Exercise:2.3

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Question 1.

Solution:

Given:
A = {1, 2, 3} and B = {4, 5, 6}
Thus, we have:
A × B = {(1, 4), (1, 5), (1, 6), (2, 4), (2, 5), (2, 6), (3, 4), (3, 5), (3, 6)}

(i) {(1, 6), (3, 4), (5, 2)}
Since it is not a subset of A × B, it is not a relation from A to B.
(ii) {(1, 5), (2, 6), (3, 4), (3, 6)}
Since it is a subset of A × B, it is a relation from A to B.
(iii) {(4, 2), (4, 3), (5, 1)}
Since it is not a subset of A × B, it is not a relation from A to B.
(iv) A × B
Since it is a subset (equal to) of A × B, it is a relation from A to B.

Question 2.

Solution:

Given:
(x, y) ∈ R ⇔ x is relatively prime to y.
Here,
2 is co-prime to 3 and 7.
3 is co-prime to 7 and 10.
4 is co-prime to 3 and 7.
5 is co-prime to 3, 6 and 7.
Thus, we get:
R = {(2, 3), (2, 7), (3, 7), (3, 10), (4, 3), (4, 7), (5, 3), (5, 6), (5, 7)}
Domain of R = {2, 3, 4, 5}
Range of R = {3, 7, 6, 10}

Question 3.

Solution:

Given:
A is the set of the first five natural numbers.

$\therefore$ A = {1, 2, 3, 4, 5}
The relation is defined as:
$(x, y) \in R \Leftrightarrow x \leq y$
Now,
R = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (2, 2), (2, 3), (2, 4), (2, 5), (3, 3), (3, 4), (3, 5), (4, 4), (4, 5), (5, 5)}
R^{-1} = {(1, 1), (2, 1), (3, 1), (4, 1), (5, 1), (2, 2), (3, 2), (4, 2), (5, 2), (3, 3), (4, 3), (5, 3), (4, 4), (5, 4), (5, 5)}
(i) Domain of R-1 = {1, 2, 3, 4, 5}
(ii) Range of R = {1, 2, 3, 4, 5}

Question 4.

Solution:

(i) R = {(1, 2), (1, 3), (2, 3), (3, 2), (5, 6)}
R^{-1} = {(2, 1), (3, 1), (3, 2), (2, 3), (6, 5)}

(ii) $R = \{(x, y) : x, y \in N, x + 2y = 8\}$
On solving $x + 2y = 8$, we get:
$x = 8 - 2y$
On putting y = 1, we get x = 6.
On putting y = 2, we get x = 4.
On putting y = 3, we get x = 2.
$\therefore$ R = {(6, 1), (4, 2), (2, 3)}
Or,
R^{-1} = {(1, 6), (2, 4), (3, 2)}

(iii) R is a relation from {11, 12, 13} to {8, 10, 12} defined by $y = x - 3$.
x belongs to {11, 12, 13} and y belongs to {8, 10, 12}.
Also, $11 - 3 = 8$ and $13 - 3 = 10$
$\therefore$ R = {(11, 8), (13,10)}
Or,
R^{-1} = {(8, 11), (10,13)}

Question 5.

Solution:

(i) A relation R from the set [2, 3, 4, 5, 6] to the set [1, 2, 3] is defined by $x = 2y$.
Putting y = 1, 2, 3 in $x = 2y$, we get:
x = 2, 4, 6
$\therefore$ R = {(2, 1), (4, 2), (6, 3)}

(ii) A relation R on the set [1, 2, 3, 4, 5, 6, 7] defined by $(x, y) \in R \Leftrightarrow$ x is relatively prime to y.
Here,

2 is relatively prime to 3, 5 and 7.
3 is relatively prime to 2, 4, 5 and 7.
4 is relatively prime to 3, 5 and 7.
5 is relatively prime to 2, 3, 4, 6 and 7.
6 is relatively prime to 5 and 7.
7 is relatively prime to 2, 3, 4, 5 and 6.
∴ R = {(2, 3), (2, 5), (2, 7), (3, 2), (3, 4), (3, 5), (3, 7), (4, 3), (4, 5), (4, 7), (5, 2), (5, 3), (5, 4), (5, 6), (5, 7), (6, 5), (6, 7), (7, 2), (7, 3), (7,4), (7, 5), (7, 6)}

(iii) A relation R on the set [0, 1, 2,…, 10] is defined by 2x + 3y = 12.

$x = \frac{12-3y}{2}$

Putting y = 0, 2, 4, we get:
x = 6, 3, 0
∴ R = {(0, 4), (3, 2), (6, 0)}

(iv) A relation R from the set A = [5, 6, 7, 8] to the set B = [10, 12, 15, 16, 18] defined by (x, y) ∈ R ⇔ x divides y.
Here,
5 divides 10 and 15.
6 divides 12 and 18.
8 divides 16.
∴ R = {(5, 10), (5, 15), (6, 12), (6, 18), (8,16)}

Question 6.

Solution:

Let R be a relation in N defined by (x, y) ∈ R ⇔ x + 2y = 8.
We have:
x = 8-2y
For y = 3, 2, 1, we have:
x = 2, 4, 6
∴ R = {(2, 3), (4, 2), (6, 1)}
And,
R−1 = {(3, 2), (2, 4), (1, 6)}

Question 7.

Solution:

Given:
A = (3, 5) and B = (7, 11)
Also,

R = {$(a, b) : a \in A, b \in B, a - b$ is odd}
a are the elements of A and b are the elements of B.

$\therefore a - b = 3 - 7, 3 - 11, 5 - 7, 5 - 11$

$\Rightarrow a - b = -4, -8, -2, -6$

Here, $a - b$ is always an even number.

So, R is an empty relation from A to B.
Hence proved.

Question 8.

Solution:

We have:
A = {1, 2} and B = {3, 4}
Now,
$n(A \times B) = n(A) \times n(B) = 2 \times 2 = 4$
There are 2^n relations from A to B, where n is the number of elements in their Cartesian product.
∴ Number of relations from A to B is $2^4 = 16$.

Question 9.

Solution:

(i) R = {(x, x + 5): x ∈ (0, 1, 2, 3, 4, 5)}
We have:
R = {(0, 0 + 5), (1, 1 + 5), (2, 2 + 5), (3, 3 + 5), (4, 4 + 5), (5, 5 + 5)}
Or, R = {(0, 5), (1, 6), (2, 7), (3, 8), (4, 9), (5, 10)}
∴ Domain (R) = {0, 1, 2, 3, 4, 5}
Range (R) = {5, 6, 7, 8, 9, 10}

(ii) R = {(x, x^3) : x is a prime number less than 10}
We have:
x = 2, 3, 5, 7
x^3 = 8, 27, 125, 343
Thus, we get:
R = {(2, 8), (3, 27), (5, 125), (7, 343)}
Domain (R) = {2, 3, 5, 7}
Range (R) = {8, 27, 125, 343}

Question 10.

Solution :

(i) R = {(a, b) : a ∈ N, a < 5, b = 4}
We have:
a = 1, 2, 3, 4
b = 4
R = {(1, 4), (2, 4), (3, 4), (4, 4)}
Domain (R) = {1, 2, 3, 4}
Range (R) = {4}

(ii) $S = \{(a,\ b)\ :\ b = |a-1|,\ a \in\ Z\ and\ |a|\ \leq 3\}$
Now,
a = -3, -2, -1, 0, 1, 2, 3
$b\ =\ |-3-1|\ = 4$
$b\ =\ |-2-1|\ = 3$
$b\ =\ |-1-1|\ = 2$
$b\ =\ |0-1|\ = 1$
$b\ =\ |1-1|\ = 0$
$b\ =\ |2-1|\ = 1$
$b\ =\ |3-1|\ = 2$
Thus, we have:
b = 4, 3, 2, 1, 0, 1, 2
Or,
S = {(-3, 4), (-2, 3), (-1, 2), (0, 1), (1, 0), (2, 1), (3, 2)}
Domain (S) = {-3, -2, -1, 0, 1, 2, 3}
Range (S) = {0, 1, 2, 3, 4}

Question 11.

Solution:

Any relation in A can be written as a set of ordered pairs.
The only ordered pairs that can be included are (a, a), (a, b), (b, a) and (b, b).

There are four ordered pairs in the set, and each subset is a unique combination of them.
Each unique combination makes different relations in A.
{ } [the empty set]
{(a, a)}
{(a, b)}
{(a, a), (a, b)}
{(b, a)}
{(a, a), (b, a)}
{(a, b), (b, a)}
{(a, a), (a, b), (b, a)}
{(b, b)}

$\{(a, a), (b, b)\}$
$\{(a, b), (b, b)\}$
$\{(a, a), (a, b), (b, b)\}$
$\{(b, a), (b, b)\}$
$\{(a, a), (b, a), (b, b)\}$
$\{(a, b), (b, a), (b, b)\}$
$\{(a, a), (a, b), (b, a), (b, b)\}$

Number of elements in the Cartesian product of A and A = $2 \times 2 = 4$
$\therefore$ Number of relations = $2^4 = 16$

Question 12.

Solution:

Given:
A = (x, y, z) and B = (a, b)
Now,
Number of elements in the Cartesian product of $A \text{ and } B = 3 \times 2 = 6$
Number of relations from A to B = $2^6 = 64$

Question 13.

Solution:

Given: $R = [(a, b) : a, b \in N \text{ and } a = b^2]$

(i) $(a, a) \in R$ for all $a \in N$.
Here, $2 \in N$, but $2 \neq 2^2$.
$\therefore (2,2) \notin R$
False

(ii) $(a, b) \in R \Rightarrow (b, a) \in R$
$\because 4 = 2^2$
$(4, 2) \in R$, but $(2,4) \notin R$.
False

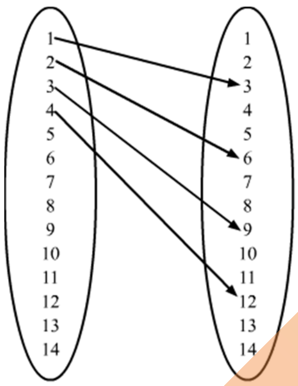
(iii) $(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R$
$\because 16 = 4^2$ and $4 = 2^2$
$\therefore (16, 4) \in R$ and $(4, 2) \in R$
Here,
$(16,2) \notin R$
False

Question 14.

Solution:

A = [1, 2, 3,…, 14]
R = {(x, y) : 3x − y = 0, where x, y ∈ A}
Or,
R = {(x, y) : 3x = y, where x, y ∈ A}
As
$3 \times 1 = 3$
$3 \times 2 = 6$
$3 \times 3 = 9$
$3 \times 4 = 12$

Or,
R = {(1, 3), (2, 6), (3, 9), (4, 12)}



Domain (R) = {1, 2, 3, 4}
Range (R) = {3, 6, 9, 12}
Co-domain (R) = A

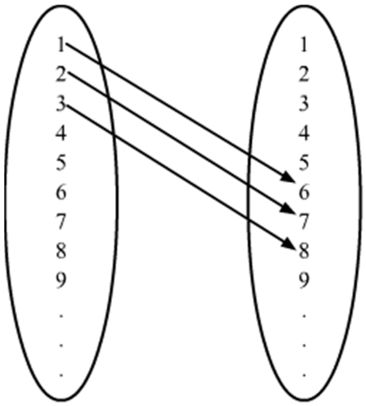
Question 15.

Solution:

R = {(x, y) : y = x + 5, x is a natural number less than 4, x, y ∈ N}

(i) ∵ x = 1, 2, 3
∴ y = 1 + 5, 2 + 5, 3 + 5
y = 6, 7, 8
Thus, we have:
R = {(1, 6), (2, 7), (3, 8)}

(ii)



Now,
Domain (R) = {1, 2, 3}
Range (R) = {6, 7, 8}

Question 16.

Solution:

A = [1, 2, 3, 5] and B = [4, 6, 9]
R = {(x, y) : the difference between x and y is odd, x ∈ A, y ∈ B}
For x = 1,
4-1 = 3 and 6-1 = 5
y = 4, 6
For x = 2,
9-2 = 7
y = 9
For x = 3,
4-3 = 1 and 6-3 = 3
y = 4, 6
For x = 5,
5-4 =1 and 6-5 =1
y = 4, 6
Thus, we have:
R = {(1, 4), (1, 6), (2, 9), (3, 4), (3, 6), (5, 4), (5, 6)}

Question 17.

Solution:

R = {(x, x^3) : x is a prime number less than 10}
x = 2, 3, 5, 7
x3 = 8, 27, 125, 343
R = {(2, 8), (3, 27), (5, 125), (7, 343)}

Question 18.

Solution:

A = [1, 2, 3, 4, 5, 6]
R = {(a, b) : a, b ∈ A, b is exactly divisible by a}

(i) Here,
2 is divisible by 1 and 2.
3 is divisible by 1 and 3.
4 is divisible by 1 and 4.
5 is divisible by 1 and 5.
6 is divisible by 1, 2, 3 and 6.
∴ R = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 2), (2, 4), (2, 6), (3, 3), (3, 6), (4, 4), (5, 5), (6, 6)}

(ii) Domain (R) = {1, 2, 3, 4, 5, 6}

(iii) Range (R) = {1, 2, 3, 4, 5, 6}

Question 19

Solution :

(i) We have:
5-2 = 3
6-2 = 4
7-2 = 5
∴ R = $\{(x, y) : y = x - 2, x \in P, y \in Q\}$

(ii) R = {(5, 3), (6, 4), (7, 5)}

(iii) Domain (R) = {5, 6, 7}
Range (R) = {3, 4, 5}

Question 20.

Solution:

R = {(a, b) : a, b ∈ Z, a − b is an integer}
We know:
Difference of any two integers is always an integer.
Thus, for all a, b ∈ Z, we get a − b as an integer.
∴ Domain (R) = Z
And,
Range (R) = Z

Question 21.

Solution:

We have:
(a, b) ∈ R1 ⇔ 1 + ab > 0
Let:
a = 1, b = -12 and c = -4
Now,

$$\left(1,-\frac{1}{2}\right)\in R_1\ and\left(-\frac{1}{2},-4\right)\in R_1\ \text{, as}\ 1+\left(-\frac{1}{2}\right)>0\ and\ 1+\left(-\frac{1}{2}\right)(-4)>0.$$

But 1+1×-4 <0.
∴ (1,-4) ∉R1
And,
(a, b) ∈ R1 and (b , c) ∈ R1
Thus, (a, c) ∈ R_1 is not true for all a, b, c ∈ R.

Question 22.

Solution:

We are given ,
(a, b) R (c, d) ⇔ a + d = b + c for all (a, b), (c, d) ∈ N × N

(i) (a, b) R (a, b) for all (a, b) ∈ N × N
$\because\ a+b = b+a\ for\ all\ a,b\in N$
$\therefore (a,b)\ R\,(a,b)\ for\ all\ a,b\in N$

(ii) (a, b) R (c, d) ⇒ (c, d) R (a, b) for all (a, b), (c, d) ∈ N × N
$(a,b)\ R\,(c,d) \Rightarrow a+d = b+c$
$\Rightarrow c+b = d+a$
$\Rightarrow (c,d)\ R\,(a,b)$

(iii) (a, b) R (c, d) and (c, d) R (e, f) ⇒ (a, b) R (e, f) for all (a, b), (c, d), (e, f) ∈ N × N
$(a,b)\ R\,(c,d)\ and\ (c,d)\ R\,(e,f)$
$\Rightarrow a+d = b+c\ and\ c+f = d+e$
$\Rightarrow a+d+c+f = b+c+d+e$
$\Rightarrow a+f = b+e$
$\Rightarrow (a,b)\ R\,(e,f)$