Class 11 RD Sharma Solutions – Chapter 22 Brief Review of Cartesian System of Rectangular Coordinates- Exercise 22.2

Last Updated : 30 Apr, 2021

Question 1. Find the locus of a point equidistant from the point (2, 4) and the y-axis.

Solution:

Let C (a, b) be any point on the locus and let A (2, 4) and B (0, b). We are given,

=> $CA = CB$

=> $CA^2 = CB^2$

Using distance formula, we get,

=> $(a - 2)^2 + (b - 4)^2 = (a - 0)^2 + (b - b)^2$

=> $a^2 + 4 - 4a + b^2 + 16 - 8b = a^2$

=> $b^2 - 4a - 8b + 20 = 0$

Replacing (a, b) with (x, y), we get the locus of our point,

=> $y^2 - 4x - 8y + 20 = 0$

Question 2. Find the equation of the locus of a point that moves such that the ratio of its distance from (2, 0) and (1, 3) is 5:4.

Solution:

Let C (a, b) be any point on the locus and let A (2, 0) and B (1, 3). We are given,

=> $CA/CB = 5/4$

=> $CA^2/CB^2 = 25/16$

Using distance formula, we get,

=> $\frac{(a-2)^2+(b-0)^2}{(a-1)^2+(b-3)^2} = \frac{25}{16}$

=> $\frac{a^2+4-4a+b^2}{a^2+1-2a+b^2+9-6b} = \frac{25}{16}$

=> $16 (a^2 + 4 - 4a + b^2) = 25 (a^2 + 1 - 2a + b^2 + 9 - 6b)$

=> $9a^2 + 9b^2 + 14a - 150b + 186 = 0$

Replacing (a, b) with (x, y), we get the equation of the locus of our point,

=> $9x^2 + 9y^2 + 14x - 150y + 186 = 0$

Therefore the locus of the point is $9x^2 + 9y^2 + 14x - 150y + 186 = 0$.

Question 3. A point moves as so that the difference of its distances from (ae, 0) and (−ae, 0) is 2a, prove that the equation to its locus is $x^2/a^2 - y^2/b^2 = 1$, where $b^2 = a^2 (e^2 - 1)$.

Solution:

Using distance formula, we get,

=> $\sqrt{(h-ae)^2+(k-0)^2}-\sqrt{(h-(-ae))^2+(k-0)^2}=2a$

=> $\sqrt{(h-ae)^2+(k-0)^2}=2a+\sqrt{(h-(-ae))^2+(k-0)^2}$

Squaring both sides, we get,

=> $(h-ae)^2+(k-0)^2=(2a+\sqrt{(h-(-ae))^2+(k-0)^2})^2$

=> $h^2 + a^2e^2 - 2aeh + k^2 = 4a^2 + (h + ae)^2 + k^2 + 4a\sqrt{(h+ae)^2+(k-0)^2}$

=> $h^2 + a^2e^2 - 2aeh + k^2 = 4a^2 + h^2 + a^2e^2 + 2aeh + k^2 + 4a\sqrt{(h+ae)^2+(k-0)^2}$

=> $-4aeh - 4a^2 = 4a\sqrt{(h+ae)^2+(k-0)^2}$

Squaring both sides again, we get,

=> $-(eh + a) = (h + ae)^2 + k^2$

=> $e^2h^2 + a^2 + 2aeh = h^2 + a^2e^2 + 2aeh + k^2$

=> $h^2 (e^2 - 1) - k^2 = a^2 (e^2 - 1)$

=> $\frac{h^2}{a^2}-\frac{k^2}{a^2(e^2-1)}=1$

As we are given, $b^2 = a^2 (e^2 - 1)$*, we get,*

=> $h^2/a^2 - k^2/b^2 = 1$

Replacing (h, k) with (x, y), we get the equation of the locus of our point,

=> $x^2/a^2 - y^2/b^2 = 1$

Hence proved.

Question 4. Find the locus of a point such that the sum of its distances from (0, 2) and (0, −2) is 6.

Solution:

=> CA + CB = 6

Using distance formula, we get,

=> $\sqrt{(a-0)^2+(b-2)^2}+\sqrt{(a-0)^2+(b-(-2))^2}=6$

=> $\sqrt{(a-0)^2+(b-2)^2}=6-\sqrt{a^2+(b+2)^2}$

Squaring both sides, we get,

=> $a^2 + b^2 + 4 - 4b = 36 + a^2 + b^2 + 4 + 4b -$ $12\sqrt{a^2+(b+2)^2}$

=> $-8b - 36 =$ $-12\sqrt{a^2+(b+2)^2}$

=> $-4\ (2b + 9) =$ $-12\sqrt{a^2+(b+2)^2}$

Squaring both sides again, we get,

=> $(2b + 9)^2 =$ $(3\sqrt{a^2+(b+2)^2})^2$

=> $4b^2 + 81 + 36b = 9a^2 + 9b^2 + 36b + 36$

=> $9a^2 + 5b^2 = 45$

Replacing (a, b) with (x, y), we get the locus of our point,

=> $9x^2 + 5y^2 = 45$

Therefore the locus of the point is $9x^2 + 5y^2 = 45$.

Question 5. Find the locus of a point which is equidistant from (1, 3) and x-axis.

Solution:

Let C (a, b) be any point on the locus and let A (1, 3) and B (a, 0). We are given,

=> CA = CB

=> $CA^2 = CB^2$

Using distance formula, we get,

$\Rightarrow a^2 + 1 - 2a + b^2 + 9 - 6b = b^2$

$\Rightarrow a^2 - 2a - 6b + 10 = 0$

Replacing (a, b) with (x, y), we get the locus of our point,

$\Rightarrow x^2 - 2x - 6y + 10 = 0$

Therefore the locus of the point is $x^2 - 2x - 6y + 10 = 0$.

Question 6. Find the locus of a point that moves such that its distance from the origin is three times is distance from x-axis.

Solution:

Let C (a, b) be any point on the locus and let A (0, 0) and B (a, 0). We are given,

$\Rightarrow CA = 3\ CB$

$\Rightarrow CA^2 = 9\ CB^2$

Using distance formula, we get,

$\Rightarrow (a - 0)^2 + (b - 0)^2 = 9\ [(a - a)^2 + (b - 0)^2]$

$\Rightarrow a^2 + b^2 = 9b^2$

$\Rightarrow a^2 = 8b^2$

Replacing (a, b) with (x, y), we get the locus of our point,

$\Rightarrow x^2 = 8y^2$

Therefore the locus of the point is $x^2 = 8y^2$.

Question 7. A (5, 3), B (3, −2) are two fixed points, find the equation to the locus a point P which moves so that the area of the triangle PAB is 9 sq. units.

=> $\frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)| = 9$

=> *|5(−2−b) + 3(b−3) + h(3+2)| = 18*

=> *|5a − 2b − 19| = 18*

=> *5a − 2b − 19 = ±18*

=> *5a − 2b − 37 = 0 or 5a − 2b − 1 = 0*

Replacing (a, b) with (x, y), we get the locus of our point,

=> *5x − 2y − 37 = 0 or 5x − 2y − 1 = 0*

Therefore the equation to the locus of the point is 5x − 2y − 37 = 0 or 5x − 2y − 1 = 0.

Question 8. Find the locus of a point such that the line segment having endpoints (2, 0) and (−2, 0) subtend a right angle at that point.

Solution:

Let C (a, b) be any point on the locus and let A (2, 0) and B (−2, 0).

We are given ∠ACB = 90°

=> $AB^2 = CA^2 + CB^2$

Using distance formula, we get,

=> $(2+2)^2 + (0-0)^2 = (a-2)^2 + (b-0)^2 + (a+2)^2 + (b-0)^2$

=> $16 = a^2 + 4 - 4a + b^2 + a^2 + 4 + 4a + b^2$

=> $2a^2 + 2b^2 + 8 = 16$

Replacing (a, b) with (x, y), we get the locus of our point,

=> $x^2 + y^2 = 4$

Therefore the locus of the point is $x^2 + y^2 = 4$.

Question 9. A (−1, 1), B (2, 3) are two fixed points, find the locus of a point P which moves so that the area of the triangle PAB is 8 sq. units.

Solution:

Let P (a, b) be any point on the locus and we have A (−1, 1) and B (2, 3). We are give n,

=> Area of the triangle PAB = 8

=> $\frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)| = 8$

=> $|-1(3-b) + 2(b-1) + a(1-3)| = 16$

=> $|-2a + 3b - 5| = 16$

=> $-2a + 3b - 5 = \pm 16$

=> $2a - 3b + 21 = 0$ or $2a - 3b - 11 = 0$

Replacing (a, b) with (x, y), we get the locus of our point,

=> $2x - 3y + 21 = 0$ or $2x - 3y - 11 = 0$

Therefore the locus of the point is $2x - 3y + 21 = 0$ or $2x - 3y - 11 = 0$.

Question 10. A rod of length l slides between two perpendicular lines. Find the locus of the point on the rod which divides it in the ratio 1:2.

lution:

Let C (h, k) be any point on the locus and let AB = l (given) be the length of the rod. Suppose, coordinates of A and B are (a, 0) and (0, b) respectively.

According to the question,

=> $h = 2a/3$

=> $a = 3h/2$ . . . . (1)

And $k = b/3$

=> $b = 3k$ . . . . (2)

Let the origin be O (0, 0). Now we know △ AOB is right-angled.

=> $AB^2 = OA^2 + OB^2$

=> $l^2 = [(a-0)^2 + (0-0)^2] + [(0-0)^2 + (b-0)^2]$

=> $a^2 + b^2 = l^2$

Using (1) and (2), we get,

=> $(3h/2)^2 + (3k)^2 = l^2$

=> $9h^2/4 + 9k^2 = l^2$

=> $9h^2 + 36k^2 = 4l^2$

Replacing (h, k) with (x, y), we get the locus of our point,

=> $9x^2 + 36y^2 = 4l^2$

Therefore the locus of the point is $9x^2 + 36y^2 = 4l^2$.

Question 11. Find the locus of the mid-point of the portion of the line x cos α + y sin α = p which is intercepted between the axes.

Solution:

We are given,

=> $\frac{x}{\frac{p}{cos}} + \frac{y}{\frac{p}{sin}} = 1$

Intercepts on x-axis and y -axis are p/cos α and p/sin α respectively.

Suppose (x, y) is the mid-point of the portion of the given line which is intercepted between the axes.

=> (x, y) = $\left(\frac{\frac{p}{cos}+0}{2}, \frac{\frac{p}{sin}+0}{2}\right) = \left(\frac{p}{2cos}, \frac{p}{2sin}\right)$

=> $x = p/2 \cos\alpha$ and $y = p/2 \sin\alpha$

=> $2 \cos\alpha = p/x$ and $2 \sin\alpha = p/y$

Squaring both sides of these, we get,

=> $4 \cos^2\alpha = p^2/x^2$ . . . . (1)

=> $4 \sin^2\alpha = p^2/y^2$ . . . . (2)

Adding (1) and (2), we get,

=> $4 \cos^2\alpha + 4 \sin^2\alpha = p^2/x^2 + p^2/y^2$

=> $p^2/x^2 + p^2/y^2 = 4$

=> $p^2 (x^2 + y^2) = 4x^2y^2$

Therefore the locus of the mid-point is $p^2 (x^2 + y^2) = 4x^2y^2$.

Question 12. If O is the origin and Q is the variable point on $y^2 = x$. Find the locus of the mid-point of OQ.

Solution:

Let P (h, k) be the point on the locus and let Q (a, b).

According to the question,

=> $h = (a+0)/2$ and $k = (b+0)/2$

=> $a = 2h$ and $b = 2k$

As point Q lies on $y^2 = x$, we get,

=> $(2k)^2 = 2h$

=> $4k^2 = 2h$

=> $2k^2 = h$

Replacing (h, k) with (x, y), we get the locus of our point,

=> $2y^2 = x$

Therefore the locus of the mid-point is $2y^2 = x$.