Chapter 2 - Relation

Exercise:2.1

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Question 1.

Solution :

(i) $\left(\frac{a}{3}+1,\ b-\frac{2}{3}\right)=\left(\frac{5}{3},\ \frac{1}{3}\right)$

By the definition of equality of ordered pairs, we have:

$\left(\frac{a}{3}+1,b-\frac{2}{3}\right)=\left(\frac{5}{3},\frac{1}{3}\right)$

$\Rightarrow\left(\frac{a}{3}+1\right)=\frac{5}{3}\ and\ \left(b-\frac{2}{3}\right)=\frac{1}{3}$

$\Rightarrow\frac{a}{3}=\frac{5}{3}-1\ and\ b=\frac{1}{3}+\frac{2}{3}$

$\Rightarrow\frac{a}{3}=\frac{2}{3}\ and\ b=1$

$\Rightarrow a=2\ and\ b=1$

(ii) (x + 1, 1) = (3, y − 2)

By the definition of equality of ordered pairs, we have:

$(x+1)=3\,and\,1=(y-2)\Rightarrow x=2\ and\ y=3$

Question 2.

Solution :

The ordered pairs (x, −1) and (5, y) belong to the set {(a, b) : b = 2a − 3}.

Thus, we have:

x = a and −1 = b such that b = 2a − 3.

∴ −1 = 2x − 3

or, 2x = 3 − 1 = 2

or, x = 1

Also,

5 = a and y = b such that b = 2a − 3.

∴ y = 2(5) − 3

or, y = 10 − 3 = 7

Thus, we get:

x = 1 and y = 7

Question 3

.

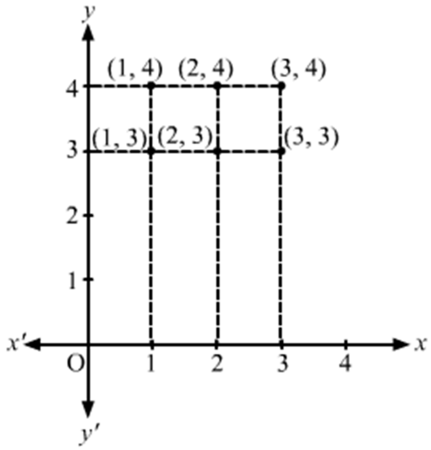
Solution :

Given:

a ∈ [−1, 2, 3, 4, 5] and b ∈ [0, 3, 6]

We know:
$-1 + 6 = 5$, $2 + 3 = 5$ and $5 + 0 = 5$
Thus, possible ordered pairs (a, b) are {(−1, 6), (2, 3), (5, 0)} such that a + b = 5.
Question 4.

Solution :
Given:
a ∈ [2, 4, 6, 9] and b ∈ [4, 6, 18, 27]
Here,
2 divides 4, 6 and 18 and 2 is less than all of them.
6 divides 18 and 6 and 6 is less than 18.
9 divides 18 and 27 and 9 is less than 18 and 27.
Now,
Set of all ordered pairs (a, b) such that a divides b and a < b = {(2, 4), (2, 6), (2, 18), (6, 18), (9, 18), (9, 27)}
Question 5.
Solution :
Given:
A = {1, 2} and B = {1, 3}
Now,
A × B = {(1, 1), (1, 3), (2, 1), (2, 3)}
B × A = {(1, 1), (1, 2), (3, 1), (3, 2)}
Question 6.
Solution :
Given:
$A = \{1, 2, 3\}$ and $B = \{3, 4\}$
Now,
$A \times B = \{(1, 3), (1, 4), (2, 3), (2, 4), (3, 3), (3, 4)\}$
To represent $A \times B$ graphically, follow the given steps:
(a) Draw two mutually perpendicular lines—one horizontal and one vertical.
(b) On the horizontal line, represent the elements of set A; and on the vertical line, represent the elements of set B.
(c) Draw vertical dotted lines through points representing elements of set A on the horizontal line and horizontal lines through points representing elements of set B on the vertical line.
The points of intersection of these lines will represent $A \times B$ graphically.



Question 7.

Solution :

Given :

A = {1, 2, 3} and B = {2, 4}

Now,

A × B = {(1, 2), (1, 4), (2, 2), (2, 4), (3, 2), (3, 4)}

B × A = {(2, 1), (2, 2), (2, 3), (4, 1), (4, 2), (4, 3)}

A × A = {(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)}

B × B = {(2, 2), (2, 4), (4, 2), (4, 4)}

We observe:

(A × B) ∩ (B × A) = {(2, 2)}

Question 8.

Solution :

Given:

n(A) = 5 and n(B) = 4

Thus, we have:

n(A × B) = 5(4) = 20

A and B are two sets having 3 elements in common.

Now,

Let:

A = (a, a, a, b, c) and B = (a, a, a, d)

Thus, we have:

(A × B) = {(a, a), (a, a), (a, a), (a, d), (a, a), (a, a), (a, a), (a, d), (a, a), (a, a), (a, a), (a, d), (b, a), (b, a), (b, a), (b, d), (c, a), (c, a), (c, a), (c, d)}

(B × A) = {(a, a), (a, a), (a, a), (a, b), (a, c), (a, a), (a, a), (a, a), (a, b), (a, c), (a, a), (a, a), (a, a), (a, b), (a, c), (d, a), (d, a), (d, a), (d, b), (d, c)}

[(A × B) ∩ (B × A)] = {(a, a), (a, a), (a, a), (a, a), (a, a), (a, a), (a, a), (a, a), (a, a)}

∴ n[(A × B) ∩ (B × A)] = 9

Question 9.

Solution :

Case (i): Let:
A = (a, b, c)
B = (e, f)
Now, we have:
A × B = {(a, e}), (a, f), (b,e), (b, f), (c, e), (c, f)}
B × A = {(e, a), (e, b), (e, c), (f, a), (f, b), (f, c)}
Thus, they have no elements in common.
Case (ii): Let:
A = (a, b, c)
B = (a, f)
Thus, we have:
A × B = {(a, a}), (a,f), (b, a), (b, f), (c,a), (c, f)}
B × A = {(a, a), (a, b), (a, c), (f, a), (f, b), (f, c)}
Here, A × B and B × A have two elements in common.
Thus, A × B and B × A will have elements in common iff sets A and B have elements in common.

Question 10.

Solution :

A is the set of all first entries in ordered pairs in A × B and B is the set of all second entries in ordered pairs in A × B.
Also,
n(A) = 3 and n(B) = 2
∴ A = {x, y, z} and B = {1, 2}

Question 11.

Solution :

Given:
A = {1, 2, 3, 4}
R = {(a, b) : a ∈ A, b ∈ A, a divides b}
We know:
1 divides 1, 2, 3 and 4.
2 divides 2 and 4.
3 divides 3.
4 divides 4.
∴ R = {(1, 1), (1, 2), (1, 3), (1, 4), (2, 2), (2, 4), (3, 3), (4, 4)}

Question 12.

Solution :

Given:
A = {−1, 1}
Thus, we have:
A × A = {(−1, −1), (−1, 1), (1, −1), (1, 1)}
And,
A × A × A = {(−1, −1, −1), (−1, −1, 1), (−1, 1, −1), (−1, 1, 1), (1, −1, −1), (1, −1, 1), (1, 1, −1), (1, 1, 1)}

Question 13.

Solution :

(i) False

Correct statement:

If P = {m, n} and Q = {n, m}, then P × Q = {(m, n), (m, m), (n, n), (n, m)}.

(ii) False

Correct statement:

If A and B are non-empty sets, then A × B is a non-empty set of an ordered pair (x, y) such that x ∈ A and y ∈ B.

(iii) True

A = {1, 2} and B = {3, 4}

Now,

(B ∩ ϕ) = ϕ

The Cartesian product of any set and an empty set is an empty set.

∴ A × (B ∩ ϕ) = ϕ

Question 14 .

Solution :

Given:

A = {1, 2}

Now,

A × A = {(1, 1), (1, 2), (2, 1), (2, 2)}

∴ A × A × A = {(1, 1, 1), (1, 1, 2), (1, 2, 1), (1, 2, 2), (2, 1, 1), (2, 1, 2), (2, 2, 1), (2, 2, 2)}

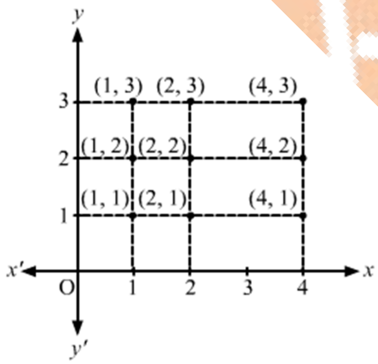
Question 15.

Solution :

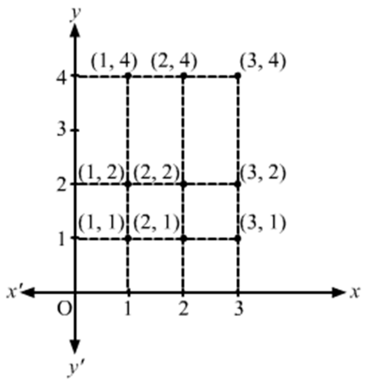
Given:

A = {1, 2, 4} and B = {1, 2, 3}

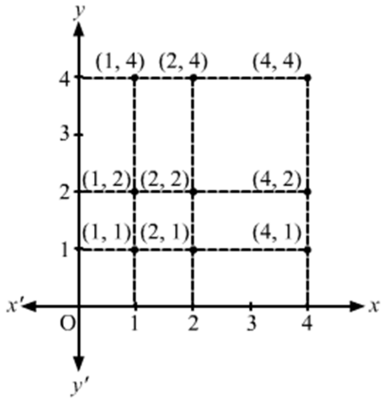
(i) A × B = {(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (4, 1), (4, 2), (4, 3)}



(ii) B × A = {(1, 1), (1, 2), (1, 4), (2, 1), (2, 2), (2, 4), (3, 1), (3, 2), (3, 4)}



(iii) A × A = {(1, 1), (1, 2), (1, 4), (2, 1), (2, 2), (2, 4), (4, 1), (4, 2), (4, 4)}



(iv) B × B = {(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)}