Exercise:2.2

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Question 1.

Solution :

Given:
$A = \{1, 2, 3\}$, $B = \{3, 4\}$ and $C = \{4, 5, 6\}$
Now,
$(A \times B) = \{(1, 3), (1, 4), (2, 3), (2, 4), (3, 3), (3, 4)\}$
$(B \times C) = \{(3, 4), (3, 5), (3, 6), (4, 4), (4, 5), (4, 6)\}$
$\therefore (A \times B) \cap (B \times C) = \{(3, 4)\}$

Question 2.

Solution :

Given:
$A = \{2, 3\}$, $B = \{4, 5\}$ and $C = \{5, 6\}$
Also,
$(B \cup C) = \{4, 5, 6\}$
Thus, we have:
$A \times (B \cup C) = \{(2, 4), (2, 5), (2, 6), (3, 4), (3, 5), (3,6)\}$
And,
$(B \cap C) = \{5\}$
Thus, we have:
$A \times (B \cap C) = \{(2, 5), (3, 5)\}$
Now,
$(A \times B) = \{(2, 4), (2, 5), (3, 4), (3, 5)\}$
$(A \times C) = \{(2, 5), (2, 6), (3, 5), (3, 6)\}$
$\therefore (A \times B) \cup (A \times C) = \{(2, 4), (2, 5), (2, 6), (3, 4), (3, 5), (3, 6)\}$

Question 3.

Solution :

Given:
$A = \{1, 2, 3\}$, $B = \{4\}$ and $C = \{5\}$

(i) $A \times (B \cup C) = (A \times B) \cup (A \times C)$
We have:
$(B \cup C) = \{4, 5\}$

LHS: $A \times (B \cup C) = \{(1, 4), (1, 5), (2, 4), (2, 5), (3, 4), (3, 5)\}$
Now,
$(A \times B) = \{(1, 4), (2, 4), (3, 4)\}$
And,
$(A \times C) = \{(1, 5), (2, 5), (3, 5)\}$
RHS: $(A \times B) \cup (A \times C) = \{(1, 4), (2, 4), (3, 4), (1, 5), (2, 5), (3, 5)\}$
$\therefore$ LHS = RHS
(ii) $A \times (B \cap C) = (A \times B) \cap (A \times C)$
We have:
$(B \cap C) = \phi$
LHS: $A \times (B \cap C) = \phi$
And,
$(A \times B) = \{(1, 4), (2, 4), (3, 4)\}$
$(A \times C) = \{(1, 5), (2, 5), (3, 5)\}$
RHS: $(A \times B) \cap (A \times C) = \phi$
$\therefore$ LHS = RHS
(iii) $A \times (B - C) = (A \times B) - (A \times C)$
We have:
$(B - C) = \phi$
LHS: $A \times (B - C) = \phi$
Now,
$(A \times B) = \{(1, 4), (2, 4), (3, 4)\}$
And,
$(A \times C) = \{(1, 5), (2, 5), (3, 5)\}$
RHS: $(A \times B) - (A \times C) = \phi$
$\therefore$ LHS = RHS

Question 4.

Solution :

Given:
$A = \{1, 2\}$, $B = \{1, 2, 3, 4\}$, $C = \{5, 6\}$ and $D = \{5, 6, 7, 8\}$

(i) $A \times C \subset B \times D$
LHS: $A \times C = \{(1, 5), (1, 6), (2, 5), (2, 6)\}$
RHS: $B \times D = \{(1, 5), (1, 6), (1, 7), (1, 8), (2, 5), (2, 6), (2, 7), (2, 8), (3, 5), (3, 6), (3, 7), (3, 8), (4, 5), (4, 6), (4, 7), (4, 8)\}$
$\therefore A \times C \subset B \times D$
(ii) $A \times (B \cap C) = (A \times B) \cap (A \times C)$
We have:
$(B \cap C) = \phi$
LHS: $A \times (B \cap C) = \phi$
Now,
$(A \times B) = \{(1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (2, 3), (2, 4)\}$
$(A \times C) = \{(1, 5), (1, 6), (2, 5), (2, 6)\}$

RHS: $(A \times B) \cap (A \times C) = \phi$
$\therefore$ LHS = RHS

Question 5.

Solution :

Given:
$A = \{1, 2, 3\}$, $B = \{3, 4\}$ and $C = \{4, 5, 6\}$

(i) $A \times (B \cap C)$
Now,

$(B \cap C) = \{4\}$
$\therefore A \times (B \cap C) = \{(1, 4), (2, 4), (3, 4)\}$

(ii) $(A \times B) \cap (A \times C)$
Now,
$(A \times B) = \{(1, 3), (1, 4), (2, 3), (2, 4), (3, 3), (3, 4)\}$
And,
$(A \times C) = \{(1, 4), (1, 5), (1, 6), (2, 4), (2, 5), (2, 6), (3, 4), (3, 5), (3, 6)\}$
$\therefore (A \times B) \cap (A \times C) = \{(1, 4), (2, 4), (3, 4)\}$

(iii) $A \times (B \cup C)$
Now,
$(B \cup C) = \{3, 4, 5, 6\}$
$\therefore A \times (B \cup C) = \{(1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 4), (2, 5), (2, 6), (3, 3), (3, 4), (3, 5), (3, 6)\}$

(iv) $(A \times B) \cup (A \times C)$
Now,
$(A \times B) = \{(1, 3), (1, 4), (2, 3), (2, 4), (3, 3), (3, 4)\}$
And,
$(A \times C) = \{(1, 4), (1, 5), (1, 6), (2, 4), (2, 5), (2, 6), (3, 4), (3, 5), (3, 6)\}$
$\therefore (A \times B) \cup (A \times C) = \{(1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 4), (2, 5), (2, 6), (3, 3), (3, 4), (3, 5), (3, 6)\}$

Question 6.

Solution :

(i) $(A \cup B) \times C = (A \times C) \cup (B \times C)$
Let (a, b) be an arbitrary element of $(A \cup B) \times C$.
Thus, we have:

$$(a,b) \in (A \cup B) \times C$$
$$\Rightarrow a \in (A \cup B) \text{ and } b \in C$$
$$\Rightarrow (a \in A \text{ or } a \in B) \text{ and } b \in C$$
$$\Rightarrow (a \in A \text{ and } b \in C) \text{ or } (a \in B \text{ and } b \in C)$$
$$\Rightarrow (a,b) \in (A \times C) \text{ or } (a,b) \in (B \times C)$$
$$\Rightarrow (a,b) \in (A \times C) \cup (B \times C)$$
$$\therefore (A \cup B) \times C \subseteq (A \times C) \cup (B \times C) \quad \ldots(i)$$

Again, let (x, y) be an arbitrary element of $(A \times C) \cup (B \times C)$.
Thus, we have:

$$(x,y) \in (A \times C) \cup (B \times C)$$
$$\Rightarrow (x,y) \in (A \times C) \text{ or } (x,y) \in (B \times C)$$
$$\Rightarrow (x \in A \;\&\; y \in C) \text{ or } (x \in B \;\&\; y \in C)$$
$$\Rightarrow (x \in A \text{ or } x \in B) \text{ or } y \in C$$
$$\Rightarrow (x \in A \cup B) \;\&\; y \in C$$
$$\Rightarrow (x,y) \in (A \cup B) \times C$$
$$\therefore (A \times C) \cup (B \times C) \subseteq (A \cup B) \times C \quad \ldots(ii)$$

From (i) and (ii), we get:
$(A \cup B) \times C = (A \times C) \cup (B \times C)$

(ii) $(A \cap \mathrm{B}) \times \mathrm{C} = (A \times C) \cap (\mathrm{B} \times C)$
Let (a, b) be an arbitrary element of $(A \cap \mathrm{B}) \times \mathrm{C}$.
Thus, we have:

$$(a,b) \in (A \cup B) \times C$$
$$\Rightarrow a \in (A \cup B) \;\&\; b \in C$$
$$\Rightarrow (a \in A \;\&\; a \in B) \;\&\; b \in C$$
$$\Rightarrow (a \in A \;\&\; b \in C) \;\&\; (a \in B \;\&\; b \in C)$$
$$\Rightarrow (a,b) \in (A \times C) \;\&\; (a,b) \in (B \times C)$$
$$\Rightarrow (a,b) \in (A \times C) \cup (B \times C)$$
$$\therefore (A \cup B) \times C \subseteq (A \times C) \cup (B \times C) \quad \ldots(iii)$$

Again, let (x, y) be an arbitrary element of $(A \times C) \cap (B \times C)$.
Thus, we have:

$(x, y) \in (A\times C)\cup(B\times C)$

$\Rightarrow (x, y) \in (A\times C)$ & $(x, y) \in (B\times C)$

$\Rightarrow (x\in A$ & $y\in C)$ & $(x\in B$ & $y\in C)$

$\Rightarrow (x\in A$ & $x\in B)$ & $y\in C$

$\Rightarrow x\in(A\cup B)$ & $y\in C$

$\Rightarrow (x, y) \in (A\cup B)\times C$

$\therefore (A\times C)\cup(B\times C)\subseteq(A\cup B)\times C \quad ...(iv)$

From (iii) and (iv), we get:

$(A \cap B) \times C = (A \times C) \cap (B \times C)$

Question 7.

Solution :

Let :

$(x,\ y) \in (A\times B)$

$\therefore\ x\in A,\ y\in B$

Now,

$\because (A\times B)\subseteq(C\times D)$

$\therefore (x, y) \in (C\times D)$

Or,

$x\in C$ *and* $y\in D$

Thus, *we have* :

$A\subseteq C$ & $B\subseteq D$