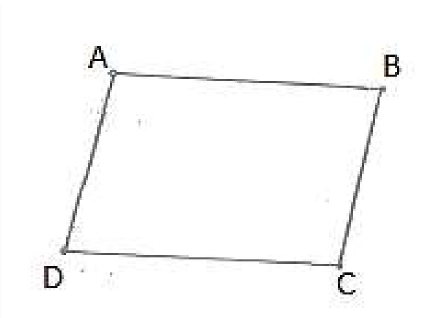
Q1) In a parallelogram ABCD, determine the sum of angles $\angle C$ and $\angle D$.

Solution:



$\angle C$ and $\angle D$ are consecutive interior angles on the same side of the transversal CD.

$\therefore \angle C + \angle D = 180^0$

Q2) In a parallelogram ABCD, if $\angle B = 135^0$, determine the measures of its other angles.

Solution:

Given $\angle B = 135^0$

ABCD is a parallelogram

$\therefore \angle A = \angle C,\ \angle B = \angle D$ and $\angle A + \angle B = 180^0$

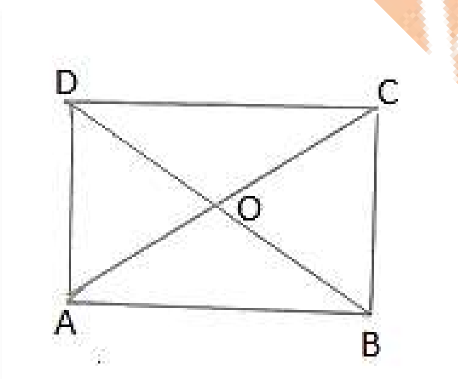
$\Rightarrow \angle A + 135^0 = 180^0$

$\Rightarrow \angle A = 45^0$

$\Rightarrow \angle A = \angle C = 45^0$ and $\angle B = \angle C = 135^0$

Q3) ABCD is a square. AC and BD intersect at O. State the measure of $\angle AOB$.

Solution:

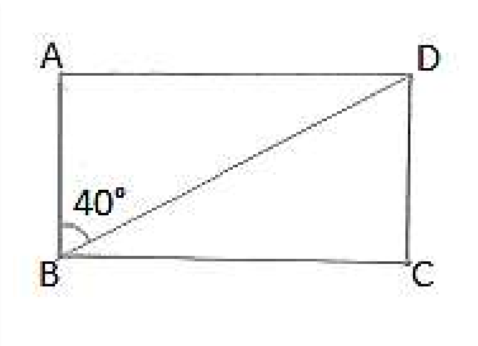


Since, diagonals of a square bisect each other at right angle.

$\therefore \angle AOB = 90^0$

Q4) ABCD is a rectangle with $\angle ABD = 40^0$. Determine $\angle DBC$

Solution:



We have,

$\angle ABC = 90^0$

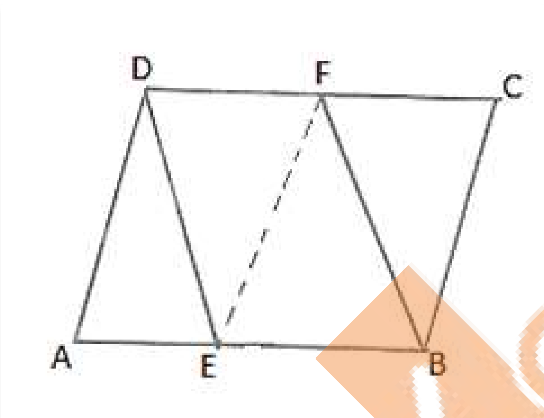
$\Rightarrow \angle ABD + \angle DBC = 90^0 \qquad [\because \angle ABD = 40^0]$

$\Rightarrow 40^0 + \angle DBC = 90^0$

$\therefore \angle DBC = 50^0$

Q5) The sides AB and CD of a parallelogram ABCD are bisected at E and F. Prove that EBFD is a parallelogram.

Solution:



Since ABCD is a parallelogram

$\therefore AB \parallel DC \text{ and } AB = DC$

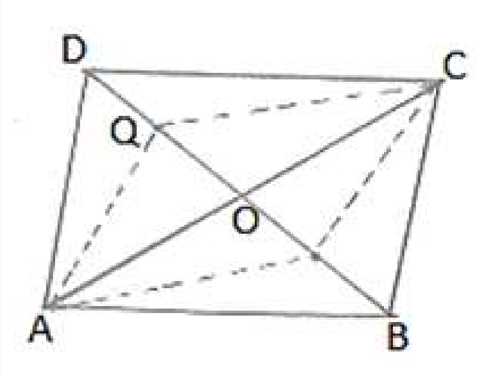
$\Rightarrow EB \parallel DF \text{ and } \frac{1}{2}AB = \frac{1}{2}DC$

$\Rightarrow EB \parallel DF \text{ and } EB = DF$

EBFD is a parallelogram.

Q6) P and Q are the points of trisection of the diagonal BD of a parallelogram ABCD. Prove that CQ is parallel to AP. Prove also that AC bisects PQ.

Solution:



We know that,

Diagonals of a parallelogram bisect each other.

Therefore, OA = OC and OB = OD

Since P and Q are point of intersection of BD.

Therefore, BP = PQ = QD

Now, OB = OD are BP = QD

=>OB − BP = OD − QD

=>OP = OQ

Thus in quadrilateral APCQ, we have

OA = OC and OP = OQ

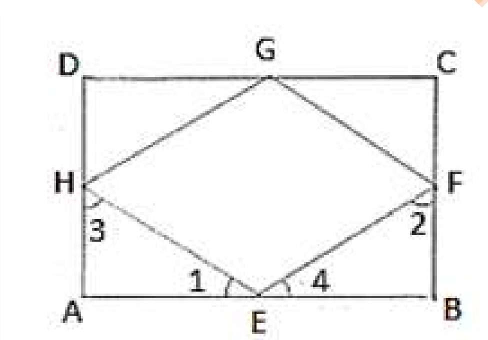
Diagonals of Quadrilateral APCQ bisect each other.

Therefore APCQ is a parallelogram.

Hence $AP \parallel CQ$.

Q7) ABCD is a square. E, F, G and H are points on AB, BC, CD and DA respectively, such that AE = BF = CG = DH. Prove that EFGH is a square.

Solution:



We have,

AE = BF = CG = DH = x (say)

BE = CF = DG = AH = y (say)

In ΔAEH and ΔBEF, we have

AE = BF

$\angle A = \angle B$

And AH = BE

So, by SAS congruency criterion, we have

$\Delta AEH \simeq \Delta BFE$

$\Rightarrow \angle 1 = \angle 2$ and $\angle 3 = \angle 4$

But $\angle 1 + \angle 3 = 90^0$ and $\angle 2 + \angle A = 90^0$

$\Rightarrow \angle 1 + \angle 3 + \angle 2 + \angle A = 90^0 + 90^0$

$\Rightarrow \angle 1 + \angle 4 + \angle 1 + \angle 4 = 180^0$

$\Rightarrow 2(\angle 1 + \angle 4) = 180^0$

$\Rightarrow \angle 1 + \angle 4 = 90^0$

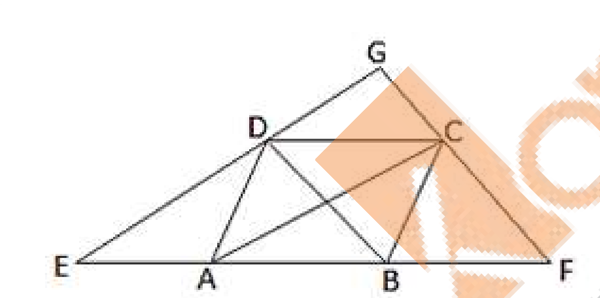
HEF=90^0

Similarly we have $\angle F = \angle G = \angle H = 90^0$

Hence, EFGH is a Square.

Q8) ABCD is a rhombus, EAFB is a straight line such that EA = AB = BF. Prove that ED and FC when produced meet at right angles.

Solution:



We know that the diagonals of a rhombus are perpendicular bisector of each other.

$\therefore OA = OC,\ OB = OD,$ and $\angle AOD = \angle COD = 90^0$

And $\angle AOB = \angle COB = 90^0$

In ΔBDE, A and O are mid-points of BE and BD respectively.

$OA \parallel DE$

$OC \parallel DG$

In ΔCFA, B and O are mid-points of AF and AC respectively.

$OB \parallel CF$

$OD \parallel GC$

Thus, in quadrilateral DOGC, we have

$OC \parallel DG$ and $OD \parallel GC$

=>DOCG is a parallelogram

$\angle DGC = \angle DOC$

$\angle DGC = 90^0$

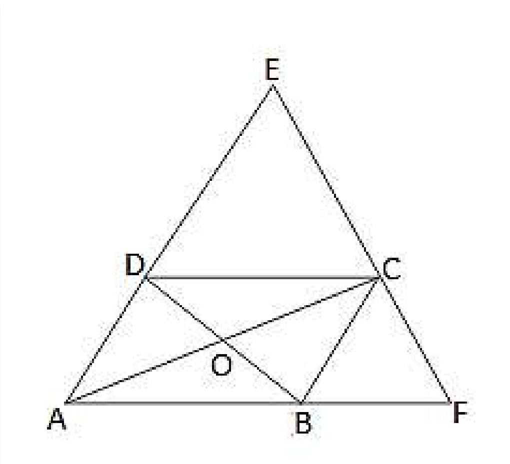
Q9) ABCD is a parallelogram, AD is produced to E so that DE = DC and EC produced meets AB produced in F. Prove that BF = BC.

Solution:

Draw a parallelogram ABCD with AC and BD intersecting at O.

Produce AD to E such that DE = DC

Join EC and produce it to meet AB produced at F.



In ΔDCE,

$\angle DCE = \angle DEC \ldots\ldots(i)$ [In a triangle, equal sides have equal angles]

$AB \parallel CD$ [Opposite sides of the parallelogram are parallel]

$\therefore AE \parallel CD$ [AB lies on AF]

$AF \parallel CD$ and EF is the Transversal.

$\angle DCE = \angle BFC \ldots\ldots(ii)$ [Pair of corresponding angles]

From (i) and (ii) we get

$\angle DEC = \angle BFC$

In ΔAFE,

$\angle AFE = \angle AEF$ [$\angle$ DEC = $\angle$ BFC]

Therefore, AE = AF [In a triangle, equal angles have equal sides opposite to them]

=>AD + DE = AB + BF

=>BC + AB = AB + BF [Since, AD = BC, DE = CD and CD = AB, AB = DE]

=> BC = BF

Hence proved.