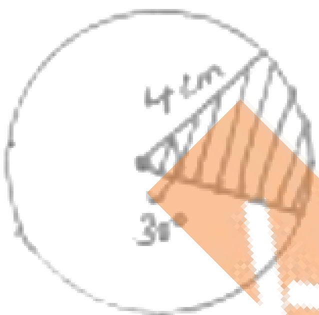
$\therefore \frac{\theta}{360°} \times 2\pi a = \frac{a\pi}{4}$

$\Rightarrow \theta = \frac{9\pi \times 360°}{4 \times 2\pi a} = 45°$

6.

Sol:

Radius = 4 cm = r



Angle subtended at centre = θ = 30°

Area of sector (shaded region)

$= \frac{\theta}{360°} \times \pi r^2$

$= \frac{30}{360} \times \frac{22}{7} \times 4 \times 4$

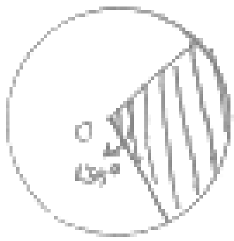
$= \frac{88}{21} cm^2$

$\therefore area\ of\ required\ sector = \frac{88}{21} cm^2$

7.

Sol:

Radius (r) = 8cm



θ = angle subtended at centre = 135°

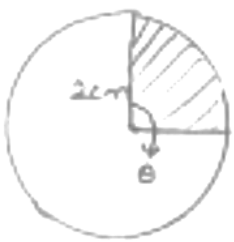
Area of sector = $\frac{x}{360°} \times \pi r^2$

$= \frac{135}{360} \times \frac{22}{7} \times 8 \times 8$

$= \frac{528}{7} cm^2$

8.

Sol:



Area of sector = $\pi\ cm^2$

Radius of circle = $2cm$

Let θ = angle subtended by arc at centre

Area of sector = $\frac{\theta}{360°} \times \pi r^2$

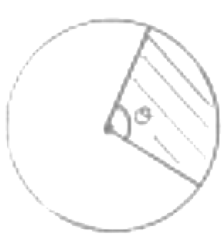
$= \frac{\theta}{360°} \times \pi \times 2 \times 2$

$= \frac{\pi\theta}{90°}$

$\frac{\pi\theta}{90°} = \pi \Rightarrow \theta = 90°$

9.

Sol:



Area of sector = 5π cm^2.

Radius (r) = 5cm

Let θ = angle subtended at centre area of sector = $\frac{\theta}{360°} \times \pi r^2$

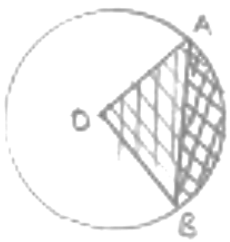
$= \frac{\theta}{360} \times \pi \times 5 \times 5 = \frac{5\pi\theta}{72°}$

$= \frac{5\pi\theta}{72°} = 5\pi$

$\Rightarrow \theta = 72°$

10.

Sol:



AB is chord AB = 4cm

OA = OB = 4cm

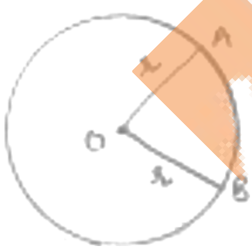
OAB is equilateral triangle ∠AOB = 60°

Area of sector (formed by chord [shaded region]) = (area of sector)

$= \frac{\theta}{360°} \times \pi r^2 = \frac{60}{360} \times \pi \times 4 \times 4 = \frac{8\pi}{3} cm^2$

11.

Sol:



Radius (r) = 35 cm

θ = angle subtended at centre = 72°

Length of arc = $\frac{\theta}{360°} \times 2\pi r$

$= \frac{72}{360} \times 2 \times \frac{22}{7} \times 35$

$= 2 \times 22 = 44 cms$

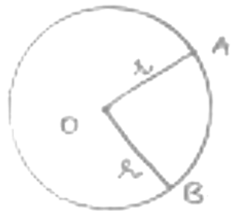
Area of sector = $\frac{\theta}{360°} \times \pi r^2$

$= \frac{72}{360} \times \frac{22}{7} \times 35 \times 35$

$= (35 \times 22) = 770\ cm^2$

12.

Sol:



Radius = OA = OB (From fig) = r

= 5.7 m

Perimeter = 27.2 m

Let angle subtended at centre = θ

Perimeter = $\left(\frac{\theta}{360°} \times 2\pi r\right) + OA + OB$

$= \frac{\theta}{360°} \times 2(5.7) \times \pi + 2(5.7)$

$= \frac{2\pi(5.7)\theta}{360°} + 11.4$

$= \frac{\pi(5.7)\theta}{180°} + 11.4 = 27.2$

$= \frac{\pi(5.7)\theta}{180°} = 15.8$

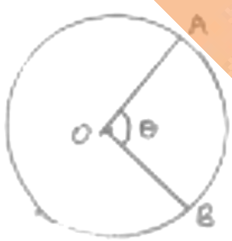
$Area\ of\ sector\ = \frac{\theta}{360°} \times \pi r^2$

$= \frac{158.8}{360} \times \frac{22}{7} \times 5.7 \times 5.7$

$= 45.048\ cm^2$

13.

Sol:



θ = angle subtended at centre

Radius (r) = 5.6m = OA $\pm$ OB

Perimeter of sector = 27.2 m

(AB arc length) + OA + OB = 27.2

$\Rightarrow \left(\frac{\theta}{360°} \times 2\pi r\right) + 5.6 + 5.6 \pm 27.2$

$\Rightarrow \frac{5.6\ \pi\ \theta}{180°} + 11.2 = 27.2$

$\Rightarrow 5.6 \times \frac{22}{7} \times \theta = 16 \times 180$

$\Rightarrow \theta = \frac{16 \times 180}{0.8 \times 22} = 163.64°$

Area of sector = $\frac{\theta}{360°} \times \pi r^2 = \frac{163.64°}{360°} \times \frac{22}{7} \times 5.6 \times 5.6$

$= \frac{163.64}{180} \times 11 \times 0.8 \times 5.6$

$= 44.8\ cm^2$

14.

Sol:

Radius of circle (r) = 21 cm

θ = angle subtended at centre = 120°

Length of its arc = $\frac{\theta}{360°} \times 2\pi r$

$= \frac{120}{360} \times 2 \times \frac{22}{7} \times 21$

= 44 cms

Area of sector = $\frac{\theta}{360°} \times \pi r^2$

$= \frac{120}{360} \times \frac{22}{7} \times 21 \times 21$

$= (22 \times 21)$

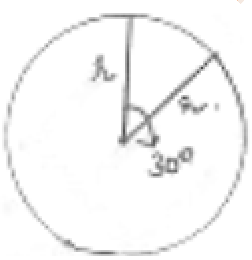
$= 462\ cm^2$

$Length\ of\ arc = 44\ cm$

Area of sector = 462 cm^2

15.

Sol:



Radius of minute hand (r) = $\sqrt{21}\ cm$

For 1hr = 60 min, minute hand completes one revolution = 360°

60 min = 360°

1 min = 6°

From 7 am to 7:05 am it is 5 min angle subtended = $5 \times 6° = 30° = \theta$

Area described = $\frac{\theta}{360°} \times \pi r^2$

$= \frac{30}{360} \times \frac{22}{7} \times 21$

$= \frac{22}{4} = 5.5 cm^2$

16.

Sol:

Radius of minute hand (r) = 10 cm

For 1 hr = 60 min, minute hand completes one revolution = 360°

60 min = 360°

1 min = 6°

From 8 am to 8:25 am it is 25 min angle subtended = 6° × 25 = 150° = θ

Area described = $\frac{\theta}{360°} \times \pi r^2$

$= \frac{150}{360} \times \frac{22}{7} \times 10 \times 10$

$= \frac{250 \times 11}{3}$

$= \frac{2750}{3}\ cm^2$

17.

Sol:

Angle subtended by sector at centre θ = 56°

Let radius be 'x' cm

Area of sector = $\frac{\theta}{360°} \times \pi r^2$

$= \frac{56}{360} \times \frac{22}{7} \times r^2$

$= \frac{22}{45} r^2$

But area of sector = 4.4cm^2 = $\frac{44}{10} cm^2$

$\frac{22}{45}\ r^2 = \frac{44}{10}$

$\Rightarrow r^2 = \frac{45 \times 44}{22 \times 10} = 9$

$\Rightarrow r = \sqrt{9}$

$= 3\ cm$

∴ radius (r) = 3cm

18.

Sol:

(i) Radius of circle (r) = 6 cm

Angle subtended at the centre = 110°

Circumference of the circle = $2\pi r$

$= 2 \times \frac{22}{7} \times 6$

$= \frac{264}{7}\ cm$

(ii) Area of circle = $\pi r^2 = \frac{22}{7} \times 6 \times 6$

$= \frac{792}{7}\ cm^2$

(iii) Length of arc = $\frac{\theta}{360°} \times 2\pi r$

$= \frac{110}{360} \times 2 \times \frac{22}{7} \times 6$

$= \frac{232}{21}\ cm$

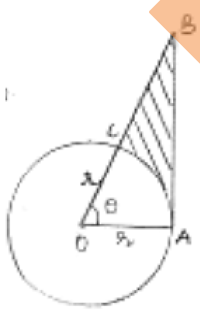
(iv) Area of sector = $\frac{\theta}{360°} \times \pi r^2$

$= \frac{110}{360} \times \frac{22}{7} \times 6 \times 6$

$= \frac{232}{7}\ cm^2$

19.

Sol:



Given angle subtended at centre of circle = θ

∠OAB = 90° [At joint of contact, tangent is perpendicular to radius]

OAB is right angle triangle

Cos $\theta = \frac{adj.side}{hypotenuse} = \frac{r}{OB} \Rightarrow OB = r\sec\theta \ldots\ldots (i)$

$\tan\theta = \frac{opp.side}{adj.\ side} = \frac{AB}{r} \Rightarrow AB = r\tan\theta \ldots\ldots (ii)$

Perimeter of shaded region = AB + BC + (CA arc)

$= r\tan\theta + (OB - OC) + \frac{\theta}{360°} \times 2\pi r$

$= r\tan\theta + r\sec\theta - r + \frac{\pi\,\theta\,r}{180°}$

$= r\left(\tan\theta + \sec\theta + \frac{\pi\theta}{180°} - 1\right)$

Area of shaded region = (area of triangle) – (area of sector)

$= \left(\frac{1}{2} \times OA \times AB\right) - \frac{\theta}{360°} \times \pi r^2$

$= \frac{1}{2} \times r \times r\tan\theta - \frac{r^2}{2}\left[\frac{\theta}{180°} \times \pi\right]$

$= \frac{r^2}{2}\left[\tan\theta - \frac{\pi\theta}{180}\right]$

20.

Sol:

(i) Radius of circle = ‘r’ cm

Angle subtended at centre = θ

Perimeter = OA + OB + (AB arc)

$= r + r + \frac{\theta}{360°} \times 2\pi r = 2r + 2r\left[\frac{\pi\theta}{360°}\right]$

But perimeter given as 50

$50 = 2r\left[1 + \frac{\pi\theta}{360°}\right]$

$\Rightarrow \frac{\pi\theta}{360°} = \frac{50}{2r} - 1$

$\Rightarrow \theta = \frac{360°}{\pi}\left[\frac{25}{r} - 1\right]$ …..(i)

(ii) Area of sector $= \frac{\theta}{360°} \times \pi r^2$

$= \frac{\frac{360°}{\pi}\left(\frac{25}{r} - 1\right)}{360°} \times \pi r^2$

$= \frac{25}{r} \times r^2 - r^2$

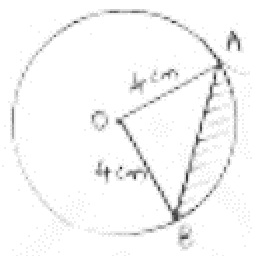
$= 25r - r^2$

$\Rightarrow A = 25r - r^2$ …..(ii)

Exercise 15.3

1.

Sol:



Radius of circle r = 4cm = OA = OB

Length of chord AB = 4cm

OAB is equilateral triangle ∠AOB = 60° ⟶ θ

Angle subtended at centre θ = 60°

Area of segment (shaded region) = (area of sector) – (area of ΔAOB)

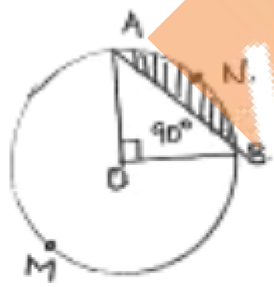
$= \frac{\theta}{360°} \times \pi r^2 = \frac{\sqrt{3}}{4}(side)^2$

$= \frac{60}{360} \times \frac{22}{7} \times 4 \times 4 = \frac{\sqrt{2}}{4} \times 4 \times 4$

$= \frac{176}{3} - 4\sqrt{3} = 58.67 - 6.92 = 51.75\ cm^2$

2.

Sol:



Radius (r) = 14cm

θ = 90°

= OA = OB

Area of minor segment (ANB)

= $(area\ of\ ANB\ sector) - (area\ of\ \Delta AOB)$

= $\frac{\theta}{360°} \times \pi r^2 - \frac{1}{2} \times OA \times OB$

= $\frac{90}{360} \times \frac{22}{7} \times 14 \times 14 - \frac{1}{2} \times 14 \times 14$

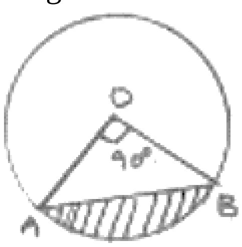
= $154 - 98 = 56cm^2$

Area of major segment (other than shaded)

= area of circle – area of segment ANB
$= \pi r^2 - 56$
$= \frac{22}{7} \times 14 \times 14 - 56$
= 616 – 56
= 560 cm^2.

3.

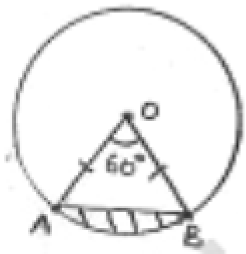
Sol:
Given radius = r = $5\sqrt{2}$ cm = OA = OB
Length of chord AB = 10cm



In ΔOAB, OA = OB = $5\sqrt{2}\ cm$ AB = 10cm
$OA^2 + OB^2 = (5\sqrt{2})^2 + (5\sqrt{2})^2 = 50 + 50 = 100 = (AB)^2$
Pythagoras theorem is satisfied OAB is right triangle
θ = angle subtended by chord = ∠AOB = 90°
Area of segment (minor) = shaded region
$= area\ of\ sector - area\ of\ \Delta OAB$
$= \frac{\theta}{360} \times \pi r^2 - \frac{1}{2} \times OA \times OB$
$= \frac{90}{360} \times \frac{22}{7} (5\sqrt{2})^2 - \frac{1}{2} \times 5\sqrt{2} \times 5\sqrt{2}$
$= \frac{275}{7} - 25 - \frac{100}{7}\ cm^2$
Area of major segment = (area of circle) – (area of minor segment)
$= \pi r^2 2 - \frac{100}{7}$
$= \frac{22}{7} \times (5\sqrt{2})^2 - \frac{100}{7}$
$= \frac{1100}{7} - \frac{100}{7} = \frac{1000}{7}\ cm^2$

4.

Sol:



Given radius (r) = 14cm = OA = OB

θ = angle at centre = 60°

In ΔAOB, ∠A = ∠B [angles opposite to equal sides OA and OB] = x

By angle sum property ∠A + ∠B + ∠O = 180°

x + x + 60° = 180° ⇒ 2x = 120° ⇒ x = 60°

All angles are 60°, OAB is equilateral OA = OB = AB

Area of segment = area of sector – area Δle OAB

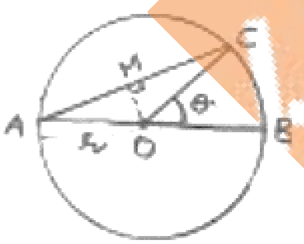
$= \frac{\theta}{360°} \times \pi r^2 - \frac{\sqrt{3}}{4} \times (-AB)^2$

$= \frac{60}{360} \times \frac{22}{7} \times 14 \times 14 - \frac{\sqrt{3}}{4} \times 14 \times 14$

$= \frac{308}{3} - 49\sqrt{3} = \frac{308-147\sqrt{3}}{3}\ cm^2$

5.

Sol:



Given AB is diameter of circle with centre O

∠COB = θ

Area of sector BOC = $\frac{\theta}{360°} \times \pi r^2$

Area of segment cut off, by AC = (area of sector) – (area of ΔAOC)

∠AOC = 180 – θ [∠AOC and ∠BOC form linear pair]

Area of sector = $\frac{(180-\theta)}{360°} \times \pi r^2 = \frac{\pi r^2}{2} - \frac{\pi\theta r^2}{360°}$

In ΔAOC, drop a perpendicular AM, this bisects ∠AOC and side AC.

Now, In ΔAMO, sin∠AOM = $\frac{AM}{DA} \Rightarrow \sin\left(\frac{180-\theta}{2}\right) = \frac{AM}{R}$

$\Rightarrow$ AM = R $\sin\left(90-\frac{\theta}{2}\right) = R.\cos\frac{\theta}{2}$

$\cos\ \angle ADM = \frac{OM}{OA}\ \Rightarrow \cos\left(90-\frac{\theta}{2}\right) = \frac{OM}{Y} \Rightarrow OM = R.Sin\frac{\theta}{2}$

Area of segment = $\frac{\pi r^2}{2} - \frac{\pi\theta r^2}{360°} - \frac{1}{2}(AC \times OM)\ [AC = 2\ AM]$

$= \frac{\pi r^2}{2} - \frac{\pi\theta r^2}{360°} - \frac{1}{2} \times \left(2\ R\cos\frac{\theta}{2}\ R\sin\frac{\theta}{2}\right)$

$= r^2\left[\frac{\pi}{2} - \frac{\pi\theta}{360°} - \cos\frac{\theta}{2}\sin\frac{\theta}{2}\right]$

Area of segment by AC = 2 (Area of sector BDC)

$r^2\left[\frac{\pi}{2} - \frac{\pi\theta}{360°} - \cos\frac{\theta}{2}\ .\sin\frac{\theta}{2}\right] = 2r^2\left[\frac{\pi\theta}{360°}\right]$

$\cos\frac{\theta}{2}\ .\sin\frac{\theta}{2} = \frac{\pi}{2} - \frac{\pi\theta}{360} - \frac{2\pi\theta}{360°}$

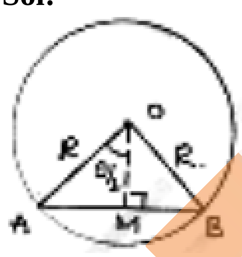
$= \frac{\pi}{2} - \frac{\pi\theta}{360°}[1+2]$

$= \frac{\pi}{2} - \frac{\pi\theta}{360°} = \pi\left(\frac{1}{2} - \frac{\theta}{120°}\right)$

$\cos\frac{\theta}{2}\ .\sin\frac{\theta}{2} = \ \pi\ \left(\frac{1}{2} - \frac{\theta}{120°}\right)$

6.

Sol:



Let radius of circle = r

Area of circle = πr^2

AB is a chord, OA, OB are joined drop OM ⊥ AB. This OM bisects AB as well as ∠AOB.

$\angle AOM = \angle MOB = \frac{1}{2}(0) = \frac{\theta}{2}$ AB = 2AM

In ΔAOM, ∠AMO = 90°

$\text{Sin}\frac{\theta}{2} = \frac{AM}{AD} \Rightarrow AM = R.\sin\frac{\theta}{2}$ AB = 2R $\sin\frac{\theta}{2}$

$\text{Cos}\frac{\theta}{2} = \frac{OM}{AD} \Rightarrow OM = R\cos\frac{\theta}{2}$

Area of segment cut off by AB = (area of sector) – (area of triangles)

$= \frac{\theta}{360} \times \pi r^2 - \frac{1}{2} \times AB\ \times OM$

$= r^2\left[\frac{\pi\theta}{360°} - \frac{1}{2}.2rsin\frac{\theta}{2}\ .R\cos\frac{\theta}{2}\right]$

$= R^2\left[\frac{\pi\theta}{360°} - \sin\frac{\theta}{2}.\cos\frac{\theta}{2}\right]$

Area of segment = $\frac{1}{2}(area\ of\ circle)$

$r^2\left[\frac{\pi\theta}{360}-\sin\frac{\theta}{2}.\cos\frac{\theta}{2}\right]=\frac{1}{8}\pi r^2$

$\frac{8\pi\theta}{360°}-8\sin\frac{\theta}{2}.\cos\frac{\theta}{2}=\pi$

$8\sin\frac{\theta}{2}.\cos\frac{\theta}{2}+\pi=\frac{\pi\theta}{45}$

Exercise 15.4

1.

Sol:

Given AB = 60m = DC [length]

BC = 28m = AD [breadth]

Radius of semicircle r = $\frac{1}{2}\times BC=14m$

Area of semicircle r = $\frac{1}{2}\times BC=14m$

Area of plot = (Area of rectangle ABCD) + (area of semicircle)

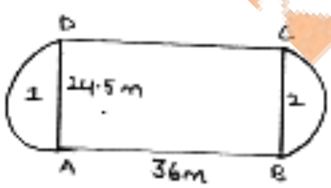
= $(length\ \times breadth)+\frac{1}{2}\pi r^2$

= $(60\times 28)+\left[\frac{1}{2}\times\frac{22}{7}\times 14\times 14\right]$

= $1680+308=1988m^2$

2.

Sol:



Let rectangular play area be ABCD

AB = CD = 36m [length]

AD = BC = 24.5 m [breadth]

Radius of the semicircle = $\frac{1}{2}(BC)=R$

= $\frac{1}{2}\times(24.5)=12.25cm$

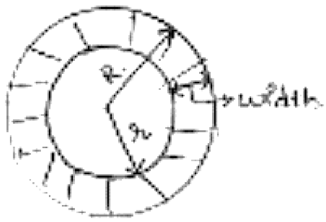
Area of playground = (Area of rectangle) + 2(Area of semicircle)

= $(AB\times BC)+\left(\frac{1}{2}\pi r^2\right)2$

$= (36 \times 24.5) + \left(\frac{1}{2} \times \frac{22}{7} \times 12.25 \times 12.25\right) 2$
$= 882 + 471.625$
$= 1353.625\ m^2$

3.

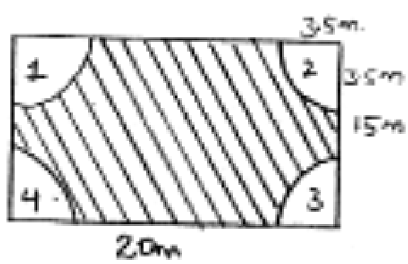
Sol:



Let inner radius = r width(d) = 14m
Outer radius = R
Outer circumference of track = 2 πr
$\therefore 2\ \pi r = 528$
$2 \times \frac{22}{7} \times R = 528 \Rightarrow R = \frac{528\times7}{2\times22} = 84\ m$
Inner radius r = R – d = 84 – 14 = 70m
Area of track = (area of outer circle) – (area of inner circles)
$= \pi R^2 - \pi r^2$
$= \pi(R^2 - r^2) = \frac{22}{7}(84^2 - 70^2)$
$= \frac{22}{7}(84 + 70)(84 - 70) = \frac{22}{7} \times 154 \times 14$
$= 6776\ m^2$
Cost of leveling $m^2 = Rs. 0.50$
Total cost of leveling track = $6776 \times \frac{1}{2} = Rs. 3388$

4.

Sol:



Length of rectangular piece l = 20m
Breadth of rectangular piece b = 15m
Radius of each quadrant r = 3.5m
Area of rectangular piece = (length × breadth) = 20 × 15 = 300m^2.

Area of quadrant each = $\frac{1}{4}(area\ of\ circle\ with\ radius\ 3.5m)$

= $\frac{1}{4} \times \pi r^2$

= $\frac{1}{4} \times \frac{22}{7} \times 3.5 \times 3.5 = \frac{38.5}{4} m^2$

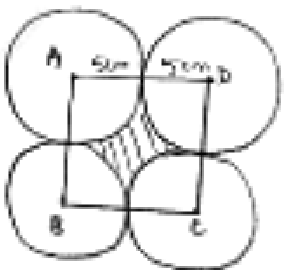
Area of remaining part = [area of rectangular piece] – 4[area of each quadrant]

= 300 – 4 [$\frac{385}{4}$] = 300 – 38.5

= $261.5m^2$

5.

Sol:



Area required shaded = (area of square ABCD) – (Area of 4 quadrant)

Side of square = 5cm + 5cm

= 10cm

Area of square = side × side

= $10cm \times 10cm = 100cm^2$

Area of quadrant = $\frac{1}{4}(area\ of\ circle\ with\ radius\ 5\ cm)$

= $\frac{1}{4} \times \pi r^2$

= $\frac{1}{4} \times \frac{22}{7} \times 5 \times 5 = (25 \times 3.14)\frac{1}{4}\ cm^2$

Area included between circles = (area of square) – 4(area of quadrant)

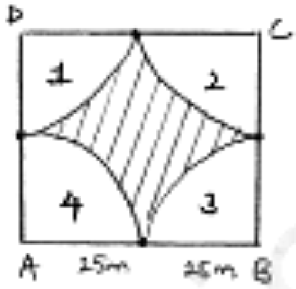
= 100 – $\left(\frac{1}{4} \times 25 \times 2.14\right)$

= 100 – 78.5

= $21.5cm^2$

6.

Sol:



Side of square plot (s) = 50m

Area grazed by four cows is area of sectors represented by 1, 2, 3 and 4.

Radius of each quadrant = 25m = r.

Area of square plot = $s^2 = 50^2 = 2500m^2$

Area of each quadrant = $\frac{1}{4}\pi r^2 = \frac{1}{4} \times \frac{22}{7} \times 25 \times 25 = (625 \times 3.14) \times \frac{1}{4}$

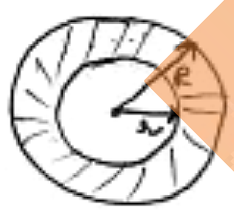
Area of ungrazed land = (area of square plot) – 4(area of quadrant)

= $2500 - 4\left(\frac{1}{4} \times 3.14 \times 625\right)$

= 2500 – 1962.5 = 537.5 m^2

7.

Sol:



Outer radius of road = R

Inner radius of road = r

Width of park road = d

R = 2 + d

Circumference of road (outer) = $2\pi R$

$2\pi R$ = 352 [from problem given]

$2 \times \frac{22}{7} \times R = 352$

R = $\frac{352 \times 7}{2 \times 22} = 56m.$

Inner radius = R – d = 56 – 7 = 49 m

Area of road = (area of circle with radius 56m) – (area of circle with radius 49m)

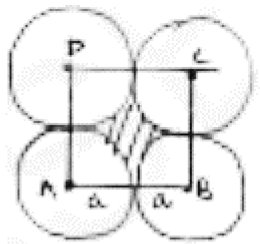
= $\pi R^2 - \pi r^2$

$= \frac{22}{7}(56^2 - 49^2) = \frac{22}{7}(56 - 49)\ (56 + 49)$

$= \frac{22}{7} \times 7 \times 105 = 2310m^2$

8.

Sol:



Let circles be with centres A, B, C, D

Join A, B, C and D then ABCD is square formed with side = (a + a) = 2a

Radius = a

Area between circles = area of square – 4(area of quadrant)

(shaded region)

$= (2a)^2 - 4\left(\frac{1}{4}\ area\ of\ circle\ with\ radius\ 'a'\right)$

$= 4a^2 - 4\left(\frac{1}{4}\right) \times a^2$

$= a^2(4 - \pi)$

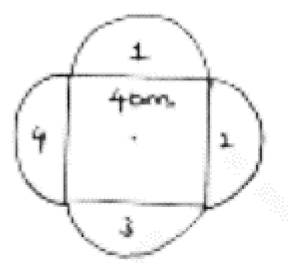
$= a^2\left(4 - \frac{22}{7}\right)$

$= \left(\frac{28-22}{7}\right)a^2 = \frac{6}{7}\ a^2$

∴ Area between circles $= \frac{6}{7}\ a^2$.

9.

Sol:



Side of water tank = 40m

Grassy plot is semicircular with radius $= \frac{side}{2} = \frac{40}{2} = 20m = r$

Area of grassy plot = 4(area od semicircular grassy plot with radius 20m)

$= 4\left[\frac{1}{2}\ (area\ of\ circle\ with\ radius)\right]$

$= 4 \times \frac{1}{2} \times \pi(20)^2$

$= 2 \times 20 \times 20 \times \pi = 800\pi\ m^2$.

Cost of turfing $1m^2$ = Rs. 1.25

Total cost of turfing the grassy plot around tank

$= 800\pi \times 1.25$

$= 1000\pi$

$= 1000 \times 3.14$

= Rs. 3140.