

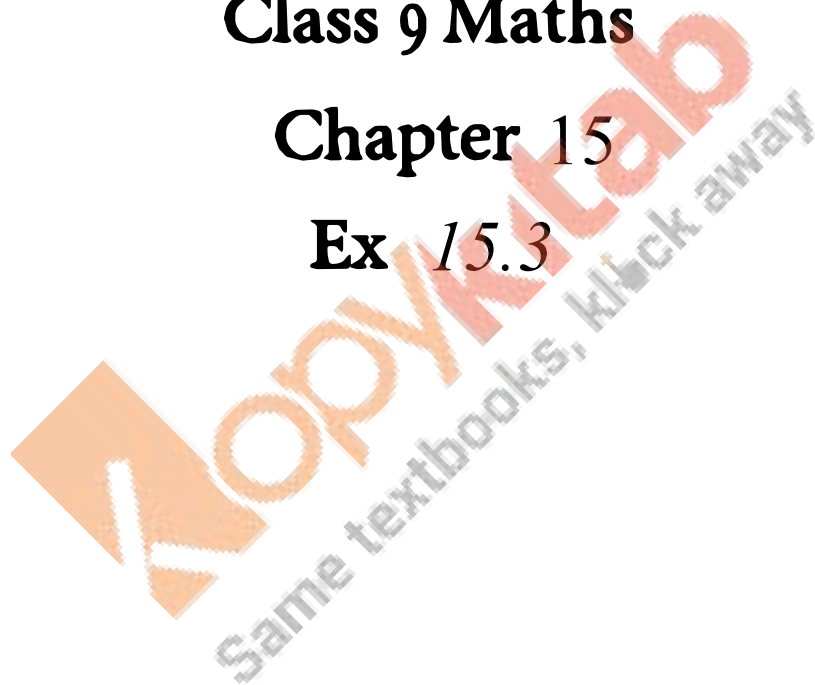
**RD SHARMA**

**Solutions**

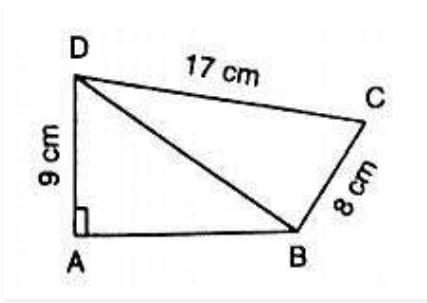
**Class 9 Maths**

**Chapter 15**

**Ex 15.3**



**Q 1. In figure, compute the area of quadrilateral ABCD.**



**Solution:**

Given:

DC = 17 cm, AD = 9 cm and BC = 8 cm

In  $\triangle BCD$  we have

$$CD^2 = BD^2 + BC^2$$

$$\Rightarrow 17^2 = BD^2 + 8^2$$

$$\Rightarrow BD^2 = 289 - 64$$

$$\Rightarrow BD = 15$$

In  $\triangle ABD$  we have

$$AB^2 + AD^2 = BD^2$$

$$\Rightarrow 15^2 = AB^2 + 9^2$$

$$\Rightarrow AB^2 = 225 - 81 = 144$$

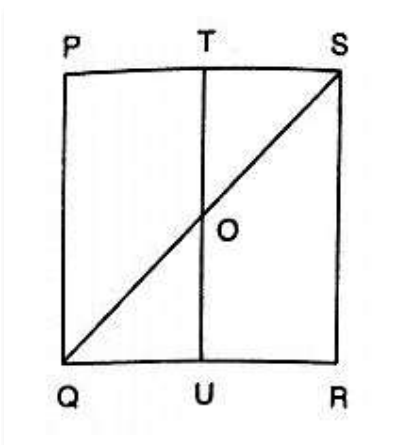
$$\Rightarrow AB = 12$$

$$\text{ar}(\text{quad } ABCD) = \text{ar}(\triangle ABD) + \text{ar}(\triangle BCD)$$

$$\text{ar}(\text{quad } ABCD) = \frac{1}{2}(12 \times 9) + \frac{1}{2}(8 \times 17) = 54 + 68 = 122 \text{ cm}^2$$

$$\text{ar}(\text{quad } ABCD) = \frac{1}{2}(12 \times 9) + \frac{1}{2}(8 \times 15) = 54 + 60 = 114 \text{ cm}^2$$

**Q2. In figure, PQRS is a square and T and U are, respectively, the midpoints of PS and QR. Find the area of  $\triangle OTS$  if PQ = 8 cm.**



**Solution:**

From the figure,

T and U are mid points of PS and QR respectively

$$\therefore TU \parallel PQ$$

$$\Rightarrow TO \parallel PQ$$

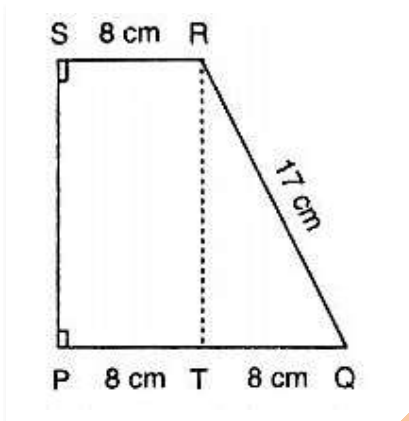
Thus, in  $\Delta PQS$ , T is the mid point of PS and  $TO \parallel PQ$

$$\therefore TO = \frac{1}{2}PQ = 4 \text{ cm}$$

$$\text{Also, } TS = \frac{1}{2}PS = 4 \text{ cm}$$

$$\therefore \text{ar}(\Delta OTS) = \frac{1}{2}(TO \times TS) = \frac{1}{2}(4 \times 4) \text{ cm}^2 = 8 \text{ cm}^2$$

**Q3. Compute the area of trapezium PQRS in figure**

**Solution:**

We have,

$$\text{ar}(\text{trap. PQRS}) = \text{ar}(\text{rect. PSRT}) + \text{ar}(\Delta QRT)$$

$$\Rightarrow \text{ar}(\text{trap. PQRS}) = PT \times RT + \frac{1}{2}(QT \times RT)$$

$$= 8 \times RT + \frac{1}{2}(8 \times RT) = 12 \times RT$$

In  $\Delta QRT$ , we have

$$QR^2 = QT^2 + RT^2$$

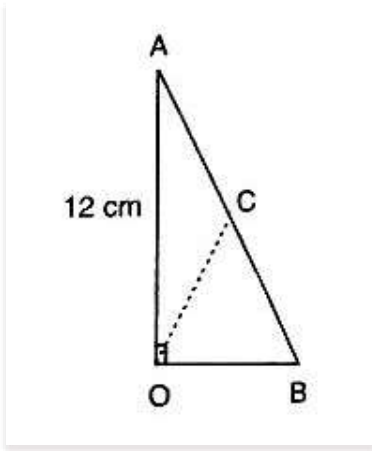
$$\Rightarrow RT^2 = QR^2 - QT^2$$

$$\Rightarrow RT^2 = 17^2 - 8^2 = 225$$

$$\Rightarrow RT = 15$$

$$\text{Hence, area of trapezium} = 12 \times 15 \text{ cm}^2 = 180 \text{ cm}^2$$

**Q4. In figure,  $\angle AOB = 90^\circ$ ,  $AC = BC$ ,  $OA = 12 \text{ cm}$  and  $OC = 6.5 \text{ cm}$ . Find the area of  $\Delta AOB$ .**



**Solution:**

Since, the midpoint of the hypotenuse of a right triangle is equidistant from the vertices

$$\therefore CA = CB = OC$$

$$\Rightarrow CA = CB = 6.5 \text{ cm}$$

$$\Rightarrow AB = 13 \text{ cm}$$

In right angled triangle OAB , we have

$$AB^2 = OB^2 + OA^2$$

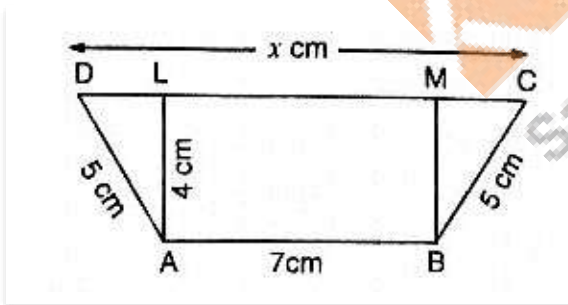
$$\Rightarrow 13^2 = OB^2 + 12^2$$

$$\Rightarrow OB^2 = 13^2 - 12^2 = 169 - 144 = 25$$

$$\Rightarrow OB = 5$$

$$\therefore \text{ar}(\triangle AOB) = \frac{1}{2}(12 \times 5) = 30 \text{ cm}^2$$

**Q5. In figure, ABCD is a trapezium in which  $AB = 7 \text{ cm}$ ,  $AD = BC = 5 \text{ cm}$ ,  $DC = x \text{ cm}$ , and distance between  $AB$  and  $DC$  is  $4 \text{ cm}$ . Find the value of  $x$  and area of trapezium ABCD.**



**Solution:**

Draw  $AL \perp DC$ ,  $BM \perp DC$  then ,

$AL = BM = 4 \text{ cm}$  and  $LM = 7 \text{ cm}$ .

In  $\triangle ADL$  , we have

$$AD^2 = AL^2 + DL^2$$

$$\Rightarrow 25 = 16 + DL^2$$

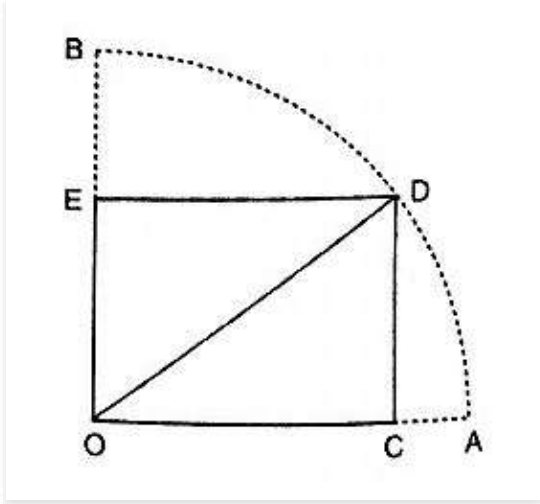
$$\Rightarrow DL = 3 \text{ cm}$$

Similarly,  $MC = \sqrt{BC^2 - BM^2} = \sqrt{25 - 16} = 3 \text{ cm}$

$\therefore x = CD = CM + ML + LD = (3 + 7 + 3) \text{ cm} = 13 \text{ cm}$

$\text{ar}(\text{trap. } ABCD) = \frac{1}{2}(AB + CD) \times AL = \frac{1}{2}(7 + 13) \times 4 \text{ cm}^2 = 40 \text{ cm}^2$

**Q 6.** In figure, OCDE is a rectangle inscribed in a quadrant of a circle of radius 10 cm. If  $OE = 2\sqrt{5} \text{ cm}$ , find the area of the rectangle.



**Solution:**

Given  $OD = 10 \text{ cm}$  and  $OE = 2\sqrt{5} \text{ cm}$

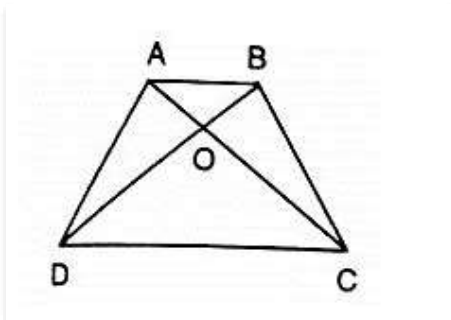
By using Pythagoras theorem

$$\therefore OD^2 = OE^2 + DE^2$$

$$\Rightarrow DE = \sqrt{OD^2 - OE^2} = \sqrt{10^2 - (2\sqrt{5})^2} = 4\sqrt{5} \text{ cm}$$

$$\therefore \text{Area of rectangle } OCDE = OE \times DE = 2\sqrt{5} \times 4\sqrt{5} \text{ cm}^2 = 40 \text{ cm}^2$$

**Q 7.** In figure, ABCD is a trapezium in which  $AB \parallel DC$ . Prove that  $\text{ar}(\triangle AOD) = \text{ar}(\triangle BOC)$



**Solution:**

Given: ABCD is a trapezium in which  $AB \parallel DC$

To prove:  $\text{ar}(\triangle AOD) = \text{ar}(\triangle BOC)$

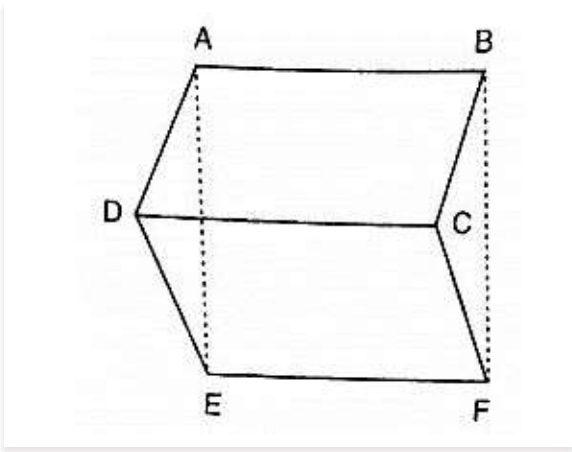
Proof:- Since,  $\triangle ADC$  and  $\triangle BDC$  are on the same base DC and between same parallels AB and DC

Then,  $\text{ar}(\triangle ADC) = \text{ar}(\triangle BDC)$

$$\Rightarrow \text{ar}(\triangle AOD) + \text{ar}(\triangle DOC) = \text{ar}(\triangle BOC) + \text{ar}(\triangle DOC)$$

$$\Rightarrow \text{ar}(\triangle AOD) = \text{ar}(\triangle BOC)$$

**Q 8.** In figure,  $ABCD$ ,  $ABFE$  and  $CDEF$  are parallelograms. Prove that  $\text{ar}(\triangle ADE) = \text{ar}(\triangle BCF)$ .



**Solution:**

Given that

$ABCD$  is parallelogram  $\Rightarrow AD = BC$

$CDEF$  is parallelogram  $\Rightarrow DE = CF$

$ABFE$  is parallelogram  $\Rightarrow AE = BF$

Thus, in  $\triangle$ s  $ADF$  and  $BCF$ , we have

$AD = BC$ ,  $DE = CF$  and  $AE = BF$

So, by SSS criterion of congruence, we have

$\triangle ADE \cong \triangle BCF$

$\text{ar}(\triangle ADE) = \text{ar}(\triangle BCF)$

**Q 9.** Diagonals  $AC$  and  $BD$  of a quadrilateral  $ABCD$  intersect each other at  $P$ . Show that :

$\text{ar}(\triangle APB) \times \text{ar}(\triangle CPD) = \text{ar}(\triangle APD) \times \text{ar}(\triangle BPC)$ .

**Solution:**

Construction: – Draw  $BQ \perp AC$  and  $DR \perp AC$

Proof:-

L.H.S

$$= \text{ar}(\triangle APB) \times \text{ar}(\triangle CDP)$$

$$= \frac{1}{2}[(AP \times BQ)] \times \left(\frac{1}{2} \times PC \times DR\right)$$

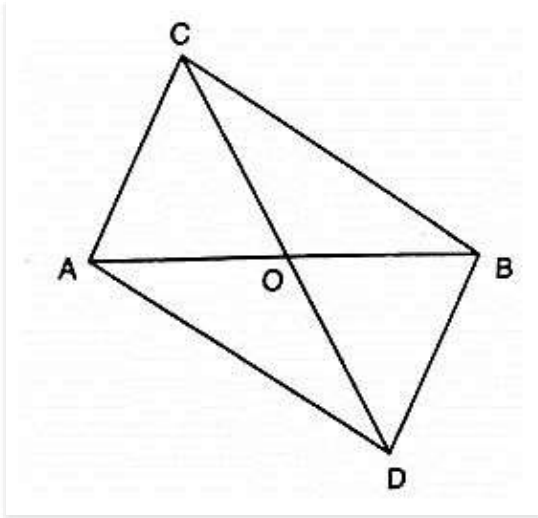
$$= \left(\frac{1}{2} \times PC \times BQ\right) \times \left(\frac{1}{2} \times AP \times DR\right)$$

$$= \text{ar}(\triangle APD) \times \text{ar}(\triangle BPC).$$

= R.H.S

Hence proved.

**Q 10.** In figure, ABC and ABD are two triangles on the base AB. If line segment CD is bisected by AB at O, show that  $\text{ar}(\Delta ABC) = \text{ar}(\Delta ABD)$ .



**Solution:**

Given that CD is bisected by AB at O

To prove:  $\text{ar}(\Delta ABC) = \text{ar}(\Delta ABD)$ .

Construction: Draw  $CP \perp AB$  and  $DQ \perp AB$ .

Proof:

$$\text{ar}(\Delta ABC) = \frac{1}{2} \times AB \times CP \dots\dots\dots (1)$$

$$\text{ar}(\Delta ABD) = \frac{1}{2} \times AB \times DQ \dots\dots\dots (2)$$

In  $\Delta CPO$  and  $\Delta DQO$

$$\angle CPO = \angle DQO \quad [\text{each } 90^\circ]$$

Given that,  $CO = OD$

$$\angle COP = \angle DOQ \quad [\text{Vertically opposite angles are equal}]$$

Then,  $\Delta CPO \cong \Delta DQO$  [By AAS condition]

$$\therefore CP = DQ \quad (3) \text{ [c. p. c. t]}$$

Compare equation (1), (2) and (3)

$$\therefore \text{ar}(\Delta ABC) = \text{ar}(\Delta ABD).$$

**Q 11.** If P is any point in the interior of a parallelogram ABCD, then prove that area of the triangle APB is less than half the area of parallelogram.

**Solution:**

Draw  $DN \perp AB$  and  $PM \perp AB$

Now,

$$\text{ar}(\parallel^{\text{gm}} ABCD) = AB \times DN, \text{ar}(\Delta APB) = \frac{1}{2}(AB \times PM)$$

Now,  $PM < DN$

$$\Rightarrow AB \times PM < AB \times DN$$

$$\Rightarrow \frac{1}{2}(AB \times PM) < \frac{1}{2}(AB \times DN)$$

$$\Rightarrow \text{ar}(\triangle APB) < \frac{1}{2}\text{ar}(\text{||}^{\text{gm}} \text{ABCD})$$

**Q 12.** If  $AD$  is a median of a triangle  $ABC$ , then prove that triangles  $ADB$  and  $ADC$  are equal in area. If  $G$  is the mid-point of the median  $AD$ , prove that  $\text{ar}(\triangle BGC) = 2\text{ar}(\triangle AGC)$ .

**Solution:**

Draw  $AM \perp BC$

Since,  $AD$  is the median of  $\triangle ABC$

$$\therefore BD = DC$$

$$\Rightarrow BD = AM = DC \times AM$$

$$\Rightarrow \frac{1}{2}(BD \times AM) = \frac{1}{2}(DC \times AM)$$

$$\Rightarrow \text{ar}(\triangle ABD) = \text{ar}(\triangle ACD) \dots \dots \dots (1)$$

In  $\triangle BGC$ ,  $GD$  is the median

$$\therefore \text{ar}(\triangle BGD) = \text{ar}(\triangle CGD) \dots \dots \dots (2)$$

In  $\triangle ACD$ ,  $CG$  is the median

$$\therefore \text{ar}(\triangle AGC) = \text{ar}(\triangle CGD) \dots \dots \dots (3)$$

From (2) and (3) we have,

$$\text{ar}(\triangle BGD) = \text{ar}(\triangle AGC)$$

$$\text{But, ar}(\triangle BGC) = 2\text{ar}(\triangle BGD)$$

$$\therefore \text{ar}(\triangle BGC) = 2\text{ar}(\triangle AGC)$$

**Q 13.** A point  $D$  is taken on the side  $BC$  of a  $\triangle ABC$ , such that  $BD = 2DC$ . Prove that  $\text{ar}(\triangle ABD) = 2\text{ar}(\triangle ADC)$ .

**Solution:**

Given that,

$$\text{In } \triangle ABC, BD = 2DC$$

$$\text{To prove: ar}(\triangle ABD) = 2\text{ar}(\triangle ADC).$$

Construction:

Take a point  $E$  on  $BD$  such that  $BE = ED$

Proof: Since,  $BE = ED$  and  $BD = 2DC$

$$\text{Then, } BE = ED = DC$$

We know that median of triangle divides it into two equal triangles.



∴ In  $\triangle ABD$ , AE is the median.

$$\text{Then, ar}(\triangle ABD) = 2\text{ar}(\triangle AED) \dots \dots \dots (1)$$

In  $\triangle AEC$ , AD is the median.

$$\text{Then, ar}(\triangle AED) = 2\text{ar}(\triangle ADC) \dots \dots \dots (2)$$

Compare equation 1 and 2

$$\text{ar}(\triangle ABD) = 2\text{ar}(\triangle ADC).$$

**Q 14. ABCD is a parallelogram whose diagonals intersect at O. If P is any point on BO, prove that :**

(i) .  $\text{ar}(\triangle ADO) = \text{ar}(\triangle CDO)$ .

(ii) .  $\text{ar}(\triangle ABP) = 2\text{ar}(\triangle CBP)$ .

**Solution:**

Given that ABCD is the parallelogram

To Prove: (i)  $\text{ar}(\triangle ADO) = \text{ar}(\triangle CDO)$ .

(ii)  $\text{ar}(\triangle ABP) = 2\text{ar}(\triangle CBP)$ .

Proof: we know that diagonals of parallelogram bisect each other

$$\therefore AO = OC \text{ and } BO = OD$$

(i) . In  $\triangle DAC$ , since DO is a median.

$$\text{Then ar}(\triangle ADO) = \text{ar}(\triangle CDO).$$

(ii) . In  $\triangle BAC$ , since BO is a median.

$$\text{Then ar}(\triangle BAO) = \text{ar}(\triangle BCO) \dots \dots \dots (1)$$

In  $\triangle PAC$ , since PO is a median.

$$\text{Then ar}(\triangle PAO) = \text{ar}(\triangle PCO) \dots \dots \dots (2)$$

Subtract equation 2 from 1.

$$\Rightarrow \text{ar}(\triangle BAO) - \text{ar}(\triangle PAO) = \text{ar}(\triangle BCO) - \text{ar}(\triangle PCO)$$

$$\Rightarrow \text{ar}(\triangle ABP) = 2\text{ar}(\triangle CBP).$$

**Q 15. ABCD is a parallelogram in which BC is produced to E such that CE = BC. AE intersects CD at F.**

(i) . **Prove that**  $\text{ar}(\triangle ADF) = \text{ar}(\triangle ECF)$ .

(ii) . **If the area of  $\triangle DFB = 3 \text{ cm}^2$ , find the area of  $\square^{gm} ABCD$ .**

**Solution:**

In triangles ADF and ECF, we have

$$\angle ADF = \angle ECF \quad [\text{Alternate interior angles, Since } AD \parallel BE]$$

$$AD = EC \quad [\text{since } AD = BC = CE]$$

$$\text{And } \angle DFA = \angle CFA \quad [\text{Vertically opposite angles}]$$

So, by AAS congruence criterion, we have

$$\triangle ADF \cong \triangle ECF$$

$$\Rightarrow \text{ar}(\triangle ADF) = \text{ar}(\triangle ECF) \text{ and } DF = CF.$$

Now,  $DF = CF$

$\Rightarrow BF$  is a median in  $\triangle BCD$ .

$$\Rightarrow \text{ar}(\triangle BCD) = 2\text{ar}(\triangle BDF)$$

$$\Rightarrow \text{ar}(\triangle BCD) = 2 \times 3 \text{ cm}^2 = 6 \text{ cm}^2$$

$$\text{Hence, area of a parallelogram} = 2\text{ar}(\triangle BCD) = 2 \times 6 \text{ cm}^2 = 12 \text{ cm}^2$$

**Q 16.** *ABCD is a parallelogram whose diagonals AC and BD intersect at O. A line through O intersects AB at P and DC at Q. Prove that  $\text{ar}(\triangle POA) = \text{ar}(\triangle QOC)$ .*

**Solution:**

In triangles POA and QOC, we have

$$\angle AOP = \angle COQ$$

$$AO = OC$$

$$\angle PAC = \angle QCA$$

So, by ASA congruence criterion, we have

$$\triangle POA \cong \triangle QOC$$

$$\Rightarrow \text{ar}(\triangle POA) = \text{ar}(\triangle QOC).$$

**Q 17.** *ABCD is a parallelogram. E is a point on BA such that  $BE = 2EA$  and F is point on DC such that  $DF = 2FC$ . Prove that AECF is a parallelogram whose area is one third of the area of parallelogram ABCD.*

**Solution:**

Draw  $FG \perp AB$

We have,

$$BE = 2EA \text{ and } DF = 2FC$$

$$\Rightarrow AB - AE = 2AE \text{ and } DC - FC = 2FC$$

$$\Rightarrow AB = 3AE \text{ and } DC = 3FC$$

$$\Rightarrow AE = \frac{1}{3}AB \text{ and } FC = \frac{1}{3}DC \dots\dots\dots(1)$$

But  $AB = DC$

$$\text{Then, } AE = FC \quad [\text{opposite sides of } \parallel^{\text{gm}}]$$

Thus,  $AE = FC$  and  $AE \parallel FC$

Then, AECF is a parallelogram

$$\text{Now, area of parallelogram AECF} = AE \times FG$$

$$\Rightarrow \text{ar}(\parallel^{\text{gm}} \text{ AECF}) = \frac{1}{3}AB \times FG \quad \text{from(1)}$$

$$\Rightarrow 3 \text{ ar} (\parallel^{\text{gm}} \text{ AECF}) = \text{AB} \times \text{FG} \dots\dots (2)$$

$$\text{And ar} (\parallel^{\text{gm}} \text{ ABCD}) = \text{AB} \times \text{FG} \dots\dots (3)$$

Compare equation 2 and 3

$$\Rightarrow 3\text{ar} (\parallel^{\text{gm}} \text{ AECF}) = \text{ar} (\parallel^{\text{gm}} \text{ ABCD})$$

$$\Rightarrow \text{ar} (\parallel^{\text{gm}} \text{ AECF}) = \frac{1}{3}\text{ar} (\parallel^{\text{gm}} \text{ ABCD})$$

**Q 18. In a triangle ABC, P and Q are respectively the mid points of AB and BC and R is the mid point of AP. Prove that :**

(i) .  $\text{ar} (\Delta \text{PBQ}) = \text{ar} (\Delta \text{ARC}).$

(ii) .  $\text{ar} (\Delta \text{PRQ}) = \frac{1}{2}\text{ar} (\Delta \text{ARC}).$

(iii) .  $\text{ar} (\Delta \text{RQC}) = \frac{3}{8}\text{ar} (\Delta \text{ABC}).$

**Solution:**

We know that each median of a triangle divides it into two triangles of equal area.

(i) . Since CR is the median of  $\Delta \text{CAP}$

$$\therefore \text{ar} (\Delta \text{CRA}) = \frac{1}{2}\text{ar} (\Delta \text{CAP}) \dots\dots (1)$$

Also , CP is the median of a  $\Delta \text{CAB}$

$$\therefore \text{ar} (\Delta \text{CAP}) = \text{ar} (\Delta \text{CPB}) \dots\dots (2)$$

From 1 and 2 , we get

$$\therefore \text{ar} (\Delta \text{ARC}) = \frac{1}{2}\text{ar} (\Delta \text{CPB}) \dots\dots (3)$$

PQ is the median of a  $\Delta \text{PBC}$

$$\therefore \text{ar} (\Delta \text{CPB}) = 2\text{ar} (\Delta \text{PBQ}) \dots\dots (4)$$

From 3 and 4, we get

$$\therefore \text{ar} (\Delta \text{ARC}) = \text{ar} (\Delta \text{PBQ}) \dots\dots (5)$$

(ii) . Since QP and QR medians of triangles QAB and QAP respectively

$$\therefore \text{ar} (\Delta \text{QAP}) = \text{ar} (\Delta \text{QBP}) \dots\dots (6)$$

$$\text{And ar} (\Delta \text{QAP}) = 2\text{ar} (\Delta \text{QRP}) \dots\dots (7)$$

From 6 and 7, we get

$$\text{ar} (\Delta \text{PRQ}) = \frac{1}{2}\text{ar} (\Delta \text{PBQ}) \dots\dots (8)$$

From 5 and 8, we get

$$\text{ar} (\Delta \text{PRQ}) = \frac{1}{2}\text{ar} (\Delta \text{ARC})$$

(iii) . Since, LR is a median of  $\Delta \text{CAP}$

$$\therefore \text{ar}(\Delta ARC) = \frac{1}{2} \text{ar}(\Delta CAD)$$

$$= \frac{1}{2} \times \frac{1}{2} \text{ar}(\Delta ABC)$$

$$= \frac{1}{4} \text{ar}(\Delta ABC)$$

Since RQ is the median of  $\Delta RBC$ .

$$\therefore \text{ar}(\Delta RQC) = \frac{1}{2} \text{ar}(\Delta RBC)$$

$$= \frac{1}{2} \{ \text{ar}(\Delta ABC) - \text{ar}(\Delta ARC) \}$$

$$= \frac{1}{2} \{ \text{ar}(\Delta ABC) - \frac{1}{4} \text{ar}(\Delta ABC) \}$$

$$= \frac{3}{8} \text{ar}(\Delta ABC)$$

**Q 19.** ABCD is a parallelogram. G is a point on AB such that AG = 2GB and E is point on DC such that CE = 2DE and F is the point of BC such that BF = 2FC. Prove that:

(i) . ar (ADEG) = ar (GBCE).

(ii) . ar ( $\Delta EGB$ ) =  $\frac{1}{6}$  ar (ABCD).

(iii) . ar ( $\Delta EFC$ ) =  $\frac{1}{2}$  ar ( $\Delta EBF$ ).

(iv) . ar ( $\Delta EGB$ ) =  $\frac{3}{2}$   $\times$  ar ( $\Delta EFC$ )

(v) . Find what portion of the area of parallelogram is the area of  $\Delta EFG$ .

**Solution:**

Given: ABCD is a parallelogram

AG = 2 GB, CE = 2 DE and BF = 2 FC

To prove:

(i) . ar (ADEG) = ar (GBCE).

(ii) . ar ( $\Delta EGB$ ) =  $\frac{1}{6}$  ar (ABCD).

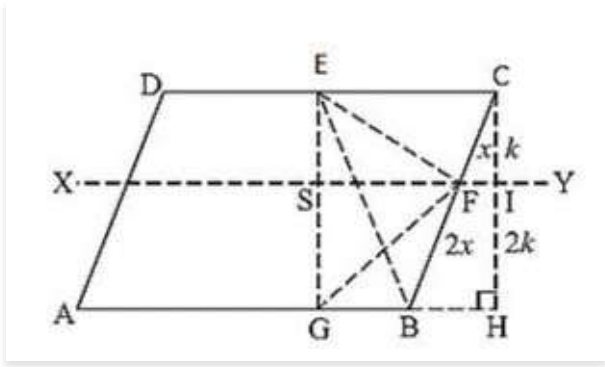
(iii) . ar ( $\Delta EFC$ ) =  $\frac{1}{2}$  ar ( $\Delta EBF$ ).

(iv) . ar ( $\Delta EGB$ ) =  $\frac{3}{2}$   $\times$  ar ( $\Delta EFC$ )

(v) . Find what portion of the area of parallelogram is the area of  $\Delta EFG$ .

Construction: Draw a parallel line to AB through point F and a perpendicular line to AB from C

Proof:



(i) . Since ABCD is a parallelogram

So,  $AB = CD$  and  $AD = BC$

Consider the two trapezium s ADEG and GBCE

Since  $AB = DC$ ,  $EC = 2DE$ ,  $AG = 2GB$

$$\Rightarrow ED = \frac{1}{3}CD = \frac{1}{3}AB \text{ and } EC = \frac{2}{3}CD = \frac{2}{3}AB$$

$$\Rightarrow AG = \frac{2}{3}AB \text{ and } BG = \frac{1}{3}AB$$

$$\text{So, } DE + AG = \frac{1}{3}AB + \frac{2}{3}AB = AB \text{ and } EC + BG = \frac{2}{3}AB + \frac{1}{3}AB = AB$$

Since the two trapezium ADEG and GBCE have same height and their sum of two parallel sides are equal

$$\text{Since Area of trapezium} = \frac{\text{sum of parallel sides}}{2} \times \text{height}$$

So,  $\text{ar}(\text{ADEG}) = \text{ar}(\text{GBCE})$ .

(ii) . Since we know from above that

$$BG = \frac{1}{2}AB. \text{ So}$$

$$\text{ar}(\triangle EGB) = \frac{1}{2} \times GB \times \text{Height}$$

$$\text{ar}(\triangle EGB) = \frac{1}{2} \times \frac{1}{3} \times AB \times \text{Height}$$

$$\text{ar}(\triangle EGB) = \frac{1}{6} \times AB \times \text{Height}$$

$$\text{ar}(\triangle EGB) = \frac{1}{6} \text{ar}(\text{ABCD}).$$

(iii) . Since height of triangle EFC and EBF are equal. So

$$\text{ar}(\triangle EFC) = \frac{1}{2} \times FC \times \text{Height}$$

$$\text{ar}(\triangle EFC) = \frac{1}{2} \times \frac{1}{2} \times FB \times \text{Height}$$

$$\text{ar}(\triangle EFC) = \frac{1}{2} \text{ar}(\triangle EBF)$$

$$\text{Hence, } \text{ar}(\triangle EFC) = \frac{1}{2} \text{ar}(\triangle EBF).$$

(iv) . Consider the trapezium in which

$$\text{ar}(\text{EGBC}) = \text{ar}(\triangle EGB) + \text{ar}(\triangle EBF) + \text{ar}(\triangle EFC)$$

$$\Rightarrow \frac{1}{2} \text{ar}(\text{ABCD}) = \frac{1}{6} \text{ar}(\text{ABCD}) + 2\text{ar}(\Delta \text{EFC}) + \text{ar}(\Delta \text{EFC})$$

$$\Rightarrow \frac{1}{3} \text{ar}(\text{ABCD}) = 3\text{ar}(\Delta \text{EFC})$$

$$\Rightarrow \text{ar}(\Delta \text{EFC}) = \frac{1}{9} \text{ar}(\text{ABCD})$$

Now from (ii) part we have

$$\text{ar}(\Delta \text{EGB}) = \frac{1}{6} \text{ar}(\Delta \text{EFC})$$

$$\text{ar}(\Delta \text{EGB}) = \frac{3}{2} \times \frac{1}{9} \text{ar}(\text{ABCD})$$

$$\text{ar}(\Delta \text{EGB}) = \frac{3}{2} \text{ar}(\Delta \text{EFC})$$

$$\therefore \text{ar}(\Delta \text{EGB}) = \frac{3}{2} \text{ar}(\Delta \text{EFC})$$

(v) . In the figure it is given that  $\text{FB} = 2\text{CF}$  . Let  $\text{CF} = x$  and  $\text{FB} = 2x$ .

Now consider the two triangles  $\text{CFI}$  and  $\text{CBH}$  which are similar triangle.

So by the property of similar triangle  $\text{CI} = k$  and  $\text{IH} = 2k$

Now consider the triangle  $\text{EGF}$  in which

$$\text{ar}(\Delta \text{EFG}) = \text{ar}(\Delta \text{ESF}) + \text{ar}(\Delta \text{SGF})$$

$$\text{ar}(\Delta \text{EFG}) = \frac{1}{2} \text{SF} \times k + \frac{1}{2} \text{SF} \times 2k$$

$$\text{ar}(\Delta \text{EFG}) = \frac{3}{2} \text{SF} \times k \dots \dots (i)$$

Now ,

$$\text{ar}(\Delta \text{EGBC}) = \text{ar}(\text{SGBF}) + \text{ar}(\text{ESFC})$$

$$\text{ar}(\Delta \text{EGBC}) = \frac{1}{2}(\text{SF} + \text{GB}) \times 2k + \frac{1}{2}(\text{SF} + \text{EC}) \times k$$

$$\text{ar}(\Delta \text{EGBC}) = \frac{3}{2}k \times \text{SF} + (\text{GB} + \frac{1}{2}\text{EC}) \times k$$

$$\text{ar}(\Delta \text{EGBC}) = \frac{3}{2}k \times \text{SF} + (\frac{1}{3}\text{AB} + \frac{1}{2} \times \frac{2}{3}\text{AB}) \times k$$

$$\frac{1}{2} \text{ar}(\Delta \text{ABCD}) = \frac{3}{2}k \times \text{SF} + \frac{2}{3}\text{AB} \times k$$

$$\Rightarrow \text{ar}(\Delta \text{ABCD}) = 3k \times \text{SF} + \frac{4}{3}\text{AB} \times k$$

[Multiply both sides by 2]

$$\Rightarrow \text{ar}(\Delta \text{ABCD}) = 3k \times \text{SF} + \frac{4}{9} \text{ar}(\text{ABCD})$$

$$\Rightarrow k \times \text{SF} = \frac{5}{27} \text{ar}(\text{ABCD}) \dots \dots (2)$$

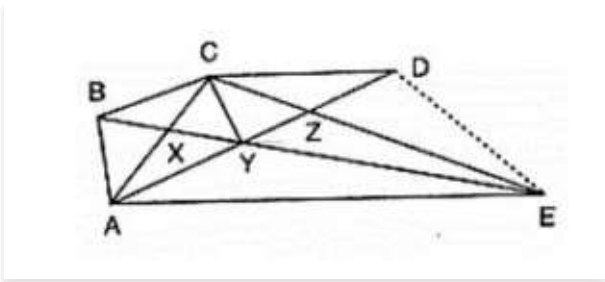
From 1 and 2 we have ,

$$\text{ar}(\Delta \text{EFG}) = \frac{3}{2} \times \frac{5}{27} \text{ar}(\text{ABCD})$$

$$\text{ar}(\Delta \text{EFG}) = \frac{5}{18} \text{ar}(\text{ABCD})$$

**Q 20. In figure,  $\text{CD} \parallel \text{AE}$  and  $\text{CY} \parallel \text{BA}$ .**

- (i) . Name a triangle equal in area of  $\triangle CBX$
- (ii) . Prove that  $\text{ar}(\triangle ZDE) = \text{ar}(\triangle CZA)$ .
- (iii) . Prove that  $\text{ar}(BCZY) = \text{ar}(\triangle EDZ)$ .



**Solution:**

Since, triangle BCA and triangle BYA are on the same base BA and between same parallel s BA and CY.

Then  $\text{ar}(\triangle BCA) = \text{ar}(\triangle BYA)$

$\Rightarrow \text{ar}(\triangle CBX) + \text{ar}(\triangle BXA) = \text{ar}(\triangle BXA) + \text{ar}(\triangle AXY)$

$\Rightarrow \text{ar}(\triangle CBX) = \text{ar}(\triangle AXY) \dots\dots\dots(1)$

Since, triangles ACE and ADE are on the same base AE and between same parallels CD and AE

Then,  $\text{ar}(\triangle ACE) = \text{ar}(\triangle ADE)$

$\text{ar}(\triangle CZA) + \text{ar}(\triangle AZE) = \text{ar}(\triangle AZE) + \text{ar}(\triangle DZE)$

$\text{ar}(\triangle CZA) = \text{ar}(\triangle DZE) \dots\dots\dots(2)$

Adding  $\text{ar}(\triangle CYZ)$  on both sides , we get

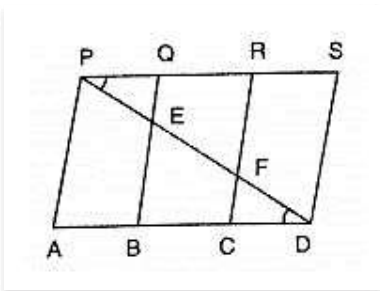
$\Rightarrow \text{ar}(\triangle CBX) + \text{ar}(\triangle CYZ) = \text{ar}(\triangle CAZ) + \text{ar}(\triangle CYZ)$

$\Rightarrow \text{ar}(BCZY) = \text{ar}(\triangle CZA) \dots\dots\dots(3)$

Compare equation 2 and 3

$\Rightarrow \text{ar}(BCZY) = \text{ar}(\triangle DZE)$

**Q 21. In figure, PSDA is a parallelogram in which  $PQ = QR = RS$  and  $AP \parallel BQ \parallel CR$ . Prove that  $\text{ar}(\triangle PQE) = \text{ar}(\triangle CRF)$ .**



**Solution:**

Given that PSDA is a parallelogram

Since,  $AP \parallel BQ \parallel CR \parallel DS$  and  $AD \parallel PS$

Therefore,  $PQ = CD$  (equ. 1)

In triangle  $BED$ ,  $C$  is the midpoint of  $BD$  and  $CF \parallel BE$

Therefore,  $F$  is the midpoint of  $ED$

$\Rightarrow EF = PE$

Smiliarly,

$EF = PE$

Therefore,  $PE = FD$  (equ . 2)

In triangles  $PQE$  and  $CFD$ , we have

$PE = FD$

Therefore,  $\angle EPQ = \angle FDC$  [Alternate angles]

So, by SAS criterion , we have

$\triangle PQE \cong \triangle DCF$

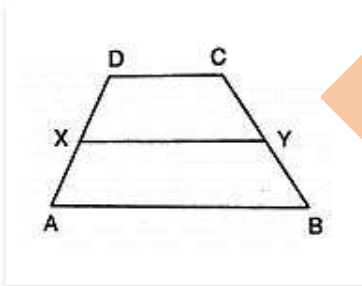
$\Rightarrow \text{ar}(\triangle PQE) = \text{ar}(\triangle DCF)$

**Q 22.** In figure,  $ABCD$  is a trapezium in which  $AB \parallel DC$  and  $DC = 40$  cm and  $AB = 60$  cm. If  $X$  and  $Y$  are , respectively , the mid points of  $AD$  and  $BC$  , prove that :

(i) .  $XY = 50$  cm

(ii) .  $DCYX$  is a trapezium

(iii) .  $\text{ar}(\text{trap. } DCYX) = \frac{9}{11} \text{ar}(XYBA)$ .



**Solution:**

(i) Join  $DY$  and produce it to meet  $AB$  produced at  $P$ .

In triangles  $BYP$  and  $CYD$  we have,

$\angle BYP = \angle CYD$  [Vertically opposite angles]

$\angle DCY = \angle PBY$  [Since ,  $DC \parallel AP$ ]

And  $BY = CY$

So, by ASA congruence criterion, we have

$(\triangle BYP) \cong (\triangle CYD)$

$\Rightarrow DY = YP$  and  $DC = BP$

$\Rightarrow Y$  is the midpoint of  $DP$



Also,  $x$  is the mid point of  $AD$

Therefore,  $XY \parallel AP$  and  $XY \parallel \frac{1}{2}AP$

$$\Rightarrow XY = \frac{1}{2}(AB + BP)$$

$$\Rightarrow XY = \frac{1}{2}(AB + DC)$$

$$\Rightarrow XY = \frac{1}{2}(60 + 40)$$

$$= 50 \text{ cm}$$

(ii) We have,  $XY \parallel AP$

$$\Rightarrow XY \parallel AB \text{ and } AB \parallel DC$$

$$\Rightarrow XY \parallel DC$$

$\Rightarrow DCYX$  is a trapezium

(iii) Since  $x$  and  $y$  are the mid points of  $AD$  and  $BC$  respectively.

Therefore, trapezium  $DCYX$  and  $ABYX$  are of the same height say  $h$  cm

Now,

$$\text{ar (trap. DCXY)} = \frac{1}{2}(DC + XY) \times h$$

$$\Rightarrow \text{ar (trap. DCXY)} = \frac{1}{2}(50 + 40) \times h \text{ cm}^2 = 45h \text{ cm}^2$$

$$\Rightarrow \text{ar (trap. ABYX)} = \frac{1}{2}(AB + XY) \times h$$

$$\Rightarrow \text{ar (trap. ABYX)} = \frac{1}{2}(60 + 50) \times h \text{ cm}^2 = 55h \text{ cm}^2$$

$$\frac{\text{ar(trap.DCYX)}}{\text{ar(trap.ABYX)}} = \frac{45h}{55h} = \frac{9}{11}$$

$$\Rightarrow \text{ar (trap. DCYX)} = \frac{9}{11} \text{ar (trap. ABYX)}$$

**Q 23. In figure  $ABC$  and  $BDE$  are two equilateral triangles such that  $D$  is the midpoint of  $BC$ .  $AE$  intersects  $BC$  in  $F$ . Prove that:**

(i) .  $\text{ar}(\triangle BDE) = \frac{1}{4} \text{ar}(\triangle ABC)$ .

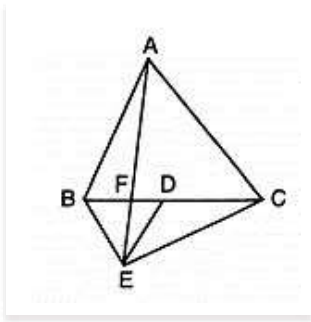
(ii) .  $\text{ar}(\triangle BDE) = \frac{1}{2} \text{ar}(\triangle BAE)$ .

(iii) .  $\text{ar}(\triangle BFE) = \text{ar}(\triangle AFD)$ .

(iv) .  $\text{ar}(\triangle ABC) = 2 \text{ar}(\triangle BEC)$ .

(v) .  $\text{ar}(\triangle FED) = \frac{1}{8} \text{ar}(\triangle AFC)$ .

(vi) .  $\text{ar}(\triangle BFE) = 2 \text{ar}(\triangle FED)$ .



**Solution:**

Given that ABC and BDE are two equilateral triangles.

Let  $AB = BC = CA = x$ . Then,  $BD = \frac{x}{2} = DE = BE$

(i) We have,

$$\text{ar}(\Delta ABC) = \frac{\sqrt{3}}{4}x^2 \text{ and } \text{ar}(\Delta BDE) = \frac{\sqrt{3}}{4}\left(\frac{x}{2}\right)^2 = \frac{1}{4} \times \frac{\sqrt{3}}{4}x^2$$

Therefore,  $\text{ar}(\Delta BDE) = \frac{1}{4} \text{ar}(\Delta ABC)$ .

(ii) . It is given that triangles, ABC and BED are equilateral triangles

$$\angle ACB = \angle DBE = 60^\circ$$

$\Rightarrow BE \parallel AC$  (Since, alternative angles are equal)

Triangles BAF and BEC are on the same base BE and between same parallels BF and AC .

Therefore,  $\text{ar}(\Delta BAE) = \text{ar}(\Delta BEC)$

$$\Rightarrow \text{ar}(\Delta BAE) = 2\text{ar}(\Delta BDE) \quad [\text{Since, ED is a median of triangle EBC ;} \\ \text{ar}(\Delta BEC) = 2\text{ar}(\Delta BDE)]$$

$$\therefore \text{ar}(\Delta BDE) = \frac{1}{2} \text{ar}(\Delta BAE)$$

(iii) Since, triangles ABC and BDE are equilateral triangles

$$\therefore \angle ABC = 60^\circ \text{ and } \angle BDE = 60^\circ$$

$$\angle ABC = \angle BDE$$

$\Rightarrow AB \parallel DE$  (since, alternate angles are equal)

Triangles BED and AED are on the same base ED and between same parallels AB and DE.

Therefore,  $\text{ar}(\Delta BED) = \text{ar}(\Delta AED)$

$$\Rightarrow \text{ar}(\Delta BED) - \text{ar}(\Delta EFD) = \text{ar}(\Delta AED) - \text{ar}(\Delta EFD)$$

$$\Rightarrow \text{ar}(\Delta BEF) = \text{ar}(\Delta AFD)$$

(iv) Since ED is the median of triangle BEC

Therefore,  $\text{ar}(\Delta BEC) = 2\text{ar}(\Delta BDE)$

$$\Rightarrow \text{ar}(\Delta BEC) = 2 \times \frac{1}{4} \text{ar}(\Delta ABC) \quad [\text{From 1, } \text{ar}(\Delta BDE) = \frac{1}{4} \text{ar}(\Delta ABC)]$$

$$\Rightarrow \text{ar}(\Delta BEC) = \frac{1}{2} \text{ar}(\Delta ABC)$$

$$\Rightarrow \text{ar}(\triangle ABC) = 2\text{ar}(\triangle BEC)$$

$$(v) \text{ar}(\triangle AFC) = \text{ar}(\triangle AFD) + \text{ar}(\triangle ADC)$$

$$\Rightarrow \text{ar}(\triangle BFE) + \frac{1}{2}\text{ar}(\triangle ABC) \quad [\text{using part (iii) , and AD is the median of triangle ABC}]$$

$$= \text{ar}(\triangle BFE) + \frac{1}{2} \times 4\text{ar}(\triangle BDE) \quad (\text{using part (i)})$$

$$= \text{ar}(\triangle BFE) = 2\text{ar}(\triangle FED) \dots (3)$$

$$\text{ar}(\triangle BDE) = \text{ar}(\triangle BFE) + \text{ar}(\triangle FED)$$

$$\Rightarrow 2\text{ar}(\triangle FED) + \text{ar}(\triangle FED)$$

$$\Rightarrow 3\text{ar}(\triangle FED) \dots (4)$$

From 2, 3 and 4 we get ,

$$\text{ar}(\triangle AFC) = 2\text{ar}(\triangle FED) + 2 \times 3\text{ar}(\triangle FED) = 8\text{ar}(\triangle FED)$$

$$\text{ar}(\triangle FED) = \frac{1}{8}\text{ar}(\triangle AFC)$$

(vi) Let h be the height of vertex E, corresponding to the side BD in triangle BDE.

Let H be the height of vertex A, corresponding to the side BC in triangle ABC

From part (i)

$$\text{ar}(\triangle BDE) = \frac{1}{4}\text{ar}(\triangle ABC)$$

$$\Rightarrow \frac{1}{2} \times BD \times h = \frac{1}{4}(\frac{1}{2} \times BC \times H)$$

$$\Rightarrow BD \times h = \frac{1}{4}(2BD \times H)$$

$$\Rightarrow h = \frac{1}{2}H \dots (1)$$

From part (iii)

$$\text{ar}(\triangle BFE) = \text{ar}(\triangle AFD)$$

$$\text{ar}(\triangle BFE) = \frac{1}{2} \times FD \times H$$

$$\text{ar}(\triangle BFE) = \frac{1}{2} \times FD \times 2h$$

$$\text{ar}(\triangle BFE) = 2(\frac{1}{2} \times FD \times h)$$

$$\text{ar}(\triangle BFE) = 2\text{ar}(\triangle EFD)$$

**Q 24. D is the midpoint of side BC of  $\triangle ABC$  and E is the midpoint of BD. If O is the midpoint of AE, Prove that  $\text{ar}(\triangle BOE) = \frac{1}{8}\text{ar}(\triangle ABC)$ .**

**Solution:**

Given that

D is the midpoint of sides BC of triangle ABC

E is the midpoint of BD and O is the midpoint of AE

Since AD and AE are the medians of triangles, ABC and ABD respectively

$$\therefore \text{ar}(\triangle ABD) = \frac{1}{2} \text{ar}(\triangle ABC) \dots \dots \dots (1)$$

$$\therefore \text{ar}(\triangle ABE) = \frac{1}{2} \text{ar}(\triangle ABD) \dots \dots \dots (2)$$

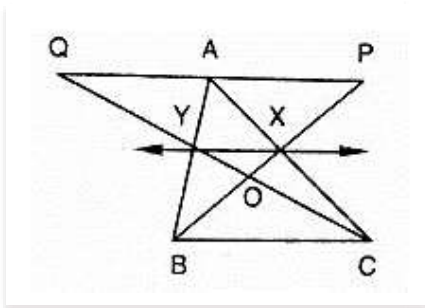
OB is the median of triangle ABE

$$\text{Therefore, } \therefore \text{ar}(\triangle BOE) = \frac{1}{2} \text{ar}(\triangle ABE)$$

From 1, 2 and 3, we have

$$\therefore \text{ar}(\triangle BOE) = \frac{1}{8} \text{ar}(\triangle ABC)$$

**Q 25. In figure, X and Y are the mid points of AC and AB respectively, QP ||BC and CYQ and BXP are straight lines . Prove that  $\text{ar}(\triangle ABP) = \text{ar}(\triangle ACQ)$ .**



**Solution:**

Since X and Y are the mid points of AC and AB respectively.

Therefore,  $XY \parallel BC$

Clearly, triangles BYC and BXC are on the same base BC and between the same parallels Xy and BC

$$\therefore \text{ar}(\triangle BYC) = \text{ar}(\triangle BXC)$$

$$\Rightarrow \text{ar}(\triangle BYC) - \text{ar}(\triangle BOC) = \text{ar}(\triangle BXC) - \text{ar}(\triangle BOC)$$

$$\Rightarrow \text{ar}(\triangle BOY) = \text{ar}(\triangle COX)$$

$$\Rightarrow \text{ar}(\triangle BOY) + \text{ar}(\triangle XOY) = \text{ar}(\triangle COX) + \text{ar}(\triangle XOY)$$

$$\Rightarrow \text{ar}(\triangle BXY) = \text{ar}(\triangle CXY) \quad (2)$$

We observed that the quadrilaterals XYAP and XYAQ are on the same base XY and between same parallels XY and PQ.

$$\therefore \text{ar}(\text{quad. XY AP}) = \text{ar}(\text{quadXY QA}) \dots \dots \dots (2)$$

Adding 1 and 2, we get

$$\therefore \text{ar}(\triangle BXY) + \text{ar}(\text{quad. XY AP}) = \text{ar}(\triangle CXY) + \text{ar}(\text{quadXY QA})$$

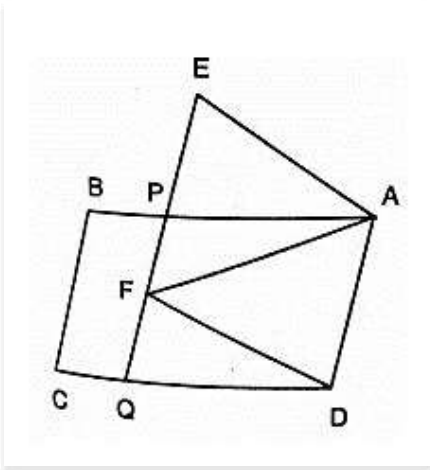
$$\Rightarrow \text{ar}(\triangle ABP) = \text{ar}(\triangle ACQ)$$

**Q 26. In figure, ABCD and Aefd are two parallelograms. Prove that**

(i) .  $PE = FQ$

(ii) .  $\text{ar}(\triangle APE) : \text{ar}(\triangle PFA) = \text{ar}(\triangle QFD) : \text{ar}(\triangle PFD)$

(iii) . ar ( $\Delta PEA$ )= ar ( $\Delta QFD$ ).



**Solution:**

Given that, ABCD and AEFD are two parallelograms

(i) . In triangles, EPA and FQD

$$\angle PEA = \angle QFD \quad \text{[corresponding angles]}$$

$$\angle EPA = \angle FQD \quad \text{[corresponding angles]}$$

$$PA = QD \quad \text{[opposite sides of parallelogram]}$$

$$\text{Then, } \Delta EPA \cong \Delta FQD \quad \text{[By AAS condition]}$$

$$\text{Therefore, } EP = FQ \quad \text{[C.P.C.T]}$$

(ii) . Since triangles, PEA and QFD stand on equal bases PE and FQ lies between the same parallels EQ and AD

$$\text{Therefore, } ar(\Delta PEA) = ar(\Delta QFD) \quad (1)$$

Since, triangles PEA and PFD stand on the same base PF and between same parallels PF and AD

$$\text{Therefore, } ar(\Delta PEA) = ar(\Delta PFD) \quad (2)$$

Divide the equation 1 by equation 2

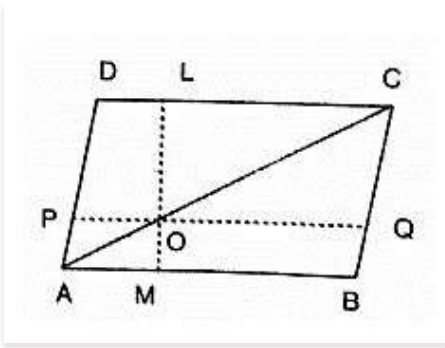
$$\frac{ar(\Delta PEA)}{ar(\Delta PEA)} = \frac{ar(\Delta QFD)}{ar(\Delta PFD)}$$

(iii) . From part (i),  $\Delta EPA \cong \Delta FQD$

$$\text{Then, } ar(\Delta PEA) = ar(\Delta QFD).$$

**Q 27. In figure, ABCD is a parallelogram . O is any point on AC. PQ ||AB and LM || AD. Prove that :**

$$ar(\parallel^{\text{gm}} DLOP) = ar(\parallel^{\text{gm}} BMOQ).$$



**Solution:**

Since a diagonal of a parallelogram divides it into two triangles of equal area

Therefore,  $\text{ar}(\triangle ADC) = \text{ar}(\triangle ABC)$

$$\Rightarrow \text{ar}(\triangle APO) + \text{ar}(\parallel^{\text{gm}} \text{DLOP}) + \text{ar}(\triangle OLC)$$

$$\Rightarrow \text{ar}(\triangle AOM) + \text{ar}(\parallel^{\text{gm}} \text{BMOQ}) + \text{ar}(\triangle OQC) \quad (1)$$

Since AO and Oc are diagonals of parallelograms AMOP and OQCL respectively.

$$\therefore \text{ar}(\triangle APO) = \text{ar}(\triangle AMO) \quad (2)$$

$$\text{And ar}(\triangle OLC) = \text{ar}(\triangle OQC) \quad (3)$$

Subtracting 2 and 3 from 1, we get

$$\text{ar}(\parallel^{\text{gm}} \text{DLOP}) = \text{ar}(\parallel^{\text{gm}} \text{BMOQ}).$$

**Q 28. In a triangle ABC, if L and M are points on AB and AC respectively such that  $LM \parallel BC$ . Prove that:**

(i) .  $\text{ar}(\triangle LCM) = \text{ar}(\triangle LBM)$ .

(ii) .  $\text{ar}(\triangle LBC) = \text{ar}(\triangle MBC)$ .

(iii) .  $\text{ar}(\triangle ABM) = \text{ar}(\triangle ACL)$ .

(iv) .  $\text{ar}(\triangle LOB) = \text{ar}(\triangle MOC)$ .

**Solution:**

(i) . Clearly triangles LMB and LMC are on the same base LM and between the same parallels LM and BC.

$$\therefore \text{ar}(\triangle LMB) = \text{ar}(\triangle LMC) \quad (1)$$

(ii) . We observe that triangles LBC and MBC are on the same base BC and between same parallels LM and BC.

$$\therefore \text{ar}(\triangle LBC) = \text{ar}(\triangle MBC) \quad (2)$$

(iii) . We have,

$$\text{ar}(\triangle LMB) = \text{ar}(\triangle LMC) \quad [\text{From 1}]$$

$$\Rightarrow \text{ar}(\triangle ALM) + \text{ar}(\triangle LMB) = \text{ar}(\triangle ALM) + \text{ar}(\triangle LMC)$$

$$\Rightarrow \text{ar}(\triangle ABM) = \text{ar}(\triangle ACL)$$

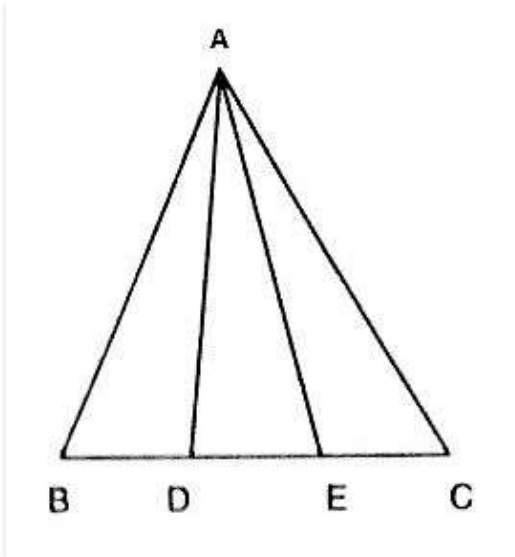
(iv) . We have,

$$\text{ar}(\triangle LBC) = \text{ar}(\triangle MBC) \quad [\text{From 1}]$$

$$\Rightarrow \text{ar}(\triangle LBC) - \text{ar}(\triangle BOC) = \text{ar}(\triangle MBC) - \text{ar}(\triangle BOC)$$

$$\Rightarrow \text{ar}(\triangle LOB) = \text{ar}(\triangle MOC).$$

**Q 29.** In figure,  $D$  and  $E$  are two points on  $BC$  such that  $BD = DE = EC$ . Show that  $\text{ar}(\triangle ABD) = \text{ar}(\triangle ADE) = \text{ar}(\triangle AEC)$ .



**Solution:**

Draw a line  $l$  through  $A$  parallel to  $BC$ .

Given that,  $BD = DE = EC$

We observed that the triangles  $ABD$  and  $AEC$  are on the equal bases and between the same parallels  $l$  and  $BC$ . Therefore, their areas are equal.

Hence,  $\text{ar}(\triangle ABD) = \text{ar}(\triangle ADE) = \text{ar}(\triangle AEC)$ .

**Q 30.** In figure,  $ABC$  is a right angled triangle at  $A$ ,  $BCED$ ,  $ACFG$  and  $ABMN$  are squares on the sides  $BC$ ,  $CA$  and  $AB$  respectively. Line segment  $AX \perp DE$  meets  $BC$  at  $Y$ . Show that

(i) .  $\triangle MBC \cong \triangle ABD$

(ii) .  $\text{ar}(BYXD) = 2\text{ar}(\triangle MBC)$

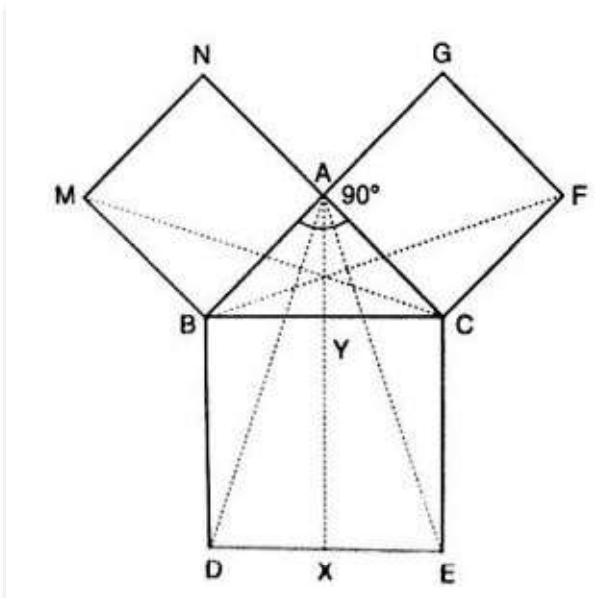
(iii) .  $\text{ar}(BYXD) = \text{ar}(ABMN)$

(iv) .  $\triangle FCB \cong \triangle ACE$

(v) .  $\text{ar}(CYXE) = 2\text{ar}(\triangle FCB)$

(vi) .  $\text{ar}(CYXE) = \text{ar}(ACFG)$

(vii) .  $\text{ar}(BCED) = \text{ar}(ABMN) + \text{ar}(ACFG)$



**Solution:**

(i) . In  $\triangle MBC$  and  $\triangle ABD$ , we have

$$MB = AB$$

$$BC = BD$$

And  $\angle MBC = \angle ABD$  [since,  $\angle MBC$  and  $\angle ABC$  are obtained by adding  $\angle ABC$  to a right angle.]

So, by SAS congruence criterion, we have

$$\triangle MBC \cong \triangle ABD$$

$$\Rightarrow \text{ar}(\triangle MBC) = \text{ar}(\triangle ABD) \dots \dots \dots (1)$$

(ii) . Clearly, triangle ABC and rectangle BYXD are on the same base BD and between the same parallels AX and BD

$$\therefore \text{ar}(\triangle ABD) = \frac{1}{2} \text{ar}(\text{rect BYXD})$$

$$\Rightarrow \text{ar}(\text{rect BYXD}) = 2\text{ar}(\triangle ABD)$$

$$\Rightarrow \text{ar}(\text{rect BYXD}) = 2\text{ar}(\triangle MBC) \dots \dots \dots (2) \quad \text{[From equ .1]}$$

(iii) . Since triangles MBC and square MBAN are on the same base Mb and between the same parallels MB and NC.

$$\therefore 2\text{ar}(\triangle MBC) = \text{ar}(\text{MBAN}) \dots \dots \dots (3)$$

From equ. 2 and 3, we have

$$\text{ar}(\text{sq. MBAN}) = \text{ar}(\text{rectBYXD})$$

(iv) . In triangles FCB and ACE, we have

$$FC = AC$$

$$CB = CE$$

And,  $\angle FCB = \angle ACE$  [since,  $\angle FCB$  and  $\angle ACE$  are obtained by adding  $\angle ACB$  to a right angle.]

So, by SAS congruence criterion, we have

$$\triangle FCB \cong \triangle ACE$$



(v) . We have,

$$\Delta FCB \cong \Delta ACE$$

$$\Rightarrow \text{ar}(\Delta FCB) = \text{ar}(\Delta ACE)$$

Clearly, triangle ACE and rectangle CYXE are on the same base CE and between same parallels CE and AX.

$$\therefore 2\text{ar}(\Delta ACE) = \text{ar}(CYXE)$$

$$\Rightarrow 2\text{ar}(\Delta FCB) = \text{ar}(\Delta CYXE) \dots \dots \dots (4)$$

(vi) . Clearly , triangle FCb and rectangle FCAG are on the same base FC and between the same parallels FC and BG.

$$\therefore 2\text{ar}(\Delta FCB) = \text{ar}(FCAG) \dots \dots \dots (5)$$

From 4 and 5, we get

$$\text{ar}(CYXE) = \text{ar}(ACFG)$$

(vii) . Applying Pythagoras theorem in triangle ACB, we have

$$BC^2 = AB^2 + AC^2$$

$$\Rightarrow BC \times BD = AB \times MB + AC \times FC$$

$$\Rightarrow \text{ar}(BCED) = \text{ar}(ABMN) + \text{ar}(ACFG)$$

