Exercise – 9.1

1.

Sol:

We have to write first five terms of given sequences

(i) $a_n = 3n + 2$

Given sequence $a_n = 3n + 2$

To write first five terms of given sequence put n = 1, 2, 3, 4, 5, we get

$a_1 = (3 \times 1) + 2 = 3 + 2 = 5$

$a_2 = (3 \times 2) + 2 = 6 + 2 = 8$

$a_3 = (3 \times 3) + 2 = 9 + 2 = 11$

$a_4 = (3 \times 4) + 2 = 12 + 2 = 14$

$a_5 = (3 \times 5) + 2 = 15 + 2 = 17$

$\therefore$ The required first five terms of given sequence $a_n = 3n + 2$ are 5, 8, 11, 14, 17.

(ii) $a_n = \frac{n-2}{3}$

Given sequence $a_n = \frac{n-2}{3}$

To write first five terms of given sequence $a_n = \frac{n-2}{3}$

put n = 1, 2, 3, 4, 5 then we get

$a_1 = \frac{1-2}{3} = \frac{-1}{3}; a_2 = \frac{2-2}{3} = 0$

$a_3 = \frac{3-2}{3} = \frac{1}{3}; a_4 = \frac{4-2}{3} = \frac{2}{3}$

$a_5 = \frac{5-2}{3} = 1$

$\therefore$ The required first five terms of given sequence $a_n = \frac{n-2}{3}$ are $\frac{-1}{3}, 0, \frac{1}{3}, \frac{2}{3}, 1$.

(iii) $a_n = 3^n$

Given sequence $a_n = 3^n$

To write first five terms of given sequence, put n = 1, 2, 3, 4, 5 in given sequence.

Then,

$a_1 = 3^1 = 3; a_2 = 3^2 = 9; a_3 = 27; a_4 = 3^4 = 81; a_5 = 3^5 = 243.$

(iv) $a_n = \frac{3n-2}{5}$

Given sequence, $a_n = \frac{3n-2}{5}$

To write first five terms, put n = 1, 2, 3, 4, 5 in given sequence $a_n = \frac{3n-2}{5}$

Then, we ger

$a_1 = \frac{3\times1-2}{5} = \frac{3-2}{5} = \frac{1}{5}$

$a_2 = \frac{3\times2-2}{5} = \frac{6-2}{5} = \frac{4}{5}$

$a_3 = \frac{3\times3-2}{5} = \frac{9-2}{5} = \frac{7}{5}$

$a_4 = \frac{3\times4-2}{5} = \frac{12-2}{5} = \frac{10}{5}$

$a_5 = \frac{3\times5-2}{5} = \frac{15-2}{5} = \frac{13}{5}$

∴ The required first five terms are $\frac{1}{5}, \frac{4}{5}, \frac{7}{5}, \frac{10}{5}, \frac{13}{5}$

(v) $a_n = (-1)^n 2^n$

Given sequence is $a_n = (-1)^n 2^n$

To get first five terms of given sequence an, put n = 1, 2, 3, 4, 5.

$a_1 = (-1)^1 . 2^1 = (-1).2 = -2$

$a_2 = (-1)^2 . 2^2 = (-1).4 = 4$

$a_3 = (-1)^3 . 2^3 = (-1).8 = -8$

$a_4 = (-1)^4 . 2^4 = (-1).16 = 16$

$a_5 = (-1)^5 . 2^5 = (-1).32 = -32$

∴ The first five terms are -2, 4, -8, 16, -32.

(vi) $a_n = \frac{n(n-2)}{2}$

The given sequence is, $a_n = \frac{n(n-2)}{2}$

To write first five terms of given sequence $a_n = \frac{n(n-2)}{2}$

Put n = 1, 2, 3, 4, 5. Then, we get

$a_1 = \frac{1(1-2)}{2} = \frac{1-1}{2} = \frac{-1}{2}$

$a_2 = \frac{2(2-2)}{2} = \frac{2.0}{2} = 0$

$a_3 = \frac{3(3-2)}{2} = \frac{3.1}{2} = \frac{3}{2}$

$a_4 = \frac{4(4-2)}{2} = \frac{4.2}{2} = 4$

$a_5 = \frac{5(5-2)}{2} = \frac{5.3}{2} = \frac{15}{2}$

∴ The required first five terms are $\frac{-1}{2}, 0, \frac{3}{2}, 4, \frac{15}{2}$.

(vii) $a_n = n^2 - n + 1$

The given sequence is, $a_n = n^2 - n + 1$

To write first five terms of given sequence a_{n1} we get put n = 1, 2, 3, 4, 5. Then we get $a_1 = 1^2 - 1 + 1 = 1$

$a_2 = 2^2 - 2 + 1 = 3$
$a_3 = 3^2 - 3 + 1 = 7$
$a_4 = 4^2 - 4 + 1 = 13$
$a_5 = 5^2 - 5 + 1 = 21$
∴The required first five terms of given sequence $a_n = n^2 - n + 1$ are 1, 3, 7, 13, 21

(viii) $a_n = 2n^2 - 3n + 1$
The given sequence is $a_n = 2n^2 - 3n + 1$
To write first five terms of given sequence a_n, we put n = 1, 2, 3, 4, 5. Then we get
$a_1 = 2.1^2 - 3.1 + 1 = 2 - 3 + 1 = 0$
$a_2 = 2.2^2 - 3.2 + 1 = 8 - 6 + 1 = 3$
$a_3 = 2.3^2 - 3.3 + 1 = 18 - 9 + 1 = 10$
$a_4 = 2.4^2 - 3.4 + 1 = 32 - 12 + 1 = 21$
$a_5 = 2.5^2 - 3.5 + 1 = 50 - 15 + 1 = 36$
∴ The required first five terms of given sequence $a_n = 2n^2 - 3n + 1$ are 0, 3, 10, 21, 36

(ix) $a_n = \frac{2n-3}{6}$
Given sequence is, $a_n = \frac{2n-3}{6}$
To write first five terms of given sequence we put n = 1, 2, 3, 4, 5. Then, we get,
$a_1 = \frac{2.1-3}{6} = \frac{2-3}{6} = \frac{-1}{6}$
$a_2 = \frac{2.2-3}{6} = \frac{4-3}{6} = \frac{1}{6}$
$a_3 = \frac{2.4-3}{6} = \frac{8-3}{6} = \frac{5}{6}$
$a_4 = \frac{2.4-3}{6} = \frac{8-3}{6} = \frac{5}{6}$
$a_5 = \frac{2.5-3}{6} = \frac{10-3}{6} = \frac{7}{6}$
∴ The required first five terms of given sequence $a_n = \frac{2n-3}{6}$ are $\frac{-1}{6}, \frac{1}{6}, \frac{1}{2}, \frac{5}{6}, \frac{7}{6}$.

2.
Sol:
We have to find the required term of a sequence when nth term of that sequence is given.
(i) $a_n = 5n - 4; a_{12}\ and\ a_{15}$
Given nth term of a sequence $a_n = 5n - 4$
To find 12th term, 15th terms of that sequence, we put n = 12, 15 in its nth term.
Then, we get
$a_{12} = 5.12 - 4 = 60 - 4 = 56$
$a_{15} = 5.15 - 4 = 15 - 4 = 71$

$\therefore$ The required terms $a_{12} = 56, a_{15} = 71$

(ii) $a_n = \frac{3n-2}{4n+5}$; a_7 and a_8

Given nth term is (a_n) = $\frac{3n-2}{4n+5}$

To find 7th, 8th terms of given sequence, we put n = 7, 8.

$a_7 = \frac{(3.7)-2}{(4.7)+5} = \frac{19}{33}$

$a_8 = \frac{(3.8)-2}{(4.8)+5} = \frac{22}{37}$

$\therefore$ The required terms $a_7 = \frac{19}{33}$ and $a_8 = \frac{22}{37}$.

(iii) $a_n = n(n-1)(n-2)$; a_5 and a_8

Given nth term is $a_n = n(n-1)(n-2)$

To find 5th, 8th terms of given sequence, put n= 5, 8 in an then, we get

$a_5 = 5(5-1).(5-2) = 5.4.3 = 60$

$a_8 = 8.(8-1).(8-2) = 8.7.6 = 336$

$\therefore$ The required terms are $a_5 = 60$ and $a_8 = 336$

(iv) $a_n = (n-1)(2-n)(3+n); a_{11}\ a_{21}\ a_3$

The given nth term is $a_n = (n+1)(2-n)(3+n)$

To find a_1, a_2, a_3 of given sequence put n = 1, 2, 3 is an

$a_1 = (1-1)(2-1)(3+1) = 0.1.4 = 0$

$a_2 = (2-1)(2-2)(3+2) = 1.0.5 = 0$

$a_3 = (3-1)(2-3)(3+3) = 2.-1.6 = -12$

$\therefore$ The required terms $a_1 = 0$, $a_2 = 0$, $a_3 = -12$

(v) $a_n = (-1)^n n; a_3, a_5, a_8$

The given nth term is, $a_n = (-1)^n.n$

To find $a_3.a_5, a_8$ of given sequence put n = 3, 5, 8, in a_n.

$a_3 = (-1)^3.3 = -1.3 = -3$

$a_5 = (-1)^5.5 = -1.5 = -5$

$a_8 = (-1)^8 = 1.8 = 8$

$\therefore$ The required terms $a_3 = -3, a_5 = -5, a_8 = 8$

3.

Sol:

We have to find next five terms of following sequences.

(i) $a_1 = 1, a_n = a_{n-1} + 2,\ n \geq 2$

Given, first term (a_1) = 1,

nth term $a_n = a_{n-1} + 2, n \geq 2$

To find 2nd, 3rd, 4th , 5th , 6th terms, we use given condition n ≥ 2 for nth term $a_n = a_{n-1} + 2$

$a_2 = a_{2-1} + 2 = a_1 + 2 = 1 + 2 = 3\ (\therefore a_1 = 1)$

$a_3 = a_{3-1} + 2 = a_2 + 2 = 3 + 2 = 5$

$a_4 = a_{4-1} + 2 = a_3 + 2 = 5 + 2 = 7$

$a_5 = a_{5-1} + 2 = a_4 + 2 = 7 + 2 = 9$

$a_6 = a_{6-1} + 2 = a_5 + 2 = a + 2 = 11$

∴ The next five terms are,

$a_2 = 3, a_3 = 5, a_4 = 7, a_5 = a, a_6 = 11$

(ii) $a_1 = a_2 = 2, a_n = a_{n-1} - 3, n > 2$

Given,

First term $(a_1) = 2$

Second term $(a_2) = 2$

n^th term $(a_n) = a_{n-1} - 3$

To find next five terms i.e., a_3, a_4, a_5, a_6, a_7 we put n = 3, 4, 5, 6, 7 is a_n

$a_3 = a_{3-1} - 3 = 2 - 3 = -1$

$a_4 = a_{4-1} - 3 = a_3 - 3 = -1 - 3 = -4$

$a_5 = a_{5-1} - 3 = a_4 - 3 = -4 - 3 = -7$

$a_6 = a_{6-1} - 3 = a_5 - 3 = -7 - 3 = -10$

$a_7 = a_{7-1} - 3 = a_6 - 3 = -10 - 3 = -13$

∴ The next five terms are, $a_3 = -1, a_4 = -4, a_5 = -7, a_6 = -10, a_7 = -13$

(iii) $a_1 = -1, a_n = \frac{a_n - 1}{n}, n \geq 2$

Given, first term $(a_1) = -1$

n^th term $(a_n) = \frac{a_n - 1}{n}, n \geq 2$

To find next five terms i.e., a_2, a_3, a_4, a_5, a_6 we put n = 2, 3, 4, 5, 6 is an

$a_2 = \frac{a_{2-1}}{2} = \frac{a_1}{2} = \frac{-1}{2}$

$a_3 = \frac{a_{3-1}}{3} = \frac{a_2}{3} = \frac{-1/2}{3} = \frac{-1}{6}$

$a_4 = \frac{a_{4-1}}{4} = \frac{a_3}{4} = \frac{-1/6}{4} = \frac{-1}{24}$

$a_5 = \frac{a_{5-1}}{5} = \frac{a_4}{5} = \frac{-1/24}{5} = \frac{-1}{120}$

∴ The next five terms are,

$a_2 = \frac{-1}{2}, a_3 = \frac{-1}{6}, a_4 = \frac{-1}{24}, a_5 = \frac{-1}{120}, a_6 = \frac{-1}{720}$

(iv) $a_1 = 4, a_n = 4\,a_{n-1} + 3, n > 1$

Given,

First term $(a_1) = 4$

n^th term $(a_n) = 4\,a_{n-1} + 3$, n > 1

To find next five terms i.e., a_2, a_3, a_4, a_5, a_6 we put n= 2, 3, 4, 5, 6 is a_n

Then, we get

$a_2 = 4a_{2-1} + 3 = 4.a_1 + 3 = 4.4 + 3 = 19\ (\because a_1 = 4)$

$a_3 = 4a_{3-1} + 3 = 4.a_2 + 3 = 4(19) + 3 = 79$

$a_4 = 4\, a_{4-1} + 3 = 4. a_3 + 3 = 4(79) + 3 = 319$
$a_5 = 4\, a_{5-1} + 3 = 4. a_4 + 3 = 4(319) + 3 = 1279$
$a_6 = 4. a_{6-1} + 3 = 4. a_5 + 3 = 4(1279) + 3 = 5119$
∴ The required next five terms are,
$a_2 = 19, a_3 = 79, a_4 = 319, a_5 = 1279, a_6 = 5119$

Exercise – 9.2

1.

Sol:
We know that if a is the first term and d is the common difference, the arithmetic progression is $a, a+d, a+2d+a+3d, ...$

(i) $-5, -1, 3, 7, ...$
Given arithmetic series is
$-5, -1, 3, 7.............$
This is in the form of $a, a+d, a+2d+a+3d, ...$ by comparing these two
$a = -5, a+d = 1, a+2d = 3, a+3d = 7, ...$
First term $(a) = -5$
By subtracting second and first term, we get
$(a+d) - (a) = d$
$-1 - (-5) = d$
$4 = d$
Common difference $(d) = 4.$

(ii) $\frac{1}{5}, \frac{3}{5}, \frac{5}{5}, \frac{7}{5}, ...$
Given arithmetic series is,
$\frac{1}{5}, \frac{3}{5}, \frac{5}{5}, \frac{7}{5}, ...$
This is in the form of $\frac{1}{5}, \frac{2}{5}, \frac{5}{5}, \frac{7}{5}, ...$
$a, a+d, a+2d, a+3d, ...$
By comparing this two, we get
$a = \frac{1}{5}, a+d = \frac{3}{5}, a+2d = \frac{5}{5}, a+3d = \frac{7}{5}$

$\boxed{First\ term\ \cos = \frac{1}{5}}$

By subtracting first term from second term, we get

$d=(a+d)-(a)$

$d=\frac{3}{5}-\frac{1}{5}$

$d=\frac{2}{5}$

$\boxed{common\ difference(d)=\frac{2}{5}}$

(iii) 0.3, 0.55, 0.80, 1.05,.........

Given arithmetic series,

0.3, 0.55, 0.80, 1.05,.........

General arithmetic series

$a, a+d, a+2d, a+3d, ...$

By comparing,

$a=0.3, a+d=0.55, a+2d=0.80, a+3d=1.05$

First term $(a)=0.3.$

By subtracting first term from second term. We get

$d=(a+d)-(a)$

$d=0.55-0.3$

$d=0.25$

Common difference (d) $=0.25$

(iv) $-1.1, -3.1, -5.1, -7.1, ...$

General series is

$a, a+d, a+2d, a+3d, ...$

By comparing this two, we get

$a=-1.1, a+d=-3.1, a+2d=-5.1, a+3d=-71$

First term $(a)=-1.1$

Common difference $(d)=(a+d)-(a)$

$=-3.1-(-1.1)$

Common difference (d) $=-2$

2.

Sol:

We know that, if first term $(a)=a$ and common difference $=d$, then the arithmetic series is, $a, a+d, a+2d, a+3d, ...$

(i) $a=4, d=-3$

Given first term $(a)=4$

Common difference $(d)=-3$

Then arithmetic progression is,

$a, a+d, a+2d, a+3d, ...$

$\Rightarrow 4, 4-3, a+2(-3), 4+3(-3), ...$

$\Rightarrow 4, 1, -2, -5, -8, ...$

(ii) $a=-1, d=\frac{1}{2}$

Given,

First term $(a)=-1$

Common difference $(d)=\frac{1}{2}$

Then arithmetic progression is,

$\Rightarrow a, a+d, a+2d, a+3d, ...$

$\Rightarrow -1, -1+\frac{1}{2}, -1+2\frac{1}{2}, -1+3\frac{1}{2}, ...$

$\Rightarrow -1, \frac{-1}{2}, 0, \frac{1}{2}, ...$

(iii) $a=-1.5, d=-0.5$

Given

First term $(a)=-1.5$

Common difference $(d)=-0.5$

Then arithmetic progression is

$\Rightarrow a, a+d, a+2d, a+3d, ...$

$\Rightarrow -1.5, -1.5-0.5, -1.5+2(-0.5), -1.5+3(-0.5)$

$\Rightarrow -1.5, -2, -2.5, -3, ...$

Then required progression is

$-1.5, -2, -2.5, -3, ...$

3.

Sol:

(i) Given,

Cost of digging a well for the first meter $(c_1)=Rs.150.$

Cost rises by $Rs.20$ for each succeeding meter

Then,

Cost of digging for the second meter $(c_2) = Rs.150 + Rs\ 20$

$= Rs\ 170$

Cost of digging for the third meter $(c_3) = Rs.170 + Rs\ 20$

$= Rs\ 210$

Thus, costs of digging a well for different lengths are

$150, 170, 190, 210, ...$

Clearly, this series is in $A \cdot p.$

With first term $(a) = 150,$ common difference $(d) = 20$

(ii) Given

Let the initial volume of air in a cylinder be V liters each time $\frac{3}{4}^{th}$ of air in a remaining i.e.,

$1 - \frac{1}{4}$

First time, the air in cylinder is $\frac{3}{4}V.$

Second time, the air in cylinder is $\frac{3}{4}V.$

Third time, the air in cylinder is $\left(\frac{3}{4}\right)^2 V.$

Therefore, series is $V, \ \frac{3}{4}V, \left(\frac{3}{4}\right)^2 V, \left(\frac{3}{4}\right)^3 V, ...$

4.

Sol:

Given sequence is

$a_n = 5n - 7$

n^{th} term of given sequence $(a_n) = 5n - 7$

$(n+1)^{th}$ term of given sequence $(a_n + 1) - a_n$

$= (5n - 2) - (5n - 7)$

$= 5$

$\therefore d = 5$

5.

Sol:
Given sequence is,
$a_n = 3n^2 - 5.$
n^{th} term of given sequence $(a_n) = 3n^2 - 5.$
$(n+1)^{th}$ term of given sequence $(a_n + 1) = 3(n+1)^2 - 5$
$= 3(n^2 + 1^2 + 2n.1) - 5$
$= 3n^2 + 6n - 2$
$\therefore$ The common difference $(d) = a_n + 1 - an$
$d = (3n^2 + 6n - 2) - (3n^2 - 5)$
$= 3a^2 + 6n - 2 - 3n^2 + 5$
$= 6n + 3$
Common difference (d) depends on $'n'$ value
$\therefore$ given sequence is not in $A.p$

6.
Sol:
Given sequence is,
$a_n = -4n + 15.$
n^{th} term is $(a_n) = -4n + 15$
$(n+1)^{th}$ term is $(a_{n+1}) = -4(n+1) + 15$
$= -4n - 4 + 15$
$= -4n + 11$
Common difference $(d) = a_{n+1} - an$
$= (-4n + 11) - (-4n + 15)$
$= -4n + 11 + 4n - 15$
$d = -4$
Common difference $(d) = a_{n+1} - an$
$= (-4n + 11) + (-4n + 15)$
$= -4n + 11 + 4n - 15$
$d = -4.$
Common difference (d) does not depend on $'n'$ value
$\therefore$ given sequence is in $A.P$

$\Rightarrow 15^{th}$ term $a_{15} = -4(15) + 15$

$= -60 + 15$

$= -45$

$a_{15} = -45$

7.

Sol:

(i) $1, -2, -5, -8, \ldots\ldots.$

Given arithmetic progression is,

$a_1 = 1, a_2 = -2, a_3 = -5, a_4 = -8 \ldots\ldots\ldots$

Common difference $(d) = a_2 - a_1$

$= -2 - 1$

$d = -3$

To find next four terms

$a_5 = a_4 + d = -8 - 3 = -11$

$a_6 = a_5 + d = -11 - 3 = -14$

$a_7 = a_6 + d = -14 - 3 = -17$

$a_8 = a_7 + d = -17 - 3 = -20$

$\therefore d = -3, a_5 = -11, a_6 = -16, a_7 = -17, a_8 = -20$

(ii) $0, -3, -6, -9, \ldots\ldots\ldots\ldots$

Given arithmetic progression is.

$0, -3, -6, a_4 = -9 \ldots\ldots\ldots\ldots$

Common difference $(d) = a_2 - a_1$

$= -3 - 0$

$d = -3$

To find next four terms

$a_5 = a_4 + d = -9 - 3 = -12$

$a_6 = a_5 + d = -12 - 3 = -15$

$a_7 = a_6 + d = -15 - 3 = -18$

$a_8 = a_7 + d = -18 - 3 = -21$

$\therefore d = -3, a_5 = -12, a_6 = -15, a_7 = -17, a_8 = -21$

(iii) $-1, \frac{1}{4}, \frac{3}{2}, \ldots\ldots\ldots$

Given arithmetic progression is,

$-1, \frac{1}{4}, \frac{3}{2}, \ldots\ldots\ldots$

$a_1 = -1, a_2 = \frac{1}{4}, a_3 = \frac{3}{2}, \ldots\ldots\ldots$

Common difference $(d) = a_2 - a_1$

$= \frac{1}{4} - (-1)$

$= \frac{1+4}{4}$

$d = \frac{5}{4}$

To find next four terms,

$a_4 = a_3 + d = \frac{3}{2} + \frac{5}{4} = \frac{6+4}{4} = \frac{11}{4}$

$a_5 = a_4 + d = \frac{11}{4} + \frac{5}{4} = \frac{16}{4}$

$a_6 = a_5 + d = \frac{16}{4} + \frac{5}{4} = \frac{21}{4}$

$a_7 = a_6 + d = \frac{21}{4} + \frac{5}{4} = \frac{26}{4}$

$\therefore d = \frac{5}{4}, a_4 = \frac{11}{4}, a_5 = \frac{16}{4}, a_6 = \frac{21}{4}, a_7 = \frac{26}{4}.$

(iv) Given arithmetic progression is,

$-1, \frac{-5}{6}, \frac{-2}{3}, \ldots\ldots\ldots$

$a_1 = -1, a_2 = \frac{-5}{6}, a_3 = \frac{-2}{3}, \ldots\ldots\ldots$

Common difference $(d) = a_2 - a_1$

$= \frac{-5}{6} - (-1)$

$= \frac{-5+6}{6}$

$= \frac{1}{6}$

To find next four terms,

$$a_4 = a_3 + d = \frac{-2}{3} + \frac{1}{6} = \frac{-4+1}{6} = \frac{-\cancel{3}^{1}}{\cancel{6}_{3}} = -\frac{1}{2}.$$

$$a_5 = a_4 + d = \frac{-1}{2} + \frac{1}{6} = \frac{-3+1}{6} = \frac{-\cancel{2}^{1}}{\cancel{6}_{3}} = -\frac{1}{2}.$$

$$a_6 = a_5 + d = \frac{-1}{3} + \frac{1}{6} = \frac{-2+1}{6} = -\frac{1}{16}.$$

$$a_7 = a_6 + d = \frac{-1}{6} + \frac{1}{6} = 0.$$

$$\therefore d = \frac{1}{6}, a_4 = -\frac{1}{2}, a_5 = -\frac{1}{3}, a_6 = -\frac{1}{6}, a_7 = 0$$

8.

Sol:

Given sequence $(a_n) = a_n + 6n$

n^{th} term $(a_n) = a + nb$

$(n+1)^{th}$ term $(a_{n+1}) = a + (n+1)b.$

Common difference (d) $= a_{n+1} - an$

$d = (a + (n+1)b) - (a + nb)$

$= \cancel{a} + \cancel{n}b + b - \cancel{a} - \cancel{nb}$

$= b$

$\therefore$ common difference (d) does not depend on n^{th} value so, given sequence I sin Ap with $(d) = b$

9.

Sol:

(i) $a_n = 3 + 4n$

Given, n^{th} term $a_n = 3 + 4n$

$(n+1)^{th}$ term $a_{n+1} = 3 + 4(n+1)$

Common difference $(d) = a_{n+1} - a_n$

$= (3 + 4(n+1)) - 3 + 4n$

$= 4.$

$d = 4$ does not depend on n value so, the given series is in $A.p$ and the sequence is

$a_1 = 3 + 4(1) = 3 + 4 = 7$

$a_2 = a_1 + d = 7 + 4 = 11; a_3 = a_2 + d = 11 + 4 = 15$

$\Rightarrow 7, 11, 15, 19, ...$

(ii) $a_n = 5 + 2n$

Given, n^{th} term $(a_n) = 5 + 2n$

$(n+1)^{th}$ term $(a_{n+1}) = 5 + 2(n+1)$

$= 7 + 2n$

Common difference $(d) = 7 + 2n - 5 - 20$

$= 2.$

$\because d = 2$ does not depend on n value given sequence is in $A.p$ and the sequence is B_1

$a_1 = 5 + 2.1 = 7$

$a_2 = 7 + 2 = 9, a_3 = 9 + 2 = 11, a_4 = 11 + 2 = 13$

$\Rightarrow 7, 9, 11, 13, ...$

(iii) $a_n = 6 - n$

Given, n^{th} term $a_n = 6 - n$

$(n+1)^{th}$ term $a_{n+1} = 6 - (n+1)$

$= 5 - n$

Common difference $(d) = a_{n+1} - a_n$

$= (5 - n) - (6 - n)$

$= -1$

$\therefore d = -1$ does not depend on n value given sequence is in $A.p$ the sequence is

$a_1 = 6 - 1 = 5, a_2 = 5 - 1 = 4, a_3 = 4 - 1 = 3, a_4 = 3 - 1 = 2$

$\Rightarrow 5, 4, 3, 2, 1, ...$

(iv) $a_n = 9 - 5n$

Given, n^{th} term $a_n = 9 - 5n$

$(n+1)^{th}$ term $a_{n+1} = 9 - 5(n+1)$

$= 4 - 5n$

Common difference $(d) = a_{n+1} - a_n$

$= (4 - 5n) - (4 - 5n)$

$= -5$

$\because d = -1$ does not depend on n value given sequence is in $A.p$

The sequence is,

$a_1 = 9 - 1.1 = 4$

$a_2 = 9 - 5.2 = -1$

$a_3 = 9 - 5.3 = -6$

$\Rightarrow 4, -1, -6, -11, \ldots\ldots\ldots$

10.

Sol:

(i) $3, 6, 12, 24, \ldots\ldots\ldots\ldots$

General arithmetic progression is $a, a+d, a+2d, a+3d, \ldots\ldots\ldots$

Common difference $(d) =$ Second term – first term

$= (d+d) - a = d$ (or)

$=$ Third term – second term

$= (a+2d) - (a+d) = d$

To check given sequence is in $A.p$ or not we use this condition.

Second term – First term = Third term – Second term

$a_1 = 3, a_2 = 6, a_3 = 12, a_4 = 24$

Second term – First term $= 6 - 3 = 3$

Third term – Second term $= 12 - 6 = 6$

This two are not equal so given sequence is not in $A.p$

(ii) $0, -4, -8, -12, \ldots\ldots\ldots\ldots$

In the given sequence

$a_1 = 0, a_2 = -4, a_3 = -8, a_4 = -12$

Check the condition

Second term – first term = third term – second term

$a_2 - a_1 = a_8 - a_2$

$-4 - 0 = -8 - (-4)$

$-4 = +8 + 4$

$-4 = -4$

Condition is satisfied $\therefore$ given sequence is in $A.p$ with common difference

$(d) = a_2 - a_1 = -4$

(iii) $\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \ldots\ldots\ldots\ldots$

In the given sequence

$a_1 = \frac{1}{2}, a_2 = \frac{1}{4}, a_3 = \frac{1}{6}, a_4 = \frac{1}{8}.$

Check the condition

$a_2 - a_1 = a_3 - a_2$

$\frac{1}{4} - \frac{1}{2} = \frac{1}{6} - \frac{1}{4}$

$\frac{1-2}{4} = \frac{4-6}{24}$

$\frac{-1}{4} = -\frac{2}{24}$

$\frac{-1}{4} \neq \frac{-1}{12}$

Condition is not satisfied

$\therefore$ given sequence not in $A.p$

(iv) $12, 2, -8, -18, \ldots\ldots\ldots\ldots$

In the given sequence

$a_1 = 12, a_2 = 2, a_3 = -8, a_4 = -18$

Check the condition

$a_2 - a_1 = a_3 - a_2$

$2 - 12 = -8 - 2$

$-10 = -10$

$\therefore$ given sequence is in $A.p$ with common difference $d = -10$

(v) $3, 3, 3, 3, \ldots\ldots\ldots\ldots$

In the given sequence

$a_1 = 3, a_2 = 3, a_3 = 3, a_4 = 3$

Check the condition

$a_2 - a_1 = a_3 - a_2$

$3 - 3 = 3 - 3$

$0 = 0$

$\therefore$ given sequence is in $A.p$ with common difference $d = 0$

(vi) $p, p+90, p+80, p+270, \ldots\ldots$ where $p = (999)$

In the given sequence

$a_1 = p, a_2 = p+90, a_3 = p+180, a_4 = p+270$

Check the condition

$a_2 - a_1 = a_3 - a_2$

$\cancel{p} + 90 - \cancel{p} = \cancel{p} + 180 - \cancel{p} - 90$

$90 = 180 - 90$

$90 = 90$

(vii) 1.0, 1.7, 2.4, 3.1….,

In the given sequence

$a_1 = 1.0, a_2 = 1.7, a_3 = 2.4, a_4 = 31$

Check the condition

$a_2 - a_1 = a_3 - a_2$

$1.7 - 1.0 = 2.4 - 1.7$

$0.7 = 0.7$

$\therefore$ The given sequence is in $A.p$ with $d = 0.7$

(viii) $-225, -425, -625, -825, ...$

In the given sequence

$a_1 = 225, a_2 = -425, a_3 = -625, a_4 = -825$

Check the condition

$a_2 - a_1 = a_3 - a_2$

$-425 + 225 = -625 + 425$

$-200 = -200$

$\therefore$ The given sequence is in $A.p$ with $d = -200$

(ix) $10, 10 + 2^5, 10 + 2^6, 10 + 2^7, ...$

In the given sequence

$a_1 = 10, a_2 = 10 + 2^5, a_3 = 10 + 2^6, a_4 = 10 + 2^7$

Check the condition

$a_2 - a_1 = a_3 - a_2$

$10 + 2^5 - 10 = 10 + 2^6 - 10 - 2^5$

$2^5 \neq 2^6 - 2^5.$

$\therefore$ The given sequence is not in $A.p$

Exercise – 9.3

1.

Sol:

(i) Given $A.p$ is

$1, 4, 7, 10, ...$

First term $(a) = 1$

Common difference (d) = second term first term

$= 4 - 1$

$= 3.$

n^{th} term in an $A.p = a + (n-1)d$

10^{th} term in an $1+(10-1)3$

$=1+9.3$

$=1+27$

$=28$

(ii) Given $A.p$ is

$\sqrt{2}, 3\sqrt{2}, 5\sqrt{2}, ...$

First term $(a) = \sqrt{2}$

Common difference $=$ Second term – First term

$= 3\sqrt{2} - \sqrt{2}$

$d = 2\sqrt{2}$

n^{th} term in an $A.p = a + (n-1)d$

18^{th} term of $A.p = \sqrt{2} + (18-1)2\sqrt{2}$

$= \sqrt{2} + 17.2\sqrt{2}$

$= \sqrt{2}(1+34)$

$= 35\sqrt{2}$

$\therefore 18^{th}$ term of $A.p$ is $35\sqrt{2}$

(iii) Given $A.p$ is

$13, 8, 3, -2, ...$

First term $(a) = 13$

Common difference $(d) =$ Second term first term

$= 8 - 13$

$= -5$

n^{th} term of an $A.p$ $a_n = a + (n-1)d$

$= 13 + (n-1) - 5$

$= 13 - 5n + 5$

$a_n = 18 - 5n$

(iv) Given $A.p$ is

$-40, -15, 10, 35, ...$

First term $(a) = -40$

Common difference $(d) =$ Second term – first term

$= -15 - (-40)$

$= 40 - 15$

$= 25$

n^{th} term of an $A.p$ $a_n = a + (n-1)d$

10^{th} term of $A.p$ $a_{10} = -40 + (10-1)25$

$= -40 + 9.25$

$= -40 + 225$

$= 185$

(v) Given sequence is

$117, 104, 91, 78, ...$

First learn can $= 117$

Common difference $(d) =$ Second term – first term

$= 104 - 117$

$= -13$

n^{th} term $a_n = a + (n-1)d$

8^{th} term $a_8 = a + (8-1)d$

$= 117 + 7(-13)$

$= 117 - 91$

$= 26$

(vi) Given $A.p$ is

$10.0, 10.5, 11.0, 11.5, ...$

First term $(a) = 10.0$

Common difference (d) = Second term – first term

$= 10.5 - 10.0$

$= 0.5$

n^{th} term $a_n = a + (n-1)d$

11^{th} term $a_{11} = 10.0 + (11-1)0.5$

$= 10.0 + 10 \times 0.5$

$= 10.0 + 5$

$= 15.0$

(vii) Given $A.p$ is

$\frac{3}{4}, \frac{5}{4}, \frac{7}{4} + \frac{9}{4}, ...$

First term $(a) = \frac{3}{4}$

Common difference $(d) =$ Second term – first term

$= \frac{5}{4} - \frac{3}{4}$

$= \frac{2}{4}$

n^{th} term $a_n = a + (n-1)d$

9^{th} term $a_9 = a + (9-1)d$

$= \frac{3}{4} + 8.\frac{2}{4}$

$= \frac{3}{4} + \frac{16}{4}$

$= \frac{19}{4}$

2.

Sol:

(i) Given $A.p$ is $3, 8, 13, ...$

First term $(a) = 3$

Common difference $(d) =$ Second term – first term

$= 8 - 3$

$= 5$

n^{th} term $(a_n) = a + (n-1)d$

Given n^{th} term $a_n = 248$

$248 = 3 + (n-1).5$

$248 = -2 + 5n$

$5n = 250$

$n = \frac{250}{5} = 50$

50^{th} term is $248.$

(ii) Given $A.p$ is $84, 80, 76, ...$

First term $(a) = 84$

Common difference $(d) = a_2 - a$

$= 80 - 84$

$= -4$

n^{th} term $(a_n) = a + (n-1)d$

Given n^{th} term is 0

$0 = 84 + (n-1) - 4$

$+84 = +4(n-1)$

$n-1 = \frac{\cancel{84}^{21}}{4} = 21$

$n = 21 + 1 = 22$

22^{nd} term is 0.

(iii) Given $A.p$ $4, 9, 14, ...$

First term $(a) = 4$

Common difference $(d) = a^2 - a$

$= 9 - 4$

$= 5$

n^{th} term $(a_n) = a + (n-1)d$

Given n^{th} term is 254

$4 + (n-1)5 = 254$

$(n-1) \cdot 5 = 250$

$n-1 = \frac{250}{5} = 50$

$n = 51$

$\therefore 51^{st}$ term is 254.

(iv) Given $A.p$

$21, 42, 63, 84, ...$

$a = 21, d = a_2 - a$

$= 42 - 21$

$= 21$

n^{th} term $(a_n) = a + (n-1)d$

Given n^{th} term $= 420$

$21 + (n-1)21 = 420$

$(n-1)21 = 399$

$n-1 = \frac{399}{21} = 19$

$n = 20$

$\therefore 20^{th}$ term is 420.

(v) Given $A.p$ is $121, 117, 113, ...$

First term $(a) = 121$

Common difference $(d) = 117 - 121$

$= -4$

n^{th} term $(a) = a + (n-1)d$

Given n^{th} term is negative i.e., $a_n < 0$

$121 + (n-1) - 4 < 0$

$121 + 4 - 4n < 0$

$125 - 4n < 0$

$4n > 125$

$n > \dfrac{125}{4}$

$n > 31.25$

The integer which comes after 31.25 is 32.

$\therefore 32^{nd}$ term is first negative term

3.

Sol:

In the given problem, we are given an $A.p$ and the Value of one of its term

We need to find whether it is a term of the $A.p$ or not so here we will use the formula

$a_n = a + (n-1)d$

(i) Here, $A.p$ is $7, 10, 13, \ldots\ldots\ldots$

$a_n = 68, a = 7$ and $d = 10 - 7 = 3$

Using the above mentioned formula, we get

$68 = 7 + (n-1)3$

$\Rightarrow 68 - 7 = 3n - 3$

$\Rightarrow 31 + 3 = 3n$

$\Rightarrow 64 = 3n$

$\Rightarrow n = \dfrac{64}{3}$

Since, the value of n is a fraction.

Thus, 68 is not the team of the given $A.p$

(ii) Here, $A.p$ is $3, 8, 13, \ldots\ldots\ldots$

$a_n = 302, a = 3$

Common difference $(d) = 8 - 3 = 5$ using the above mentioned formula, we get

$302 = 3 + (n-1)5$

$\Rightarrow 302 - 3 = 5n - 5$

$\Rightarrow 299 = 5n - 5$
$\Rightarrow 5n = 304$
$\Rightarrow n = \frac{305}{5}$

Since, the value of $'n'$ is a fraction. Thus, 302 is not the term of the given $A.p$

(iii) Here, $A.p$ is $11, 8, 5, 2, ...$

$a_n = -150, a = 1$ and $d = 8 - 11 = -3$

Thus, using the above mentioned formula, we get

$-150 = 11 + (x-1)(-3)$
$\Rightarrow -150 - 11 = -34 + 3$
$\Rightarrow -161 - 3 = -34$
$\Rightarrow -34 = -164$
$\Rightarrow n = \frac{164}{3}$

Since, the value of n is a fraction. Thus, -150 is not the term of the given $A.p$

4. How many terms are there in the AP?

(i) $7, 10, 13, ...43$

(ii) $-1, \frac{-5}{6}, \frac{-2}{3}, \frac{-1}{2}, ..., \frac{10}{3}.$

(iii) $7, 13, 19, ...05$

(iv) $18, 15\frac{1}{2}, 13, ...-47$

Sol:

(i) $7, 10, 13, ...43$

From given $A.p$

$a = 7, d = 10 - 7 = 3, a_n = a + (n-1)d.$

Let, $a_n = 43$ (last term)

$7 + (n-1)3 = 43$

$(n-1) = \frac{\cancel{26}^{12}}{\cancel{3}} = 12$

$n = 13$

$\therefore 13$ terms are there in given $A.p$

(ii) $-1, \frac{-5}{6}, \frac{-2}{3}, \frac{-1}{2}, ..., \frac{10}{3}.$

From given $A.p$

$a=-1, d=-\frac{5}{6}+1, \quad a_n=a+(n-1)d$

$=\frac{1}{6}$

Let, $a_n=\frac{10}{3}$ (last term)

$-1+(n-1)\frac{1}{6}=\frac{10}{3}$

$(n-1)\times\frac{1}{6}=\frac{10}{3}$

$(n+1)=\frac{13\times \cancel{6}^{2}}{\cancel{3}}=26$

$n=27$

$\therefore 27$ terms are there in given $A.p$

(iii) $7,13,19,..........05$

From the given $A.p$

$a=7, d=13-7=6, a_n=a+(n-1)d$

Let, $a_n=205$ (last term)

$7+(n-1)6=205$

$(n-1).6=198$

$n-1=33$

$n=34$

$\therefore 34$ terms are there in given $A.p$

(iv) $18,15\frac{1}{2},13,.........-47$

From the given $A.p.,$

$a=18, d=15\frac{1}{2}-18=\frac{31}{2}-18=15\cdot5-18=-2\cdot5$

$a_n=a+(n-1).d$

Let $a_n=-47$ (last term)

$18+(n-1),2.5=-47$

$12.5(n-1)=+65$

$n-1=\frac{65}{2\times5}=\frac{65\times10}{25}=26$

$n=27$

$\therefore 27$ terms are there in given $A.p$

5.

Sol:

Given

First term $(a) = 5$

Common difference $(d) = 3$

Last term $(1) = 80$

To calculate no of terms in given $A.p$

$a_n = a + (n-1)d$

Let $a_n = 80,$

$80 = 5 + (n-1) \cdot 3$

$75 = (n-1) \cdot 3$

$n - 1 = \frac{75}{3} = 25$

$n = 26$

$\therefore$ There are 26 terms.

6.

Sol:

Given, $a_6 = 19, a_{17} = 41$

$\Rightarrow a_6 = a + (6-1)d$

$19 = a + 5d$ ……….(1)

$\Rightarrow a_{17} = a + (17-1) \cdot d$

$41 = a + 16d$ ………..(2)

Subtract (1) from (2)

$$\begin{array}{l} a + 16d = 41 \\ \underline{a + 5d = 19} \\ 0 + 11d = 22 \end{array}$$

$d = \frac{22}{11} = 2$

Substitute $d = 2$ in (1)

$19 = a + 5(2)$

$9 = a$

$\therefore 40^{th}$ term $a_{40} = a + (40-1) \cdot d$

$= 9 + 39 \cdot 2$
$= 9 + 78$
$= 87$
$\therefore a_{40} = 87$

7.

Sol:
Given
9^{th} term of an $A.p$ $a_9 = 0, a_n = a + (n-1)d$
$a + (a-1) \cdot d = 0$
$a + 8d = 0$
$a = -8d$
We have to prove
24^{th} term is double the 19^{th} term $a_{29} = 2 \cdot a_{19}$
$a + (29-1)d = 2[a + (1a-1).d]$
$a + 28d = 2[a + 18a]$
Put $a = -8d$
$-8d + 28d = 2[-8d + 18d]$
$20d = 2 \times 10d$
$20d = 20d$
Hence proved

8.

Sol:
Given,
10 times of 10^{th} term is equal to 15 times of 15^{th} term.
$10a_{10} = 15.a_{15}$
$10[a + (10-1)d] = 15[a + (15-1).d] (\therefore a_n = a + (n-1)d)$
$10(a + 9d) = 15(a + 14 \cdot d)$
$a + 9d = \frac{\cancel{15}}{\cancel{10}}(a + 14d)$
$a - \frac{3}{2}a = \frac{42d}{2} - 9d$
$-\frac{1}{2}a = \frac{\cancel{24}^{12}}{\cancel{2}} \cdot d$

$-a = +24 \cdot d$

$a = -24 \cdot d$

We have to prove 25^{th} term of $A.p$ is 0

$a_{25} = 0$

$a + (25-1)d = 0$

$a + 24d = 0$

Put $a = -24d$

$-24 \times d + 24d = 0$

$0 = 0$

Hence proved.

9.

Sol:

Given,

$a_{10} = 41, a_{18} = 73, a_n = a + (n-1) \cdot d$

$\Rightarrow a_{10} = a + (10-1) \cdot d$

$41 = a + 9d$ ....(1)

$\Rightarrow a_{18} = a + (18-1)d$

$73 = a + 17d$ ....(2)

Subtract (1) from (2)

(2) (1)

$$\begin{array}{r} a + 17d = 73 \\ a + 9d = 41 \\ \hline 0 + 8d = 32 \end{array}$$

$d = \frac{32}{8} = 4$

Substitute $d = 4$ in (1)

$a + 9 \cdot 4 = 41$

$a = 41 - 36$

$a = 5$

26^{th} term $a_{26} = a + (26-1)d$

$= 5 + 25.4$

$= 5 + 100$

$= 105$

$\therefore 26^{th}$ term $a_{26} - 105.$

10.

Sol:

Given

24^{th} term is twice the 10^{th} term

$a_{24} = 2\ a_{10}$

Let, first term of a square $= a$

Common difference $= d$

n^{th} term $a_n = a + (n-1)d$

$$\cancel{a} + (24-1)d = (a + (10-1).d)2$$

$$a + 23d = 2(a + 9d)$$

$$(23-18)d = a$$

$$a = 5d$$

We have to prove

72^{nd} term is twice the 34^{th} term

$a_{12} = 2a_{34}$

$$a + (12-1)d = 2\left[a + (34-1)d\right]$$

$$a + 71d = 2a + 66d$$

Substitute $a = 5d$

$$5d + 71d = 2(5d) + 66d$$

$$76d = 10d + 66d$$

$$76d = 76d$$

Hence proved.

11.

Sol:

Given

$(m+1)^{th}$ term is twice the $(m+1)^{th}$ term.

First term $= a$

Common difference $= d$

n^{th} term $a_n = a + (n-1).d$

$$a_{m+1} = 2\ a_n + 1$$

$$a + (m+1-1)\cdot d = 2(a + (n+1-1)\cdot d)$$

$$a + md = 2(a + nd)$$

$$a = (m-2n)d$$

We have to prove

$(3m+1)^{th}$ term is twice the $(m+n+1)^{th}$ term

$a_{3m+1} = 2 \cdot a_{m+n+1}$

$a + (3m+1-1) \cdot d = (a + (m+n+1-1) \cdot d)$

$a + 3m \cdot d = 2a + 2(m+n)d$

Substitute $a = (m-2n) \cdot d$

$(m-2n)\cancel{d} + 3m\cancel{d} = 2(m-2n)\cancel{d} + 2(m+n) \cdot \cancel{d}$

$4m - 2n = 4m - 4n + 2n$

$4m - 2n = 4m - 2n$

Hence proved.

12.

Sol:

Given,

First sequence is $9, 7, 5, ...$

$a = 9, d = \neq -9 = -2, a_n = a + (n+1)d$

$a_n = 9 + (n-1).-2$

Second sequence is $15, 12, 9, ...$

$a = 15, d = 12 - 15 = -3, a_n = a + (n-1)d$

$a_n = 15 + (n-1) - 3$

Given an. a_n are equal

$9 - 2(n-1) = 15 - 3(n-1)$

$3(n-1) - 2(n-1) - 15 - 9$

$n - 1 = 6$

$n = 7$

$\therefore 7^{th}$ term of two sequence are equal

13.

Sol:

(i) $3, 5, 7, 9, ...2d$

First term $(a) = 3$

Common difference $(d) = 5 - 3 = 2$

12^{th} term from the end is can be considered as (1) last term = first term and common differnce $= d^1 = -d$ n^{th} term from the end = last term $+(n-1)-d$

12^{th} term from end $= 201+(12-1)(-2)$

$= 201-22$

$= 179$

(ii) $3, 8, 13, ...253$

First term $= a = 3$

Common difference $d = 8-3 = 5$

Last term $(1) = 253$

n^{th} term of a sequence on $= a+(n-1)\cdot d$

To find n^{th} term from the end, we put last term (1) as $'a'$ and common difference as $-d$

n^{th} term from the end = last term $+(n-1)\cdot -d$

12^{th} term from the end $= 253+(12-1)-5$

$= 253-55$

$= 198$

$\therefore 12^{th}$ term from the end $= 198$

(iii) $1, 4, 7, 10, ...88$

First term $a = 1$

Common differnce $d = 4-1 = 3$

Last term $(1) = 88$

n^{th} term $a_n = a+(n+1)\cdot d$

n^{th} term from the end = last term $+(n-1)\cdot -d$

12^{th} term from the end $= 88+(12-1)\cdot -3$

$= 88-33$

$= 55$

$\therefore 12^{th}$ term from the end $= 55.$

14.

Sol:

Given,

4^{th} term of an $A.p$ is three times the times the first term

$a_4 = 3.\ a$

n^{th} term of a sequence $a_n = a+(n-1)\cdot d$

$a+(4-1)\cdot d=3a$

$a+3d=3a$

$3d=2a$

$a=\frac{3}{2}d.$ ....(1)

Seventh term exceeds twice the third term by 1.

$a_7+1=2.a_3$

$a+(7-1)\cdot d+1=2(\alpha+\beta-1\cdot d)$

$a+6d+1=2a+4d$

$a=2d+1$ ....(2)

By equating (1), (2)

$\frac{3}{2}d=2d+1$

$\frac{3}{2}d-2d=1$

$\frac{3d-4d}{2}=1$

$-d=2$

$d=-2$

Put $d=-2$ in $a=\frac{3}{2}d$

$=\frac{3}{2}.\ x$

$=-3$

$\therefore$ First term $a=-3$, common difference $d=-2$.

15.

Sol:

Given

$a_6=12, a_8=22$

n^{th} term of an $A.p$ $a_n=a+(n-1)d$

$a_6=a+(n-1)\cdot d=a+(6-1)d=a+5d=12$ ....(1)

Subtracting (1) from (2)

$$\begin{array}{ll} & a+7d=22 \\ (2)\ (1) \Rightarrow & \underline{a+5d=12} \\ & 0+2d=10 \\ & \quad d=5 \end{array}$$

Put $d=5$ in
$$a+5d=12$$
$$a+55=12$$
$$a=12-25$$
$$a=-13$$

Second term $a_2 = a+(2-1)\cdot d$

$=a+d$

$=-13+5$

$a_1=-8$

n^{th} term $a_n = a+(n-1)d$

$=-13+(n-1)-5$

$a_n=-18+5n$

n^{th} term $a_n = a+(n-1)d$

$=-13+(n-1)-5$

$a_n=-18+5n$

$\therefore a_2=-8, a_n=-18+5n$

16.

Sol:

We observe that 12 is the first two-digit number divisible by 3 and 99 is the last two digit number divisible by 3. Thus, the sequence is

12,15,18,.................99

This sequence is in $A.p$ with

First term $(9)=12$

Common difference $(d)=15-12=3$

n^{th} term $a_n=99$

n^{th} term of an $A.p(a_n)=a+(n-1)\cdot d$

$99=12+(n-1)\cdot 3$

$99-12=n-1\cdot 3$

$\frac{87}{3}=n=1$

$n=30$

$\therefore 30$ term are there in the sequence

17.

Sol:

Given

No. of terms $= n = 60$

First term $(a) = 7$

Last term $a_{10} = 125$

$a_{60} = a + (60-1)\cdot d \qquad (\therefore a_n = a + (n-1)d)$

$125 = 7 + 59\cdot d$

$118 = 59d$

$d = \frac{118}{59} = 2$

52^{nd} term $a_{32} = a + (32-1)d$

$= 7 + 31.2$

$= 7 + 62$

$= 69$

18.

Sol:

Given

$a_4 + a_8 = 24$

$a_6 + a_{10} = 34$

$\Rightarrow a + (4-1)d + a + (18-1)d = 24$

$2a + 10d = 24$

$a + 5d = 12 \qquad \text{…………(1)}$

$\Rightarrow a_6 + a_{10} = 34$

$a + (6-1)d + a + (10-1)d = 34$

$2a + 14d = 34$

$a + 7d = 17 \qquad \text{…………(2)}$

Subtract (1) from (2)

$a + 7d = 17$

$\underline{a + 5d = 12}$

$2d = 5$

$d = \frac{5}{2}$

Put $d=\frac{5}{2}$ in $a+5d=12$

$a=12-5\cdot\frac{5}{2}$

$a=\frac{24-25}{2}=\frac{-1}{2}$

$\therefore a=-\frac{1}{2}, d=\frac{b}{2}$

19.

Sol:

Given,

$a_{30}-a_{20}=a+(30-1)d-(a+(20-1)d)(\because a_n=a+(n-1)d)$

$=a+29d-a-19d$

$=10d$

(i) $-9,-14,-19,-24,.......$

Common difference (d) = second tern – first term

$=-14-(-9)$

$=-14+9$

$d=5$

Then $a_{30}-a_{20}=10d$

$=10.5$

$a_{30}-a_{20}=50$

(ii) $a, a+d, a+2d, a+3d$

First term $(a)=a$

Common difference $(d)=d$

$a_{30}-a_{20}=a+(30-1)d-(a+(20-1)d)$

$=a+29d-0-19d$

$a_{30}-a_{20}=10d$

20.

Sol:

Given,

$a_{30}-a_{20}=a+(30-1)d-(a+(20-1)d)(\because a_n=a+(n-1)d)$

$=a+29d-a-19d$

$= 10d$

(i) $-9, -14, -19, -24, \ldots\ldots..$

Common difference (d) = second term – first term

$= -14 - (-9)$

$= -14 + 9$

$d = 5$

Then $a_{30} - a_{20} = 10d$

$= 10.5$

$a_{30} - a_{20} = 50$

(ii) $a, a+d, a+2d, a+3d, \ldots\ldots\ldots..$

First term $(a) = a$

$a_{30} - a_{20} = a + (30-1)d - (a + (20-1)d)$

$= a + 29d - a - 19d$

$a_{30} - a_{20} = 10d$

21.

Sol:

General arithmetic progression

$a, a+d, a+2d, \ldots\ldots..$

$a_b - a_k = a + (n-1)d - (a + (k-1)d) \left(\because a_n = a + (n-1)d\right)$

$= a + (n-1)d - 2\cancel{a} + (k-1)d$

$a_n - a_k = (n-k)d$ ……………(1)

(i) Given

11^{th} term $a_n = 5$

13^{th} term $a_{13} = 79$

By using (1) put $n = 13, k = 11$

$a_n - a_k = (n-k)\cdot d$

$79 - 5 = (13-11)\cdot d$

$74 = 2 \times d$

$d = \frac{74}{2} = 37$

(ii) Given

$a_{10} - a_5 = 200$

From (1) $a_{10} - a_5 = (10-5)d$

$200 = 5 \cdot d$

$d = \dfrac{200}{5} = 40 \Rightarrow d = 40$

(iii) Given

$a_{20} \neq 10 = a_{18}$

$a_{20} - a_{18} = 10$

By (1) $a_n - a_k = (n-k) \cdot d$

$a_{20} - a_{18} = (20-18) \cdot d$

$10 = 2 \cdot d$

$d = \dfrac{10}{2} = 5$

$\therefore d = 5$

22.

Sol:

(i) $25, 50, 75, 100 \ldots\ldots\ldots : c = 1000$

First term $(a) = 25$

Common difference $(d) = 50 - 25 = 25$

n^{th} term $a_n = a + (n-1) \times d$

Given, $a_n = 1000$

$1000 = 25 + (n-1) \cdot 25$

$975 = (n-1) \times 25$

$n - 1 = \dfrac{975}{25} = 39$

$n = 40$

(ii) Given sequence $-1, -3, -5, -7, \ldots\ldots : x = -151$

First term $(a) = -1$

Common difference $(d) = -3 - (-1) = -3 + 1 = -2$

n^{th} term $a_n = a + (n-1)d$

Given $a_n = -151,$

$$-151 = -1 + (n-1) - 2$$

$$-150 = 1(n+1)2$$

$$n+1 = \frac{150}{2} = 75$$

$$n = 74$$

(iii) Given sequence is

$$5\frac{1}{2}, 11, 16\frac{1}{2}, 22, \ldots\ldots\ldots : x = 550$$

First term (a) $= 5\frac{1}{2} = \frac{11}{2}$

$$= \frac{22-11}{2}$$

$$= \frac{11}{2}$$

n^{th} term $a_n = a + (n-1)d$

$$550 = \frac{11}{2} + (n-1) \cdot \frac{11}{2}$$

$$550 = \frac{11}{2}[1 + n - 1]$$

$$n = 550 \times \frac{2}{11}$$

$$n = 100$$

(iv) Given sequence is

$$1, \frac{21}{11}, \frac{31}{11}, \frac{41}{11}, \ldots\ldots, x = \frac{141}{11}$$

First term $(a) = 1$

Common difference $(d) = \frac{21}{11} - 1$

$$= \frac{21-11}{11}$$

$$= \frac{10}{11}$$

n^{th} term $a_n = a + (n-1) \times d$

$$\frac{171}{11} = 1 + (n-1) . \frac{10}{11}$$

$$\frac{171}{11} - 1 = (n-1)\frac{10}{11}$$

$$\frac{171-11}{11}=(n-1)\frac{10}{11}$$

$$\frac{160}{11}=(n-1).\frac{10}{11}$$

$$n-1=\frac{160}{11}\times\frac{11}{10}$$

$$n=17$$

23.

Sol:

First term of a sequence is a

Last term $=1$

Total no. of terms $=n$

Common difference $=d$

m^{th} term from the beginning $a_m=a+(n-1)\cdot d$

m^{th} term from the end = last term $+(n-1)-d$

$a_n-m+1=1-(n-1)\times d$

$\Rightarrow a_m+a_n-m+1=a+(n-1)d+(l-(n-1)d)$

$=a+(n-1)d+l-(n-1)d$

$a_m+a_n-m+1=a+l$

Hence proved

24.

Sol:

Given, $a_3=16$

$a+(3-1)d=16$

$a+2d=16.$ ……..(1)

And $a_7-12=a_5$

$a+(7-1)d-12=a+(5-1)d$ $\quad(\therefore a_n=a+(n-1)d)$

$\cancel{a}+6d-12=\cancel{a}+4d$

$2d=+12$

$d=+\frac{12}{2}=+6$

Put $d=-6$ in (1)

$a+2(+6)=16$

$a+12=6$
$a=284$
Then the sequence is $a, a+d, a+2d, a+3d, ...$
$\Rightarrow 28, 4, 10, 16, 22, ...$

25.
Sol:
Given,
$a+=32$
$a+(7-1)d=32$
$a+6d=32 \qquad ....(1)$
And $a_{13}=62$
$a+(13-1)d=62$
$a+12d=62 \qquad ....(2)$
Subtract (1) from (2)

$$(2)-(1) \Rightarrow \begin{array}{r} a+12d=62 \\ a+6d=32 \\ \hline 0+6d=32 \end{array}$$

$d=\frac{30}{6}=5$
Put $d=5$ in $a+6d=32$
$a+6\cdot 5=32$
$a=2$
Then the sequence is $a, a+d, a+2d, a+3d, ...$
$\Rightarrow 2, 7, 12, 17, ...$

26.
Sol:
Given $A.p$ is $3, 10, 17, ...$
First term $(a)=3$, Common difference $(d)=10-3$
$=7$
Let, n^{th} term of $A.p$ will be 84 more than 13^{th} term
$a_n=84+a_{13}$
$\cancel{a}+(n-1)d=\cancel{a}+(13-1)d+84$
$(n-1)7=12.7+84$
$(n-1)\cdot 7=168$

$n-1=\frac{168}{7}=24$

$n=25$

Hence 25^{th} term of given $A.p$ is 84 more than 13^{th} term

27.

Sol:

Let the two $A.p$ is be $a_1, a_2, a_3, ...$ and $b_1, b_2, b_3, ...$

$a_n = a_1 + (n-1)d$ and $b_n = b_1 + (n-1)\cdot d$

Since common difference of two equations is same given difference between 100^{th} terms is 100

$a_{100} - b_{100} = 100$

$a_1 + (9\cancel{9})d - b_1 - 99d = 100$

$a_1 - b_1 = 100$ …………..(1)

Difference between. 1000^{th} terms is

$a_{1000} - b_{1000} = a_1 + (1000-1)d - (b_1 + (1000-1)d)$

$= a_1 + 9\cancel{9}9d - b_1 - 9\cancel{9}9d$

$= a_1 - b_1$

$=100$ (from (1))

$\therefore$ Hence difference between 1000^{th} terms of two $A.p$ is 100.

28.

Sol:

Given two $A.p$ is are

$63, 65, 67..........$ and $3, 10, ...$

First term of sequence 1 is $a_1 = 63$

Common difference $d_1 = 65 - 63$

$= 2.$

n^{th} term $(a_n) = a_1 + (n-1)d$

$= 63 + (n-1)d$

First term of sequence 2 is $b_1 = 3.$

Common difference $d_2 = 10 - 3$

$= 7$

n^{th} term $(b_n) = b_1 + (n-1)d_2$

$= 3 + (n-1) \cdot 7$

Let n^{th} terms of two sequence is equal

$63 + (n-1)2 = 3 + (n-1) \times 7$

$60 = 5(n-1)$

$n - 1 = \frac{60}{5} = 12$

$n = 13$

$\therefore$ 13^{th} term of both the sequence are equal.

29.

Sol:

Multiple of 4 after 10 is 12 and multiple of 4 before 250 is $\frac{250}{4}$ remainder is 2, so,

$250 - 2 = 248$

248 is the last multiple of 4 before 250.

The sequence is

$12, ..., 248$

With first term $(a) = 12$

Last term $(1) = 248$

Common difference $(d) = 4$

n^{th} term $a_n = a + (n-1) \cdot d$

Here, n^{th} term $a_n = 248$

$248 = 12 + (n-1) \times 4$

$236 = (n-1) \times 4$

$n - 1 = \frac{236}{4} = 59$

$n = 60$

$\therefore$ There are 60 terms between 10 and 250 which are multiples of 4

30.

Sol:

The three digit numbers are $100, ...999$ 105 is the first 3 digit number which is divisible by 7 when we divide 999 with 7 remainder is 5. So, 999 – 5 = 994 is the last three digits divisible by 7 so, the sequence is

$105, ..., 994$

First term $(a) = 105$

Last term $(1) = 994$

Common difference $(d) = -7$

Let there are n numbers in the sequence

$a_n = 994$

$a + (n-1)d = 994$

$a + (n-1)d = 994$

$105 + (n-1)7 = 994$

$(n-1) \cdot 7 = 889$

$n - 1 = \frac{889}{7} = 127$

$n = 128$

$\therefore$ there are 128 numbers between 105, 994 which are divisible by 7

31.

Sol:

Given sequence

$8, 14, 20, 26, ...$

Let n^{th} term is 72 more than its 41^{st} term

$a_n = a_{41} + 72$

For the given sequence

$a = 8, d = 14 - 8 = 6$

$a + (n-1)d = 8 + (a+1)6 + 72$

$8 + (n-1)6 = 8 + (90) \cdot 6 + 72$

$(n-1)6 = 312$

$n - 1 = \frac{312}{6} = 52$

$n = 53$

$\therefore 53^{rd}$ term is 72 more than 41^{st} term

32.

Sol:

Given $A.p$ is $9, 12, 15, ...$

For this $a = 9, d = 12 - 9 = 3$

Let n^{th} term is 39 more than its 36^{th} term

$a_n = 39 + a_{36}$

$\cancel{a} + (n-1)3 = 39 + \cancel{9} + (36-1)\cdot 3 \qquad (\therefore a_n = a + (n-1)d)$

$(n-1)3 = 39 + 35\cdot 3$

$(n-1)\times 3 = 144$

$n - 1 = \dfrac{144}{3} = 48$

$n = 49$

$\therefore 49^{th}$ term is 39 more than its 36^{th} term

33.

Sol:

Given $A.p$ is $7, 10, 13, \ldots\ldots\ldots 184$

$a = 7, d = 10 - 7 = 3, l = 184$

n^{th} term from the end $= l + (n-1) - d$

8^{th} term from the end $= 184 + (8-1) - 3$

$= 184 - 21$

$= 163$

$\therefore 8^{th}$ term from the end -163

34.

Sol:

Given $A.p$ is $8, 10, 12, \ldots\ldots\ldots 126$

$a = 8, d = 10 - 8 = 2, l = 126$

n^{th} term from the end $= l + (n-1) - d$

10^{th} term from the end $= 126 + (10-1) - 2$

$= 126 - 18$

$= 108$

$\therefore 10^{th}$ term from the end $= 108$

35.

Sol:

Given, $a_4 + a_8 = 24$

$(a + (4-1)d) + (a + (8-1)d) = 24 (\therefore a_n = a + (n-1)d)$

$2a + 10d = 24$

$a+5d=12$ ………(1)

And $a_6+a_{10}=44$

$a+(6-1)+a+(10-1)d=44$ $\left(\because a_n=a+(n-1)d\right)$

$2a+14d=44$

$a+7d=22$ ………(2)

Subtract (1) from (2)

$$\begin{array}{l} a+7d=22 \\ \underline{a+5d=12} \\ 0+2d=10 \end{array}$$

$(2)-(1) \Rightarrow$

$d=5$

Put $d=5$ in (1) $a+5\cdot 5=12$

$a=-13$

36.

Sol:

Given $A.p$ is

$3,15,27,39,........$

Let n^{th} term is 120 more than 21^{st} term

Then $a_n=120+a_{21}$

For the given sequence

$a=3, d=15-3=12$

$\cancel{a}+(n-1)d=120+\cancel{a}+(21-1)d$

$(n-1)12=120+20(2)$

$(n-1)12=360$

$(n-1)=\dfrac{360}{12}=30$

$n=31$

$\therefore$ 31^{st} term is 120 more than 21^{st} term

37.

Sol:

Given

17^{th} term of an $A.p$ is 5 more than twice its 8^{th} term

$a_{17}=5+2a_8$

$a+(17-1)d=5+2\left(a+(8-1)\cdot d\right)$

$a+16d=5+2a+14d$

$a+5=2d \quad \ldots\ldots\ldots\ldots(1)$

And 11^{th} term of the $A.p$ is 43

$a_{11}=43$

$a+(11-1)d=43$

$a_{11}=43$

$a+(11-1)d=43$

$a+10d=43 \quad \ldots\ldots\ldots\ldots(2)$

$$(2)-(1) \Rightarrow \begin{array}{r} a+10d=43 \\ a-2d=+5 \\ \hline a+12d=48 \end{array}$$

$d=\dfrac{48}{12}=4$

Put $d=4$ in (1)

$a+5=2(4)$

$a=3$

$\therefore n^{th}$ term of given sequence is an $=a=1+(n-1)d$

$=3+(n-1)4$

$=3+4n-4$

$=4n-1$

$\therefore n^{th}$ term of given sequence $a_n=4n-1$

Exercise – 9.4

1.

Sol:

Given,

Sum of three terms of on A.P is 21.

Product of first and the third term exceeds the second term by 6.

Let, the three numbers be a-d, a, a+d, with common difference d: then,

(a – d) + a + (a + d) = 21

3a = 21

a = $\frac{21}{3}=7$

and (a – d) (a + d) = a + 6

$a^2-d^2=a+6$

Put a = 7 $\Longrightarrow 7^2-d^2=7+6$

49 – 13 = d^2

$d = \pm 6$

∴ The three terms are a – d, a, a + d, i.e., 1, 7, 13.

2.

Sol:

Let, the three numbers are a – d, a, a + d.

Given,

(a – d) + a + (a + d) = 27

3a = 27

a = $\frac{27}{3} = 9$

and, $(a - d)(a)(a + d) = 648$

$(a^2 - d^2)(a) = 648$

Put a = 9, then

$(9^2 - d^2)\ 9 = 648$

$81 - d^2 = \frac{648}{9} = 72$

$d^2 = 81 - 72$

$d^2 = 9$

d = 3

∴ The three terms are a – d, a, a + d i.e. 6, 9, 12.

3.

Sol:

Let, the four numbers be a – 3d, a – d, a + d, a + 3d, with common difference 2d.

Given, sum is 50.

(a – 3d) + (a – d) + (a + d) + (a + 3d) = 50

4a = 50

a = 12.5

greater number is 4 time the least

(a + 3d) = 4(a – 3d)

a + 3d = 4a – 12d

15d = 3a

Put a = 12.5

d = $\frac{3}{15} \times 12.5$

d = 2.5

∴ The four numbers are a – 3d, a – d, a + 3d i.e., 12.5 – 3(2.5), 12.5 – 2.5, 12.5 + 2.5, 12.5 + 3(2.5)

⟹ 5, 10, 15, 20

4.

Sol:

A quadrilateral has four angles. Given, four angles are in A.P with common difference 10.

Let, the four angles be, a – 3d, a – d, a + d, a + 3d with common difference = 2d.

2d = 10

d = $\frac{10}{2}$ = 5

In a quadrilateral, sum of all angles = 360°

(a – 3d) + (a – d) + (a + d) + (a + 3d) = 360

4a = 360

a = 360/4 = 90°

∴ The angles are a – 3d, a – d, a + d, a + 3d with a = 90, d = 5

i.e. 90 – 3(5), 90 – 5, 90 + 3(5)

⟹ 75°, 85°, 95°, 105°.

5.

Sol:

2, 4, 6, or 6, 4, 2.

6.

Sol:

Given,

8x + 4, 6x – 2, 2x + 7 are are A.P.

If the numbers a, b, c are in A.P. then condition is 2b = a + c.

Then, 2(6x – 2) = 8x + 4 + 2x + 7

12x – 4 = 10 + 11

2x = 15

x = $\frac{15}{2}$

7.

Sol:

Given numbers

x + 1, 3x, 4x + 2 are in AP

If a, b, c are in AP then 2b = a + c

Then 2(3x) = x + 1 + 4x + 2

6x = 5x + 3

x = 3

8.

Sol:

We have to show, (a – b)2, (a2 + b2) and (a + b)2 are in AP.

If they are in AP. Then they have to satisfy the condition

2b = a + c

$2(a^2 + b^2) = (a - b)^2 + (a + b)^2$

$2a^2 + 2b^2 = a^2 + 2ab + b^2 + a^2 + 2ab + b^2$

$2a^2 + 2b^2 = 2a^2 + 2b^2$.

They satisfy the condition means they are in AP.

Exercise – 9.5

1.

Sol:

In an A.P let first term = a, common difference = d, and there are n terms. Then, sum of n terms is,

$S_n = \frac{n}{2}\{2a + (n-1)d\}$

(i) Given progression is,

50, 46, 42, ……to 10 term.

First term (a) = 50

Common difference (d) = 46 – 50 = −4

n^{th} term = 10

Then $S_{10} = \frac{10}{2}\{2.50 + (10-1) - 4\}$

$= 5\{100 - 9.4\}$

$= 5\{100 - 36\}$

$= 5 \times 64$

$\therefore S_{10} = 320$

(ii) Given progression is,

1, 3, 5, 7, …..to 12 terms

First term difference (d) = 3 – 1 = 2

n^{th} term = 12

Sum of n^{th} terms $S_{12} = \frac{12}{2} \times \{2.1 + (12-1).2\}$

$= 6 \times \{2 + 22\} = 6.24$

$\therefore S_{12} = 144$.

(iii) Given expression is

$3, \frac{9}{2}, 6, \frac{15}{2}, \ldots \ldots to\ 25\ terms$

First term (a) = 3

Common difference (d) = $\frac{9}{2} - 3 = \frac{3}{2}$

Sum of nth terms $S_n, given\ n = 25$

$S_{25} = \frac{n}{2}(2a + (n-1).d)$

$S_{25} = \frac{25}{2}\left(2.3 + (25-1).\frac{3}{2}\right)$

$= \frac{25}{2}\left(6 + 24.\frac{3}{2}\right)$

$= \frac{25}{2}(6 + 36)$

$= \frac{25}{2}(42)$

$\therefore S_{25} = 525$

(iv) Given expression is,

41, 36, 31, …… to 12 terms.

First term (a) = 41

Common difference (d) = 36 – 41 = − 5

Sum of nth terms $S_n, given\ n = 12$

$S_{12} = \frac{n}{2}(2a(n-1).d)$

= $\frac{12}{6}(2.41 + (12-1).-5)$

= $6(82 + 11.(-5))$

= 6(27)

= 162

∴ S_{12} = 162.

(v) a + b, a – b, a – 3b, …… to 22 terms

First term (a) = a + b

Common difference (d) = a – b – a – b = −2b

Sum of nth terms $S_n = \frac{n}{2}\{2a(n-1).d\}$

Here n = 22

$S_{22} = \frac{22}{2}\{2.(a+b) + (22-1).-2b\}$

= $11\{2(a+b) - 22b\}$

= 11 {2a – 20b}

= 22a – 440b

∴ S_{22} = 22a – 440b

(vi) $(x - y)^2, (x^2 + y^2), (x + y)^2, \ldots \ldots ..\ to\ n\ terms$

First term (a) = $(x - y)^2$

Common difference (d) = $x^2 + y^2 - (x - y)^2$

= $x^2 + y^2 - (x^2 + y^2 - 2xy)$

= $x^2 + y^2 - x^2 + y^2 + 2xy$

= 2xy

Sum of n^{th} terms $S_n = \frac{n}{2}\{2a(n-1).d\}$

$= \frac{n}{2}\{2(x-y)^2 + (n-1).2xy\}$

$= n\{(x-y)^2 + (n-1)xy\}$

$\therefore S_n = n\{(x-y)^2 + (n-1).xy\}$

(vii) $\frac{x-y}{x+y}, \frac{3x-2y}{x+y}, \frac{5x-3y}{x+y}, \ldots..to\ n\ terms$

$First\ term\ (a) = \frac{x-y}{x+y}$

Common difference (d) = $\frac{3x-2y}{x+y} - \frac{x-y}{x+y}$

$= \frac{3x-2y-x+y}{x+y}$

$= \frac{2x-y}{x+y}$

Sum of n terms $S_n = \frac{n}{2}\{2a + (n-1).d\}$

$= \frac{n}{2}\left\{2.\frac{x-y}{x+y} + (n-1).\frac{2x-y}{x+y}\right\}$

$= \frac{n}{2(x+y)}\{2(x-y) + (n-1)(2x-y)\}$

$= \frac{n}{2(x+y)}\{2x - 2y + 2nx - ny - 2x + y\}$

$= \frac{n}{2(x+y)}\{n(2x-y) - y\}$

$\therefore S_n = \frac{n}{2(x+y)}\ \{n(2x-y) - y\}$

(viii) Given expression $-26, -24, -22, \ldots\ldots$ To 36 terms

First term (a) = −26

Common difference (d) = −24 – (−26) = −24 + 26 = 2

Sum of n terms $S_n = \frac{n}{2}\{2a + (n-1)d\}$

Sum of n terms $S_n = \frac{36}{2}\{2.-26 + (36-1)2\}$

$= 18[-52 + 70]$

$= 18.18$

$= 324$

$\therefore S_n = 324$

2.

Sol:

Given AP is 5, 2, -1, -4, -7, …..

a = 5, d = 2 – 5 = −3

$S_n = \frac{n}{2}\{2a + (n-1)d\}$

$= \frac{n}{2}\{2.5 + (n-1) - 3\}$

$= \frac{n}{2}\{10 - 3(n-1)\}$

$= \frac{n}{2}\{13 - 3n\}$

$\therefore S_n = \frac{n}{2}(13 - 3n)$

3.

Sol:

Given nth term $a_n = 5 - 6n$

Put n = 1, $a_1 = 5 - 6.1 = -1$

We know, first term (a_1) = −1

Last term (a_n) = 5 – 6n = 1

Then $S_n = \frac{n}{2}(-1 + 5 - 6n)$

$= \frac{n}{2}(4 - 6n) = \frac{n}{2}(2 - 3n)$

4.

Sol:

Given AP is 25, 22, 19, ……

First term (a) = 25, d = 22 – 25 = −3.

Given, $S_n = \frac{n}{2}(2a + (n - 1)d)$

$116 = \frac{n}{2}(2 \times 25 + (n - 1) - 3)$

$232 = n\big(50 - 3(n - 1)\big)$

232 = n(53 – 3n)

$232 = 53n - 3n^2$

$3n^2 - 53n + 232 = 0$

(3n – 29) (n – 8) = 0

∴ n = 8

$\Longrightarrow a_8 = 25 + (8 - 1) - 3$

∴ n = 8, $a_8 = 4$

= 25 – 21 = 4

5.

Sol:

(i) Given sequence, 18, 16, 14, …

a = 18, d = 16 – 18 = −2.

Let, sum of n terms in the sequence is zero

$S_n = 0$

$\frac{n}{2}(2a + (n - 1)d) = 0$

$\frac{n}{2}(2.18 + (n - 1) - 2) = 0$

$n\big(18 - (n - 1)\big) = 0$

$n(19 - n) = 0$

n = 0 or n = 19

(ii) $\because n = 0$ is not possible. Therefore, sum of 19 numbers in the sequence is zero.

Given, a = -14, a 5 = 2

a + (5 – 1)d = 2

−14 + 4d = 2

$4d = 16 \implies d = 4$

Sequence is −14, −10, -6, −2, 2, ……

Given $S_n = 40$

40 = $\frac{n}{2}\{2(-14) + (n-1)4\}$

80 = $n(-28 + 4n - 4)$

80 = $n(-32 + 4n)$

4(20) = 4n(−8 + n)

$n^2 - 8n - 20 = 0$

$(n - 10)(n + 2) = 0$

$n = 10 \text{ or } n = -2$

∴ Sum of 10 numbers is 40 (Since -2 is not a natural number)

(iii) Given AP 9, 17, 25, …….

a = 9, d = 17 – 9 = 8, and $S_n = 636$

636 = $\frac{n}{2}(2.9 + (n-1)8)$

1272 = $n(18 - 8 + 8n)$

1272 = n(10 + 8n)

2 × 636 = 2n(5 + 4n)

636 = 5n + $4n^2$

$4n^2$ + 5n – 636 = 0

(4n + 53) (n – 12) = 0

∴ n = 12 (Since n $\frac{-53}{4}$ is not a natural number)

Therefore, value of n is 12.

(iv) Given AP, 63, 60, 57, ……

a =63, d = 60 – 63 = −3 $S_n = 693$

$S_n = \frac{n}{2}(2a + (n-1)d)$

693 = $\frac{n}{2}(2.63 + (n-1) - 3)$

1386 = n(126 – 3n + 3)

1386 = (129 – 3n)n

$3n^2 - 129n + 1386 = 0$

$n^2 - 43n + 462 = 0$

n = 21, 22

$\therefore$ Sum of 21 or 22 term is 693

6.

Sol:

Given, a = 17, l = 350, d = 9

l = $a_n = a + (n - 1)d$

350 = 17 + (n – 1)9

333 = (n – 1)9

n – 1 = $\frac{333}{9} = 37$

n = 38

$\therefore$ 38 terms are there

$S_n = \frac{n}{2}\{a + l\}$

= $\frac{38}{2}\{17 + 350\}$

= 19.367

$\therefore S_n = 6973$

7.

Sol:

Given, $a_3 = 7$ and $3a_3 + 2 = a_7$

$a_7 = 3.7 + 2$

$a_7 = 21 + 2 = 23$

$\therefore a_n = a + (n - 1)d$

$a_3 = a(3 - 1)d \ and \ a_7 = a + (7 - 1)d$

$7 = a + 2d \ldots..(i) \qquad 23 = a + 6d \quad \ldots.(ii)$

$Subtract \ (i) from \ (ii)$

$(ii) - (i) \Longrightarrow$ a + 6d = 23

a + 2d = 7

4d = 16

d = 4

Put d = 4 in (i) $\Longrightarrow$ 7 = a + 2.4

a = 7 – 8 = -1

Given to find sum of first 20 terms.

$S_{20} = \frac{20}{2}\{-2 + (10 - 1)4\}$

= $10(-2 + 76)$

$\therefore S_{20} = 740$

8.

Sol:

Given a = 2, l = 50, $S_n = 442$

$S_n = \frac{n}{2}(a + 1)$

442 = $\frac{n}{2}(2 + 50)$

442 = $\frac{n}{2}.52$

∴ n = $\frac{442}{26} = 17$

Given, $a_n = 1 = 50$

50 = 2 + (17 – 1) d

48 = $16 \times d$

d = $\frac{48}{16} = 3$

∴ d = 3

9.

Sol:

Given, $a_{12} = -13, a + a_2 + a_3 + a_4 = 24$

$S_4 = \frac{4}{2}(2a + 3d) = 24$

2a + 3d = $\frac{24}{2} = 12 \quad \dots.(i)$

$\Longrightarrow$ a + $(12 - 1)d = -13$

a + 11d = -13 ... ... (ii)

Subtract (i) from (ii) $\times$ 2

2 $\times$ (ii) – (i) $\Longrightarrow$ 2a + 22d = -28

$\underline{2a + 3d \;=\; 12}$

198d = -38

d = $\frac{-38}{19} = -2$

put d = -2 in (ii)

a + 11 (-2) = -13

a = $-13 + 22$

a = 9

Given to find sum of first 10 terms.

$S_{10} = \frac{10}{2}\{2(a) + (10 - 1) - 2\}$

= 5(18 – 18)

= 0

∴ $S_{10} = 0$

10.

Sol:

Given, d = 22, $a_{22} = 149$

$a + (22 - 1)\ d$ = 149

$a = -313$

Given, to find $S_{22} = \frac{22}{2}[2a + (22-1)d]$

= $11[2(-313) + 21.22]$

= $11[-626 + 462]$

= 11 – 164

= -1804

∴ $S_{22} = -1804$

11.

Sol:

The numbers between 1 and 100 which are divisible by 3 are 3, 6, 9, ….99.

In this sequence, a = 3, d = 3, $a_n = 99$

99 = $a + (n-1)d$

99 = 3 + (n – 1)3

99 = 3[1 + n – 1]

n = $\frac{99}{3} = 33$

∴ There are 33 numbers in the given sequence

$S_{33} = \frac{33}{2}(2.3 + (33-1)3)\ \left(\therefore S_n = \frac{n}{2}(2a + (n-1)d)\right)$

= $\frac{33}{2}(6 + 96)$

$= \frac{33}{2} \times 102$

= 1683

∴ Sum of all natural numbers between 1 and 100, which are divisible by 3 is 1683.

12.

Sol:

The sequence is, 1, 3, 5, …..n.

In this first term (a) = 1, common difference (d) = 2

$S_n = \frac{n}{2}(2a + (n-1)d)$

$= \frac{n}{2}(2.1 + (n-1)2)$

$= \frac{n}{2} \times 2(1 + n - 1)$

$= n^2.$

∴ Sum of first n odd natural numbers is n^2.

13.

Sol:

(i) Odd numbers between 0 and 50 are 1, 3, 5, ….., 49

In this a = 1, d = 2, l = 49 = a_n

49 = 1 + (n – 1)2 ($\therefore a_n = a + (n – 1)d$)

48 = (n – 1)2

n – 1 = $\frac{48}{2} = 24$

n = 25.

∴ There are 25 terms

$S_{25} = \frac{25}{2}(1 + 49)$ $\left(\therefore S_n = \frac{n}{2}(a + 1)\right)$

$= \frac{25}{2} \times 50 = 625$

∴ Sum of all odd numbers between 0 and 50 is 625.

(ii) Odd numbers between 100 and 200 are 101, 103, …. 199

In this a = 101, d = 2, l = a_n = 199

199 = 101 + (n – 1)2

n – 1 = $\frac{98}{2} = 49$

n = 50

∴ There are 50 terms.

$S_{50} = \frac{50}{2}(101 + 199)$ $\left(\therefore S_n = \frac{n}{2}(a + 1)\right)$

= $\frac{50}{2} \times 300$

$= 7500$

∴ Sum of all odd numbers between 100 and 200 is 7500.

14.

Sol:

Odd integers between 1 and 1000 which are divisible by 3 are 3, 6, 9, 15 …… 999.

In this a = 3, d = 5, l = $a_n = 999$

999 = 3 + (n – 1)5 $(\therefore a_n = a + (n - 1)d)$

999 = 3[1 + (n – 1)2]

∴ 2n – 1 = $\frac{999}{3} = 333 \Rightarrow n = \frac{334}{2} = 167$

∴ There are 333 numbers 167.

$S_{167} = \frac{333}{2}[3 + 999]$

= $\frac{333}{2} \times 100$ = 83667

$\therefore S_{167} = 83667$

∴ Sum of all odd integers between 1 and 4000 which are divisible by 3 is 83667.

15.

Sol:

The numbers between 84 and 719, which are multiples of 5 are 85, 90, 95,…..715.

In this, a = 85, d = 5, $a_n = l = 715$

715 = 85 + (n – 1)5 $\quad (\therefore a_n = a + (n-1)d)$

630 = (n – 1)5

$n - 1 = 126$

n = 127

$\therefore S_n = \frac{127}{2}(85 + 115) \quad \left(\therefore S_n = \frac{n}{2}(a + 1)\right)$

$= \frac{127}{2} \times 800 = 50800$

∴ Sum of all integers between 84 and 719, which are multiples of 5 is 50800.

16.

Sol:

Numbers between 50 and 500, which are divisible by 7 are 56, 63, ….., 497.

In this a = 56, d = 7, l = $a_n = 497$

497 = 56 + (n – 1)7

441 = (n – 1)7

n – 1 = $\frac{441}{7} = 63$

n = 64

∴ There are 64 terms.

$S_{64} = \frac{64}{2}(56 + 497)$

$= 32 \times 553 = 17696$

∴ Sum of all integers between 50 and 500, which are divisible by 7 is 17696.

17.

Sol:

Even integers between 101 and 999 are 102, 104, …..998

a = 102, d = 2, $a_n = l = 998$

998 = 102 + (n – 1)× 2 $(\therefore a_n = a + (n-1)d)$

896 = (n – 1)(2)

n – 1 = 448

n = 449.

∴ 449 terms are there

$S_{449} = \frac{449}{2}[102 + 998]$

$=\frac{449}{2}\times 1100 = 246950$

∴ Sum of all even integers between 101 and 999 is 24690

18.

Sol:

Integers between 100 and 550 which are divisible by 9 are 108, 117, …., 549.

In this a = 108, d = 9, $a_n = l = 549$

549 = 108 + (n – 1)× 9 $(\therefore a_n = a + (n+1)d)$

441 = (n – 1) × 9

n – 1 = $\frac{449}{9} = 49$

n = 50.

$\therefore S_{50} = \frac{50}{2}\{108 + 549\}$ $\left(\therefore S_n = \frac{n}{2}(10 + 1)\right)$

= 25 × 657

= 16425

∴ Sum of all integers between 100 and 550, which are divisible by 9 is 16425.

19.

Sol:

Given, a = 22, d = −4, $S_n = 64$

$S_n = \frac{n}{2}(2a + (n-1)d)$

64 = $\frac{n}{2}\times(2.22 + (n-1) - 4)$

64 = n(24 – 2n)

64 = 2n (12 – n)

12n – n^2 = $\frac{64}{2} = 32$

$n^2 - 12n + 32 = 0$

$(n-4)(n-8) = 0$

∴ n = 4 or 8

20.

Sol:

Given, $a_5 = 30, a_{12} = 65$

⟹ 30 = a + (5 – 1)d

30 = a + 4d …..(i)

⟹ 65 = a + (12 – 1)d

65 = a + 11d …..(ii)

(ii) – (i) ⟹ a + 11d = 65

a + 4d = 30

$0 + 7d = 35$

$d = \frac{35}{7} = 5$

put d = 5 in …(i) $\Longrightarrow$ 80 = a + 4 (5)

a = 30 – 20 = 10

$S_{20} = \frac{20}{2}(2(10) + (20-1)5) \quad \left(\therefore S_n = \frac{n}{2}(20 + (n-1)d)\right)$

= $10[20 + 95]$

= 10×115

= 1150

$\therefore$ Sum of first 20 terms $S_{20} = 1150$

21.

Sol:

(i)

Given AP, 2; 6, 10, 14, ……

a = 2, d = 4, $S_n = S_{11} = \frac{11}{2}(2.2 + (11-1).4) \quad \left(\therefore S_n = \frac{n}{2}(2a + (n-1)d)\right)$

$= \frac{11}{2}(4 + 40)$

$= \frac{11}{2} \times 49$

$\therefore S_{11} = 242$

(ii)

Given AP $-6, 0, 6, 12, \ldots\ldots$

a = -6, $d = 6$, $S_n = \frac{n}{2}(2a + (n-1)d)$

$S_n = S_{13} = \frac{13}{2}(2 \times -6 + (13-1) \times 6)$

$= \frac{13}{2}(-12 + 72)$

$= \frac{13}{2} \times 60$

= 390

$\therefore S_{13} = 890$

(iii)

Given, $a_2 = 2$ and $a_4 = 8$

a + d = 2 ….(i) a + 3d = 8 ….(ii)

(ii) – (i) $\Longrightarrow$ a + 3d = 8

<u>a + d = 2</u>

2d = 6

d = 3

put d = 3 in ….(i) $\Longrightarrow$ a + d = 2

a + 3 = 2

a = −1

$S_{51} = \frac{51}{2}(2 \times -1 + (51 - 1) \times 3)$ $\left(\therefore S_n = \frac{n}{2}(2a + (n-1)d)\right)$

$= \frac{51}{2}(-2 + 50 \times 3)$

$= \frac{51}{2} \times 148$

= 3774.

$\therefore S_n = 3774$

22.

Sol:

The first 15 multiples of 8 are 8, 16, 24, ……

a = 8, d = 8, n = 15

$S_{15} = \frac{15}{2}(28 + (15 - 1) \times 8)$ $\left(\therefore S_n = \frac{n}{2}(2a + (n-1)d)\right)$

$= \frac{15}{2}(16 + 112)$

$= \frac{15}{2} \times 128$

$= 960$

$\therefore \mathit{Sum\ of\ first}\ 15\ \mathit{multiples\ of}\ 8\ \mathit{is}\ 960.$

Given, $a_2 = 2\ and\ a_4 = 8$

a + d = 2 ….(i) a + 3d = 8 …..(ii)

(ii) – (i) ⟹ a + 3d = 8

$\underline{a + d = 2}$

2d = 6

d = 3

Put d = 3 in ….(i) ⟹ a + d = 2

a + 3 = 2

a = −1

$S_{51} = \frac{51}{2}(2 \times -1 + (51 - 1 \times 3))$ $\left(\therefore S_n = \frac{n}{2}(20 + (n-1)d)\right)$

$= \frac{51}{2}(-2 + 50 \times 3)$

$= \frac{51}{2} \times 148$

$= 3774$

$= 44550$

∴ Sum of all 3 – digit natural numbers which are multiples of 11 is 44550.

23.

Sol:

(i) $2 + 4 + 6 + \ldots\ldots + 200$

$a = 2$, $d = 4 - 2 = 2$, $l = 200 = a_n$

$\therefore S_n = \frac{n}{2}(a + l) \text{ and } a_n = a + (n-1)d$

$200 = 2 + (n - 1)2$

$198 = (n - 1)2$

$n - 1 = \frac{198}{2} = 99$

$n = 100$

$S_n = \frac{100}{2}(2 + 200)$

$= 50 \times 202$

$= 10100$

(ii) $3 + 11 + 19 + \ldots\ldots + 803$

$a = 3$, $d = 11 - 3 = 8$, $l = a_n = 803$

$803 = 3 + (n - 1)8$

$\frac{800}{8} = n - 1$

$n = 101$

$S_n = \frac{101}{2}(3 + 803)$

$= \frac{101}{2} \times 806$

$= 504$

$S_n = 504$

(iii) $34 + 32 + 30 + \ldots\ldots + 10$

$a = 34$, $d = -2$, $l = a_n = 10$

$10 = 34 + (n - 1) \times 2$

$+24 = 2(n - 1)$

$n - 1 = 12$

$n = 13$

$\therefore S_{13} = \frac{13}{2}(34 + 10)$

$= \frac{13}{2} \times 44$

$= 286$

(iv) $25 + 28 + 31 + \ldots\ldots + 100$

$a = 25$, $d = 8$, $l = a_n = 100$

$100 = 25 + (n - 1) \times 3$

$75 = (n - 1) \times 3$

$n - 1 = 25$

$n = 26$

24.

Sol:

(i) Given $a_n = 3 + 4n$

Put n = 1, $a_1 = 3 + 4(1) = 7$

Put n = 15, $a_{15} = 3 + 4(15) = 63 = 8$

Sum of 15 terms, $S_{15} = \frac{15}{2}(7 + 63)$ $\left(\because S_n = \frac{n}{2}(a + l)\right)$

$= \frac{15}{2} \times 70$

$\therefore S_{15} = 525$

(ii) Given $b_n = 5 + 2n$

Put n = 1, $b_1 = 5 + 2(1) = 7$

Put n = 15, $b_{15} = 5 + 2(15) = 35 = l$

Sum of 15 terms , $S_{15} = \frac{15}{2}(7 + 35)$ $\left(\because S_n = \frac{n}{2}\ (a + l)\right)$

$= \frac{15}{2} \times 42$

$= 315$

$\therefore S_{15} = 315$

(iii) Given, $Y_n = 9 - 5n$

Put n = 1, $y_1 = 9 - 5.1 = -4$

Put n = 15, $y_{15} 9 - 5.15 = 9 - 75 = -66 = (l)$

$\therefore S_{15} = \frac{15}{2}(-4 - 66)$ $\left(\because S_n = \frac{n}{2}(a + l)\right)$

$= \frac{15}{2} \times -70$

$= -465$

$\therefore S_{15} = -465$

25.

Sol:

Given, n^{th} term $a_n = A_n + B$

Put n = 1, $a_1 = A + B$

Put n = 20, $a_{20} = 20\,A + B = (l)$

$\therefore S_{20} = \frac{20}{2}(A + B + 20A + B)$ $\left(\because S_n = \frac{n}{2}(a + l)\right)$

$= 10\,(21A + 2B)$

$= 210A + 20B$

$\therefore S_n = 210A + 2B$

26.

Sol:

Given, n^{th} term $a_n = 2 - 3n$

Put n = 1, $a_1 = 2 - 3.1 = -1$
Put n = 25, $a_{15} = l = 2 - 3.15 = -43$
$\therefore$ 25 = $\frac{25}{2}$ $(-1 - 43) = \frac{25}{2}(-44) = -925$
$\therefore S_{25} = -925$

27.
Sol:
Given, $a_n = 7 - 3n$
Put n = 1, $a_1 = 7 - 3.1 = 4$
Put n = 25, $a_{25} = l = 7 - 3.25 = -68$
$\therefore S_{25} = \frac{25}{2}(4 - 68) \quad \left(\therefore S_n = \frac{n}{2}(a + l)\right)$
= $\frac{25}{2} \times -64$
$= -800$
$\therefore S_{25} = -800$

28.
Sol:
Given, $a_2 = 14 \Longrightarrow a + d = 14 \ldots\ldots(i)$
$a_3 = 18 \Longrightarrow a + 2d = 18 \ldots\ldots(ii)$
(ii) – (i) $\Longrightarrow$ a + 2d = 18
a + d = 14
0 + d = 4
Put d = 4 is (i) a + 4 = 4
a = 10
$\therefore S_{50} = \frac{51}{2}\{2.10 + (51 - 1) \times 4\} \quad \left(S_n = \frac{n}{2}\{2a + (n - 1)d\}\right)$
$= \frac{51}{2}\{20 + 200\}$
$= \frac{51}{2} \times 220$
$= 5610$
$\therefore S_{51} = 5610$

29.
Sol:
Given, $S_7 = 49$
$\frac{7}{2}(2a + (7 - 1)d) = 49 \quad \left(\therefore S_n = \frac{n}{2}\{2a + (n - 1)d\}\right)$
$\frac{7}{2}(2a + 6d) = 49$
$\frac{7}{2} \times 2(a + 3d) = 49$

$a + 3d = \frac{49}{7} = 7 \ldots.(i)$ and

$S_{17} = 289$

$\frac{17}{2}(2a + (17 - 1)d) = 289$

$\frac{17}{2} \times 2(a + 8d) = 289$

$a + 8d = \frac{289}{17} = 17$ …..(ii)

$Subtract\ (i) from\ (ii)$

$$\begin{array}{r} a + 8d = 17 \\ \underline{a + 3d = \ 7} \\ 5d = 10 \\ d = \ 2 \end{array}$$

put d = 2, in (i) $\Longrightarrow$ a + 3 × 2 = 7

a = 1

$\therefore S_n = \frac{n}{2}\{2.1 + (n - 1).2\}$ $\quad \left(\therefore S_n = \frac{n}{2}(2a + (n - 1)d)\right)$

$= n\{1 + n - 1\}$

$\therefore S_n = n^2.$

30.

Sol:

Given, a = 5, l = 45, Sum of terms = 400

$\therefore S_n = 400$

$\frac{n}{2}\{5 + 45\} = 400$

$\frac{n}{2} = 50 = 400$

$n = 40 \times \frac{2}{5}$

$\therefore n = 16$

16^{th} term is 45

$a_{16} = 45 \Longrightarrow 5 + (16 - 1) \times d = 45 = 15 \times d = 40$

d = $\frac{408}{15} = \frac{8}{3}$

$\therefore$ n = 16, d = $\frac{8}{3}$

31.

Sol:

Given, sum of n terms $S_n = \frac{3n^2}{2} + \frac{13}{2}n$

Let, $a_n = S_n - S_{n-1}$ $\quad$ ($\therefore$ Replace n by (n – 1) is S_n to get $S_{n-1} = \frac{3(n-1)^2}{2} + \frac{13}{2}\ (n - 1)$)

$a_n = \frac{3n^2}{2} + \frac{13}{2}n - \frac{3(n-1)^2}{2} - \frac{13}{2}(n - 1)$

$= \frac{3}{2}\{n^2 - (n - 1)^2\} + \frac{13}{2}\{n - (n - 1)\}$

$= \frac{3}{2}\{n^2 - n^2 + 2n - 1\} + \frac{13}{2}\{1\}$

$= 3n + \frac{10}{2} = 3n + 5$

Put n = 25, $a_{25} = 3(25) + 5 = 75 + 5 = 80$

∴ 25th term $a_{25} = 80$

32.

Sol:

(i) Given a = 5, d = 3, $a_n = 50$

$a_n = 50$

$a + (n - 1)d = 50$

5 + (n – 1)3 = 50

(n – 1)3 = 45

n – 1 = $\frac{45}{3} = 15$

n = 16

Some of n terms $S_n = \frac{n}{2}[a + l]$

$= \frac{16}{2}[5 + 50]$

$= 8 \times 55$

= 440

(ii) Given, $a_n = 4, d = 2, S_n = -14$

a + (n – 1).2 = 4 and $\frac{n}{2}[2a + (n - 1).2] = -14$

a + 2n = 6 $n[2a + 2n - 2] = -14$

(or)

$\frac{n}{2}[a + a_n] = -14$

$\frac{n}{2}[a + 4] = -14$

$n[6 - 2n + 4] = -28$

$n[10 - 2n] = -28$

$2n^2 - 10n - 28 = 0$

$2(n^2 - 5n - 14) = 0$

(n + 2) (n – 7) = 0

N = −2, n = 7

∵ n = −2 is not a natural number. So, n = 7.

(iii) Given, a = 3, n = 8, S_n = 192.

$S_n = \frac{n}{2}[2a + (n - 1)d]$

$192 \times 2 = 8[6 + (8 - 1)d]$

$\frac{192 \times 2}{8} = 6 + 7d$

$48 = 6 + 7d$

7d = 42

d = 6

(iv) Given, $a_n = 28, S_n = 144, n = 9$

$S_n = \frac{n}{2}[a + l]$

144 = $\frac{9}{2}[a + 28]$

144 = $\frac{2}{9} = a + 28$

a + 28 = 32

a = 4

(v) Given, a = 8, 62 and S_n = 210

$S_n = \frac{n}{2}[a + l]$

210 = $\frac{n}{2}[8 + 62]$

210 × 2 = n[70]

n = $\frac{210 \times 2}{70} = 6$

a + (n – 1) d = 62

8 + (6 – 1) d = 62

5d = 54

d = 10.8

(vi) Given

a = 2, d = 8 and S_n = 90

90 = $\frac{n}{2}[4 + (n-1)8]$ $\left(\therefore S_n = \frac{n}{2}[2a + (n-1)d]\right)$

180 = n [4 + 8n – 8]

$8n^2 - 4n - 180 = 0$

$4(2n^2 - n - 45) = 0$

$2n^2 - n - 45 = 0$

(2n + 1) (n – 5) = 0

∵ n = $-\frac{1}{2}$ is not a natural no. n = 5

$a_n = 2 + 4(8)$ $(\therefore a_n = a + (n-1)d)$

$a_n = 32$

33.

Sol:

Let ‘a’ be the money he saved in first year

⟹ First year he saved the money = Rs a

He saved Rs 100 more than, he did in preceding year.

⟹ Second year he saved the money = Rs (a + 100)

⟹ Third year he saved the money = Rs. (a + 2 (100))

So, the sequence is a, a + 100, a + 2(100), ……, This is in AP with common difference (d) = 100.

$\Longrightarrow$ Sum of money he saved in 10 years $S_{10} = 16{,}500\ rupees$

$S_n = \frac{n}{2}(2a + (n-1)d)$

$S_{10} = \frac{10}{2}(2a + (10-1).100)$

16,500 = 5 (2a + 9 × 100)

2a + 900 = $\frac{16500}{5} = 3300$

2a = 2400

a = $\frac{2400}{2} = 1200$

∴ He saved the money in first year (a) = Rs. 1200

34.

Sol:

Given

Saving in 1st yr (a_1) = Rs 32

Saving in 2nd yr (a_2) = Rs 36

Increase in salary every year (d) = Rs 4

Let in n years his saving will be Rs 200

$\Longrightarrow S_n = 200$

$\Longrightarrow \frac{n}{2}[2a + (n-1)d] = 200$

$\Longrightarrow \frac{n}{2}[64 + 4n - 4] = 200$

$\Longrightarrow \frac{n}{2}[4n + 60] = 200$

$\Longrightarrow 2n^2 + 30n = 200$

$\Longrightarrow n^2 + 15 - 100 = 0$ [Divide by 2]

$\Longrightarrow n^2 + 20n - 5n - 100 = 0$

$\Longrightarrow$ n(n + 20) – 5(n + 20) = 0

$\Longrightarrow$ (n + 20)(n – 5) = 0

If n + 20 = 0 or n – 5 = 0

n = −20 or n = 5 (Rejected as n cannot be negative)

∴ In 5 years his saving will be Rs 200

35.

Sol:

Given

A man arranges to pay off a debt of Rs 3600 by 40 annual installments which form an A.P i.e., sum of all 40 installments = Rs 3600

$S_{40} = 3600$

Let, the money he paid in first installment is a, and every year he paid with common difference = d

Then,

S40 = 3600 $\quad\quad (\therefore S_n = \frac{n}{2}[2a + (n-1)d])$

$\frac{40}{x}[2a + (40-1)d] = 3600$

$2a + 39d = \frac{3600}{20} = 180 \ldots\ldots (i)$

but,

He died by leaving one third of the debt unpaid that means he paid remaining money in 30 installments.

∴ The money he paid in 30 installments = 3600 $- \frac{3600}{3} = 3600 - 1200$

$\therefore S_{30} = 2400$

$S_{30} = 2400$

$= \frac{30}{2}[2a + (30-1)d] = 2400 \quad \left(\therefore S_n = \frac{n}{2}(2a + (n-1)d)\right)$

2a + 2ad = $\frac{2400}{15} = 160 \;\ldots.(ii)$

(i) – (ii) ⟹ 2a + 39d = 180

2a + 29d = 160

0 + 10 d = 20

d = $\frac{20}{10} = 2$

put d = 2 in (ii) 2a + 29 (2) = 160

2a = 102

a = $\frac{102}{2} = 51$

∴ The value of his first installment = 51.