Quadrilaterals 14.1

1.

Sol:

Given

Three angles are

$110°, 50°$ and $40°$

Let fourth angle be x

We have,

Sum of all angles of a quadrilaterals $= 360°$

$110° + 50° + 40° + x° = 360°$

$\Rightarrow x = 360° - 200°$

$\Rightarrow x = 160°$

Required fourth angle $= 160°$

2.

Sol:

Let the angles of the quadrilateral be

$A = x, B = 2x, C = 4x$ and $D = 5x$ then,

$A + B + C + D = 360°$

$\Rightarrow x + 2x + 4x + 5x = 360°$

$\Rightarrow 12x = 360°$

$$\Rightarrow x = \frac{360°}{12}$$

$\Rightarrow \quad x = 30°$

$\therefore A = x = 30°$

$B = 2x = 60°$

$C = 4x = 30°(4) = 120°$

$D = 5x = 5(30°) = 150°$

3.

Sol:

In ΔDOC

$\angle 1 + \angle COD + \angle 2 = 180°$ [Angle sum property of a triangle]

$\Rightarrow \angle COD = 180 - \angle 1 - \angle 2$

$\Rightarrow \angle COD = 180 - \angle 1 + \angle 2$

$$\Rightarrow \angle COD = 180 - \left[\frac{1}{2}\angle C + \frac{1}{2}\angle D\right]$$

[$\because$ OC and OD are bisectors of $\angle C$ and $\angle D$ represents]

$\Rightarrow \angle COD = 180 - \frac{1}{2}(\angle C + \angle D)]$ .....(1)

In quadrilateral $ABCD$

$\angle A + \angle B + \angle C + \angle D = 360°$

$\angle C + \angle D = 360 - \angle A + \angle B$ ......(2)

$\Rightarrow \angle COD = 180 - (\angle C + \angle D)]$ [Angle sum property of quadrilateral]

Substituting (ii) in (i)

$\Rightarrow \angle COD = 180 - \frac{1}{2}(360 - (\angle A + \angle B))$

$\Rightarrow \angle COD = 180 - 180 + \frac{1}{2}(\angle A + \angle B)$

$\Rightarrow \angle COD = \frac{1}{2}(\angle A + \angle B)$

4.

Sol:

Let the common ratio between the angle is 't' so the angles will be $3t, 5t, 9t$ and $13t$ respectively

Since the sum of all interior angles of a quadrilateral is $360°$

$\therefore 3x + 5x + 9x + 13x = 360°$

$\Rightarrow 30x = 360°$

$\Rightarrow x = 12°$

Hence, the angles are

$3x = 3 \times 12 = 36°$

$5x = 5 \times 12 = 60°$

$9x = 9 \times 12 = 108°$

$13x = 13 \times 12 = 156°$

Quadrilaterals 14.2

1.

Sol:

We know that

Opposite sides of a parallelogram are equal

$\therefore 3x-2=50-x$

$\Rightarrow 3x+x=50+2$

$\Rightarrow 4x=52$

$\Rightarrow x=13°$

$\therefore (3x-2)° = (3\times 13-2) = 37°$

$(50-x)° = (50-13°) = 37°$

Adjacent angles of a parallelogram are supplementary

$\therefore x+37=180°$

$\therefore x=180°-37°=143°$

Hence, four angles are $37°, 143°, 37°, 143°$

2.

Sol:

Let the measure of the angle be x

$\therefore$ The measure of the angle adjacent is $\frac{2x}{3}$

WKT the adjacent angle of a parallelogram is supple mentary

Hence $x+\frac{2x}{3}=180°$

$2x+3x=540°$

$\Rightarrow 5x=540°$

$\Rightarrow x=108°$

Adjacent angles are supplementary

$\Rightarrow x+108°=180°$

$\Rightarrow x=180°-108° = 72°$

$\Rightarrow x=72°$

Hence, four angles are $180°, 72°, 108°, 72°$

3.

Sol:

Let the smallest angle be x

Then, the other angle is $(3x-24)$

$Now,\ x+2x-24=180°$

$3x-24=180°$

$\Rightarrow 3x=180+24$

$\Rightarrow x=\frac{204}{3}=68°$

$\Rightarrow x=68°$

$\Rightarrow 2x-24°=2\times 68°-24°=136°-24°$

$\Rightarrow$ Hence four angles are $68°, 112°, 68°, 112°$

4.

Sol:

Let the shorter side be x

$\therefore$ Perimeter $=x+65+6\cdot 5+x$ [sum of all sides]

$22=2(x+6\cdot 5)$

$11=x+6\cdot 5$

$\Rightarrow x=11-6\cdot 5=4\cdot 5cm$

$\therefore$ Shorter side $=4\cdot 5cm$

5.

Sol:

In a parallelogram $ABCD$

Adjacent angles are supplementary

So, $\angle D+\angle C=180°$

$135°+\angle C=180° \Rightarrow \angle C=180°-135°$

$\angle C=45°$

In a parallelogram opposite sides are equal

$\angle A=\angle C=45°$

$\angle B=\angle D=135°$

6.

Sol:

In a parallelogram ABCD

$\angle A=70°$ [$\because$ Adjacent angles supplementary]

$\angle A=\angle B=180°$

$70°+\angle B=180°$ [$\because \angle A=70°$]

$\angle B=180°-70°$

$=110°$

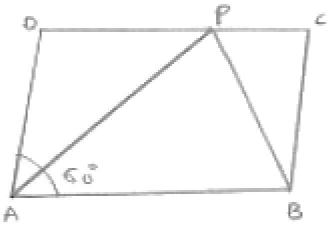
In a parallelogram opposite sides are equal

$\angle A = \angle C = 70°$

$\angle B = \angle D = 110°$

7.

Sol:



AP bisects $\angle A$

Then, $\angle AP = \angle PAB = 30°$

Adjacent angles are supplementary

Then, $\angle A + \angle B = 180°$

$\angle B + 60° = 180°$ $\qquad$ $\angle A = 60°$

$\angle B = 180° - 60°$

$\angle B = 120°$

BP bisects $\angle B$

Then, $\angle PBA \quad \angle PBC = 30°$

$\angle PAB = \angle APD = 30°$ [Alternative interior angles]

$\therefore AD = DP$ [$\because$ Sides opposite to equal angles are in equal length]

Similarly

$\angle PBA = \angle BPC = 60°$ [Alternative interior angle]

$\therefore PC = BC$

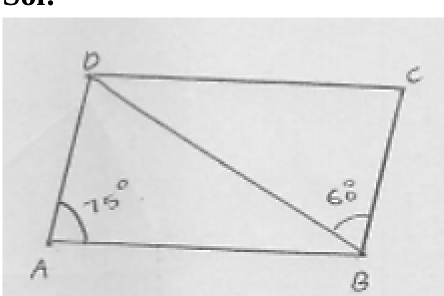
$DC = DP + PC$

$DC = AD + BC$ $\quad [\because DP = AD, PC = BC]$

$DC = 2AD$ [$\because AD = BC$ Opposite sides of a parallelogram are equal]

8.

Sol:



To find $\angle CDB$ *and* $\angle ADB$

[Alternative interior angle $AD \parallel BC$ *and* BD is the transversal]

In a parallelogram $ABCD$

$\angle A = \angle C = 75^\circ$

[$\because$ Opposite side angles of a parallelogram are equal]

In $\angle BDC$

$\angle CBD + \angle C + \angle CDB = 180^\circ$ [Angle sum property]

$\Rightarrow 60^\circ + 75^\circ + \angle CDB = 180^\circ$

$\Rightarrow \angle CDB = 180^\circ - (60^\circ + 75^\circ)$

$\Rightarrow \angle CDB = 45^\circ$

Hence $\angle CDB = 45^\circ, \angle ADB = 60^\circ$

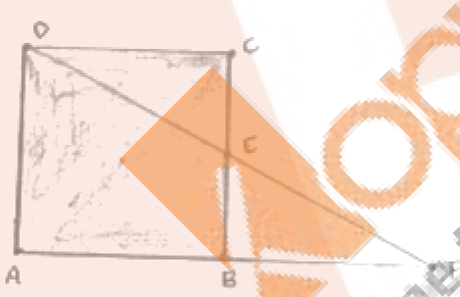
9.

Sol:

In ΔBEF and ΔCED

$\angle BEF = \angle CED$ [Verified opposite angle]

$BE = CE$ [$\because$ E is the mid-point of BC]



$\angle$EBF = $\angle$ECD [$\therefore$ Alternate interior angles are equal]

$\because \nabla BEF \cong \Delta CED$ [A, S, A congruence]

$\because BF = CD$ [C.P.C.T]

$AF = AB + AF$

$AF = AB + AB$

$AF = 2AB$

10.

Sol:

(i) False

(ii) True

(iii) False

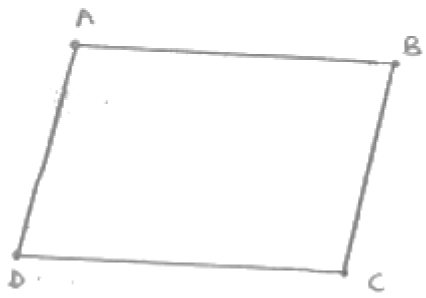
(iv) False

(v) True
(vi) False
(vii) False
(viii) True

Quadrilaterals 14.3

1.

Sol:



$\angle C$ and $\angle D$ are consecutive interior angles on the same side of the transversal CD

$\therefore \angle C + \angle D = 180°$

2.

Sol:

given $\angle B = 135°$

$ABCD$ is a parallelogram

$\therefore \angle A = \angle C, \angle B = \angle D$ and $\angle A + \angle B = 180°$

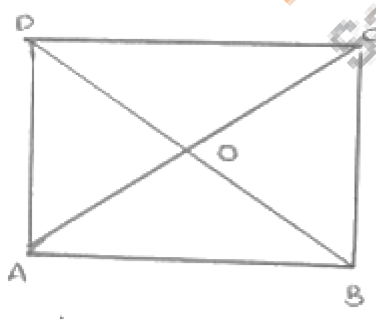
$\angle A + \angle B = 180°$

$\angle A = 45°$

$\Rightarrow \angle A = \angle C = 45°$ and $\angle B = \angle C = 135°$

3.

Sol:

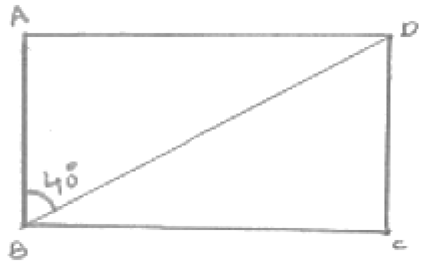


Since, diagonals of square bisect each other at right angle

$\therefore \angle ADB = 90°$

4.

Sol:



We have,

$\angle ABC = 90°$

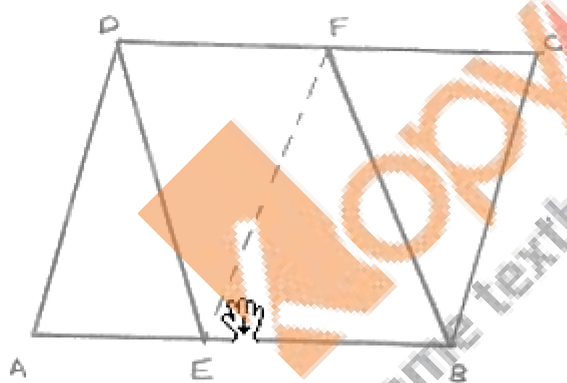
$\Rightarrow \angle ABD + \angle DBC = 90°$ $[\because \angle ABD = 40°]$

$\Rightarrow 40 + \angle DBC = 90°$

$\therefore \angle DBC = 50°$

5.

Sol:



Since ABCD is a parallelogram

$\therefore AB \parallel DC$ and $AB = DC$

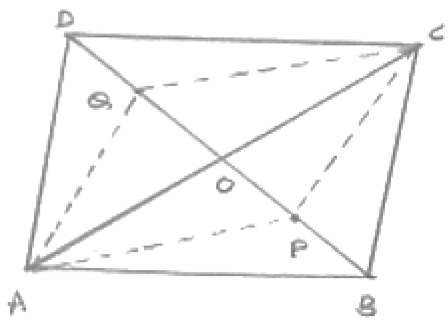
$\Rightarrow EB \parallel DF$ and $\frac{1}{2}AB = \frac{1}{2}DC$

$\Rightarrow EB \parallel DF$ and $EB = DF$

$EBFD$ is a parallelogram

6.

Sol:



WKT,

Diagonals of a parallelogram bisect each other

$\therefore OA = OC$ and $OB = OD$

Since P and Q are point of intersection of BD

$\therefore BP = PQ = QD$

Now, $OB = OD$ and $BP = QD$

$\Rightarrow OB - BP = OD - QD$

$\Rightarrow OP = OQ$

Thus in quadrilateral APCQ, we have

$OA = OC$ and $OP = OQ$

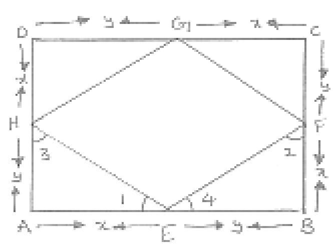
$\Rightarrow$ diagonals of quadrilateral APCQ bisect each other

$\therefore APCQ$ is a parallelogram

Hence $AP \parallel CQ$

7.

Sol:



We have

$AE = BF = CG = DH = x(say)$

$\therefore BE = CF = DG = AH = y(say)$

In $\Delta's\ AEH\ and\ BEF$, we have

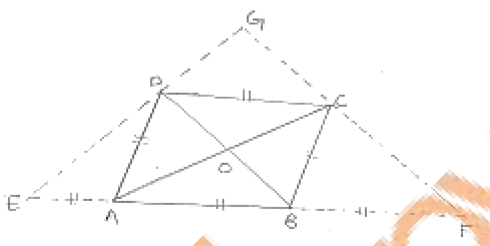
$AE = BF$

$\angle A = \angle B$

And $AH = BE$
So, by SAS configuration criterion, we have
$\Delta AEH \cong \Delta BFE$
$\Rightarrow \angle 1 = \angle 2 \text{ and } \angle 3 = \angle 4$
But $\angle 1 + \angle 3 = 90° \text{ and } \angle 2 + \angle A = 90°$
$\Rightarrow \angle 1 + \angle 3 + \angle 2 + \angle A = 90° + 90°$
$\Rightarrow \angle 1 + \angle 4 + \angle 1 + \angle 4 = 180°$
$\Rightarrow 2(\angle 1 + \angle 4) = 180°$
$\Rightarrow \angle 1 + \angle 4 = 90°$
Similarly we have $\angle F = \angle G = \angle H = 9°$
Hence, $EFGH$ is a square

8.

Sol:



We know that the diagonals of a rhombus are perpendicular bisector of each othee
$\therefore OA = OC, OB = OD, \angle AOD = \angle COD = 90°$
And $\angle AOB = \angle COB = 90°$
In $\Delta BDE, A \text{ and } O$ are midpoints of BE and BD respectively
$OA \parallel DE$
$OC \parallel DG$
In $\Delta CFA, B \text{ and } O$ are midpoints of AF and AC respectively
$\therefore OB \parallel CF$
$OD \parallel GC$
Thus, in quadrilateral $DOCG$, we have
$OC \parallel DG \text{ and } OD \parallel GC$
$\Rightarrow DOCG$ is a parallelogram
$\angle DGC = \angle DOC$
$\angle DGC = 90°$

9.

Sol:
Draw a parallelogram $ABCD$ with AC and BD intersecting at O

Produce AD to E such that $DE = DC$

Join EC and produce it to meet AB produced at F

In ΔDCE

$\therefore \angle DCE = \angle DEC \quad ....CD$

[In a triangle, equal sides have equal angles opposite]

$AB \parallel CD$ (Opposite sides of the parallelogram are parallel)

$\therefore AE \parallel CD \quad (AB \text{ Lies on } AF)$

$AF \parallel CD$ and EF is the transversal.

$\therefore \angle DCE = \angle BFC \quad .....(2)$ [Pair of corresponding angles]

From (1) and (2), we get

$\angle DEC = \angle BFC$

In ΔAFE,

$\angle AFE = \angle AEF \quad (\angle DEC = \angle BFC)$

$\therefore AE = AF$ (In a triangle, equal angles have equal sides opposite to them)

$\Rightarrow AD + DE = AB + BF$

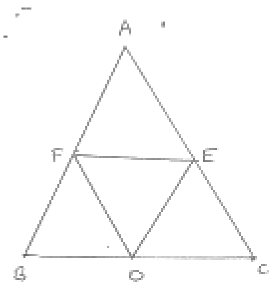
$\Rightarrow BC + AB = AB + BF \quad [\because AD = BC, DE = CD \text{ and } CD = AB, AB = DE]$

$\Rightarrow BC = BF$.

Quadrilaterals – 14.4

1.

Sol:



Given that

$AB = 7cm, BC = 8cm, AC = 9cm$

In ΔABC

$\therefore F$ *and* E are the midpoint of AB and AC

$\therefore EF = \frac{1}{2} BC$ [Mid-points the orem]

Similarly

$DF = \frac{1}{2} AC, DE = \frac{1}{2} AB$

Perimeter of $\Delta DEF = DE + EF + DF$

$= \frac{1}{2} AB + \frac{1}{2} BC \frac{1}{2} AC$

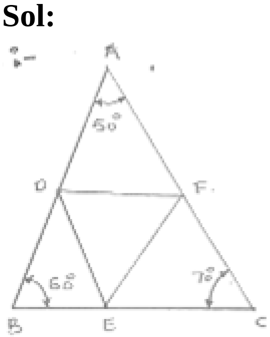
$= \frac{1}{2} \times 7 + \frac{1}{2} \times 8 + \frac{1}{2} \times 9$

$= 3 \cdot 5 + 4 + 4 \cdot 5 = 12cm$

$\therefore$ Perimeter of $\Delta DEF = 12cm$

2.

Sol:



In ΔABC

D and E are midpoints of AB and BC

By midpoint theorem

$\therefore DE \parallel AC, DE = \frac{1}{2} AC.$

F is the midpoint of AC

Then, $DE = \frac{1}{2} AC = CF$

In a quadrilateral DECF

$DE \parallel AC, DE = CF$

Hence $DECF$ is a parallelogram

$\therefore \angle C = \angle D = 70°$ [Opposite sides of parallelogram]

Similarly

$BEFD$ is a parallelogram, $\angle B = \angle F = 60°$

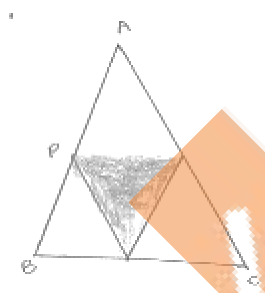
$ADEF$ is a parallelogram, $\angle A = \angle E = 50°$

$\therefore$ Angles of ΔDEF

$\angle D = 70°, \angle E = 50°, \angle F = 60°$

3.

Sol:



In ΔABC

R and P are the midpoint of AB and BC

$\therefore RP \parallel AC, RP = \frac{1}{2} AC$ [By midpoint theorem]

In a quadrilateral

[A pair of side is parallel and equal]

$RP \parallel AQ, RP = AQ$

$\therefore RPQA$ is a parallelogram

$AR = \frac{1}{2} AB = \frac{1}{2} \times 30 = 15cm$

$AR = QP = 15$ [$\because$ Opposite sides are equal]

$\Rightarrow RP = \frac{1}{2} AC = \frac{1}{2} \times 21 = 10 \cdot 5cm$ [$\because$ Opposite sides are equal]

Now,

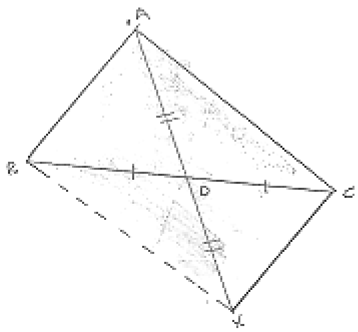
Perimeter of $ARPQ = AR + QP + RP + AQ$

$= 15 + 15 + 10 \cdot 5 + 10 \cdot 5$

$= 51 cm$

4.

Sol:



In a quadrilateral $ABXC$, we have

$AD = DX$ $\quad\quad$ $[Given]$

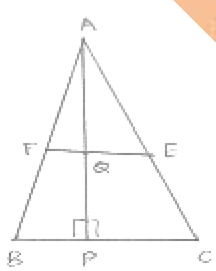
$BD = DC$ $\quad\quad$ $[Given]$

So, diagonals AX and BC bisect each other

$\therefore ABXC$ is a parallelogram

5.

Sol:



In ΔABC

E and F are midpoints of $AB \phi AC$

$\therefore EF \parallel FE, \frac{1}{2} BC = FE$ $\quad\quad$ [$\because$ By mid-point theorem]

In ΔABD

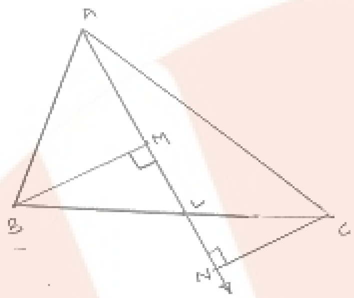
F is the midpoint of AB and $FQ \parallel BP$ $\quad\quad$ $[\because EF \parallel BC]$

$\therefore Q$ is the midpoint of AP $\quad\quad$ [By converse of midpoint theorem]

Hence, $AQ = QP$

6.

Sol:



In B

Given that

In ΔBLM *and* ΔCLN

$\angle BML = \angle CNL = 90°$

$BL = CL$ [L is the midpoint of BC]

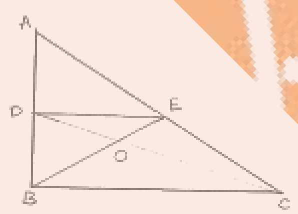
$\angle MLB = \angle NLC$ [vertically opposite angle]

$\therefore \Delta BLM = \Delta CLN$ $(A \cdot L \cdot A \cdot S)$

$\therefore LM = LN$ [Corresponding plats parts of congruent triangles]

7.

Sol:



In right $\Delta ABC, \angle B = 90°$

By using Pythagoras theorem

$AC^2 = AB^2 + BC^2$

$\Rightarrow 15^2 = 9^2 + BC^2$

$\Rightarrow BC = \sqrt{15^2 - 9^2}$

$\Rightarrow BC = \sqrt{225 - 81}$

$\Rightarrow BC = \sqrt{144}$

$= 12cm$

In ΔABC

D *and* E are midpoints of AB *and* AC

$\therefore DE \parallel BC, DE = \frac{1}{2}BC$ [By midpoint theorem]

$AD = OB = \frac{AB}{2} = \frac{9}{2} = 4\cdot 5cm$ [$\because D$ is the midpoint of AB]

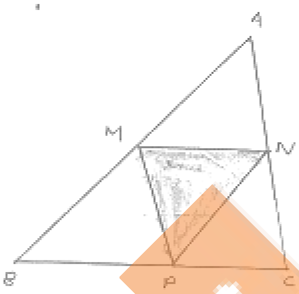
$DE = \frac{BC}{2} = \frac{12}{2} = 6cm$

Area of $\Delta ADE = \frac{1}{2} \times AD \times DE$

$= \frac{1}{2} \times 4\cdot 5 \times 6 = 13\cdot 5cm^2$

8.

Sol:



Given $MN = 3cm, NP = 3\cdot 5cm$ *and* $MP = 2\cdot 5cm$

To find BC, AB *and* AC

In ΔABC

M and N are midpoints of AB and AC

$\therefore MN = \frac{1}{2}BC, MN \parallel BC$ [By midpoint theorem]

$\Rightarrow 3 = \frac{1}{2}BC$

$\Rightarrow 3 \times 2 = BC$

$\Rightarrow BC = 6cm$

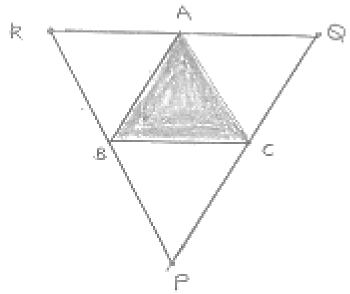
Similarly

$AC = 2MP = 2(2\cdot 5) = 5cm$

$AB = 2NP = 2(3\cdot 5) = 7cm$

9.

Sol:



Clearly ABCQ and ARBC are parallelograms

$\therefore BC = AQ \text{ and } BC = AR$

$\Rightarrow AQ = AR$

$\Rightarrow A$ is the midpoint of QR

Similarly B and C are the midpoints of PR and PQ respectively

$\therefore AB = \frac{1}{2}PQ, BC = \frac{1}{2}QR, CA = \frac{1}{2}PR$

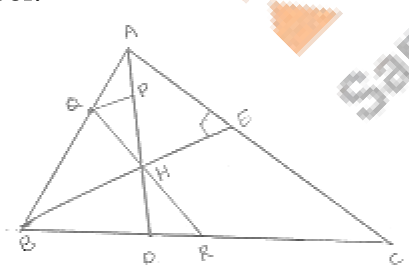
$\Rightarrow PQ = 2AB, QR = 2BC \text{ and } PR = 2CA$

$\Rightarrow PQ + QR + RP = 2(AB + BC + CA)$

$\Rightarrow$ Perimeter of $\Delta PQR = 2$ [Perimeter of ΔABC]

10.

Sol:



Given

$BE \perp AC$ and P, Q and Rare respectively midpoint of $AH, AB \text{ and } BC$

To prove:

$\angle PQRD = 90°$

Proof: In $\Delta ABC, Q \text{ and } R$ are midpoints of AB and BC respectively

$\therefore QR \parallel AC$ ......(i)

In $\Delta ABH, Q \text{ and } P$ are midpoints of AB and AH respectively

$\therefore QP \parallel BH$

$\Rightarrow QP \parallel BE$ ......(ii)

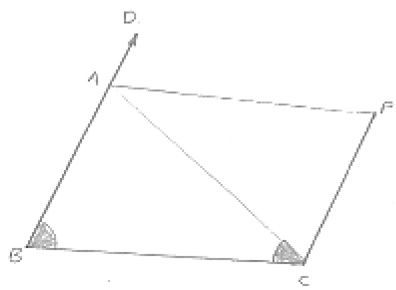
But, $AC \perp BE$ $\therefore$ from equation (i) and equation (ii) we have

$QP \perp QR$

$\Rightarrow \angle PQR = 90°$, hence proved.

11.

Sol:



Given

$AB = AC$ and $CD \parallel BA$ *and* AP is the bisector of exterior

$\angle CAD$ of ΔABC

To prove:

(i) $\angle PAC = \angle BCA$

(ii) $ABCD$ is a parallelogram

Proof:

(i) We have, [Opposite angles of equal sides of triangle are equal]

AB=AC

$\Rightarrow$ ∠ACB=∠ABC

Now, $\angle CAD = \angle ABC + \angle ACB$

$\Rightarrow \angle PAC + \angle PAD = 2\angle ACB$ $(\because \angle PAC = \angle PAD)$

$\Rightarrow 2\angle PAC = 2\angle ACB$

$\Rightarrow \angle PAC = \angle ACB$

(ii) Now,

$\angle PAC = \angle BCA$

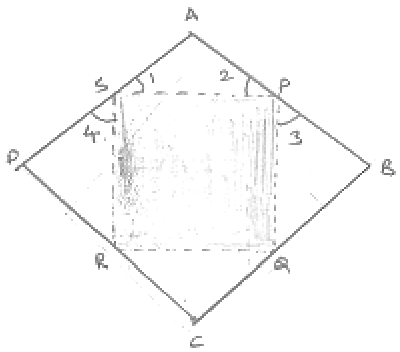
$\Rightarrow AP \parallel BC$

And, $CP \parallel BA$ [Given]

$\therefore ABCD$ is a parallelogram

12.

Sol:



Given,

A kite $ABCD$ having $AB = AD$ and $BC = CD \cdot P, Q, R, S$ are the midpoint of sides $AB \cdot BC, CD$ and DA respectively PQ, QR, RS and spare joined

To prove:

$PQRS$ is a rectangle

Proof:

In $\Delta ABC, P$ and Q are the midpoints of AB and BC respectively

$\therefore PQ \parallel AC$ and $PQ = \frac{1}{2}AC$ ....(i)

In $\Delta ADC, R$ and S are the midpoint of CD and AD respectively

$\therefore RS \parallel AC$ and $RS = \frac{1}{2}AC$ .....(ii)

From (i) and (ii), we have

$PQ \parallel RS$ and $PQ = RS$

Thus, in quadrilateral PQRS, a pair of opposite sides are equal and parallel. So, PQRS is a Parallelogram. Now, we shall prove that one angle of parallelogram PQRS it is a right angle

Since $AB = AD$

$\Rightarrow \frac{1}{2}AB = AD\left(\frac{1}{2}\right)$ [$\because P$ and S are the midpoints of B and AD respectively]

$\Rightarrow AP = AS$ ...(iii)

$\Rightarrow \angle 1 = \angle 2$ ....(iv)

Now, in ΔPBQ and ΔSDR, we have

$PB = SD$ [$\because AD = AB \Rightarrow \frac{1}{2}AD = \frac{1}{2}AB$]

$BQ = DR$ $\therefore PB = SD$

And $PQ = SR$ [$\because PQRS$ is a parallelogram]

So by SSS criterion of congruence, we have

$\Delta PBQ \cong \Delta SOR$

$\Rightarrow \angle 3 = \angle 4$ [CPCT]

Now, $\angle 3 + \angle SPQ + \angle 2 = 180°$

And $\angle 1 + \angle PSR + \angle 4 = 180°$

$\therefore \angle 3 + \angle SPQ + \angle 2 = \angle 1 + \angle PSR + \angle 4$

$\Rightarrow \angle SPQ = \angle PSR \quad (\angle 1 = \angle 2 \text{ and } \angle 3 = \angle 4)$

Now, transversal PS cuts parallel lines SR and PQ at S and p respectively

$\therefore \angle SPQ + \angle PSR = 180°$

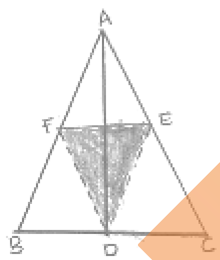
$\Rightarrow 2\angle SPQ = 180° = \angle SPQ = 90° \quad [\because \angle PSR = \angle SPQ]$

Thus, PQRS is a parallelogram such that $\angle SPQ = 90°$

Hence, PQRS is a parallelogram

13.

Sol:



Since D, E and F are the midpoints of sides BC, CA and AB respectively

$\therefore AB \parallel DF$ and $AC \parallel FD$

$AB \parallel DF$ and $AC \parallel FD$

$ABDF$ is a parallelogram

$AF = DE$ and $AE = DF$

$\frac{1}{2}AB = DE$ and $\frac{1}{2}AC = DF$

$DE = DF \quad (\because AB = AC)$

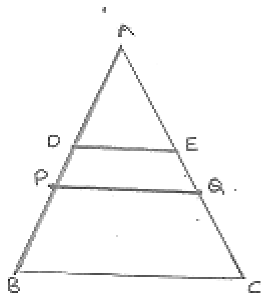
$AE = AF = DE = DF$

$ABDF$ is a rhombus

$\Rightarrow AD$ and FE bisect each other at right angle.

14.

Sol:



Let P and Q be the midpoints of AB and AC respectively

Then $PQ \parallel BC$ such that

$$PQ = \frac{1}{2}BC \qquad ......(i)$$

In ΔAPQ D and E are the midpoint of AP *and* AQ are respectively

$$\therefore DE \parallel PQ \text{ and } DE = \frac{1}{2}PQ \qquad ....(ii)$$

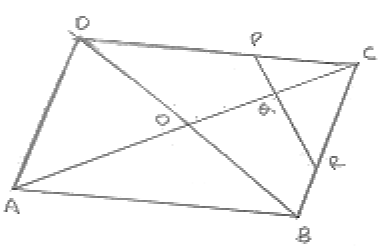
From (1) and (2) $DE = \frac{1}{2}PQ = \frac{1}{2}PQ = \frac{1}{2}\left(\frac{1}{2}BC\right) \qquad ....(iii)$

$$DE = \frac{1}{4}BC$$

Hence proved.

15.

Sol:



Join B and D, suppose AC and BP out at 0

Then $OC = \frac{1}{2}AC$

Now,

$$CQ = \frac{1}{4}AC$$

$$\Rightarrow CQ = \frac{1}{2}\left[\frac{1}{2}AC\right]$$

$$= \frac{1}{2} \times OC$$

In $\Delta DCO, P$ *and* Q are midpoints of DC and OC respectively

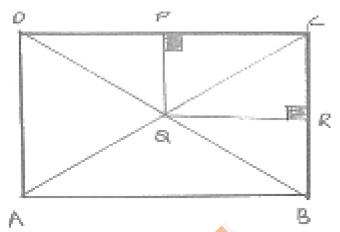
$\therefore PQ \parallel PO$

Also in $\Delta COB, Q$ is the midpoint of OC and $QR \parallel OB$

$\therefore R$ is the midpoint of BC

16.

Sol:



(i) In $\Delta ADC, Q$ is the midpoint of AC such that

$PQ \parallel AD$

$\therefore P$ is the midpoint of DC

$\Rightarrow DP = DC$ [Using converse of midpoint theorem]

(ii) Similarly, R is the midpoint of BC

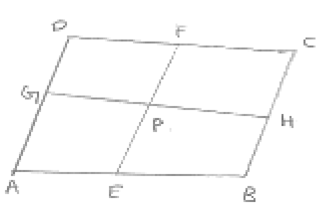
$\therefore PR = \frac{1}{2}BD$

[Diagonals rectangle are equal $\therefore BD = AC$]

$PR = \frac{1}{2}AC$

17.

Sol:



Since E and F are midpoints of AB and CD respectively

$\therefore AE = BE = \frac{1}{2}AB$

And $CF = DF = \frac{1}{2}CD$

But, $AB = CD$

$\therefore \frac{1}{2}AB = \frac{1}{2}CD$

$\Rightarrow BE = CF$

Also, $BE \parallel CF$ $\quad [\because AB \parallel CD]$

$\therefore BEFC$ is a parallelogram

$\Rightarrow BC \parallel EF$ and $BF = PH$ $\quad ....(i)$

Now, $BC \parallel EF$

$\Rightarrow AD \parallel EF$ $\quad$ [$\because BC \parallel AD$ as $ABCD$ is a parallel]

$\Rightarrow AEFD$ is parallelogram

$\Rightarrow AE = GP$

But is the midpoint of AB

$\therefore AE = BE$

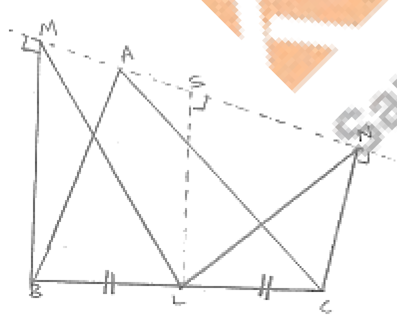
$\Rightarrow GP = PH$

18.

Sol:

To prove $LM = LN$

Drawls perpendicular to line MN



$\therefore$ The lines BM, LS and CN being the same perpendiculars, on line MN are parallel to each other

According to intercept theorem,

If there are three or more parallel lines and the intercepts made by them on a transversal or equal. Then the corresponding intercepts on any other transversal are also equal

In the drawn figure, MB and LS and NC are three parallel lines and the two transversal line are MN and BC

We have, $BL = LC$ (As L is the given midpoint of BC)

$\therefore$ using intercept theorem, we get

$MS = SN \quad ....(i)$

Now in ΔMLS and LSN

$MS = SN$ using $\quad ....(1)$

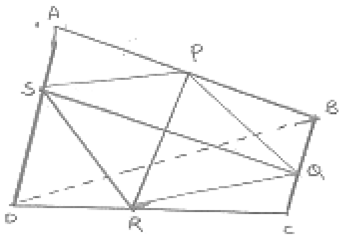
$\angle LSM = \angle LSN = 90° LS \perp MN$ and $SL = LS$ common

$\therefore \Delta MLS \cong \Delta LNS$ $CSAS$ congruency theorem

$\therefore LM = LN \quad (CPCT)$

19.

Sol:



Let ABCD is a quadrilateral in which P, Q, R and S are midpoints of sides AB, BC, CD and DA respectively join PQ, QR, RS, SP and BD

In ΔABD S and P are the midpoints of AD and AB respectively

So, by using midpoint theorem we can say that

$$SP \parallel BD \text{ and } SP = \frac{1}{2}BD \quad ......(1)$$

Similarly in ΔBCD

$$QR \parallel BD \text{ and } QR = \frac{1}{2}BD \quad .....(2)$$

From equation (1) and (2) we have

$SP \parallel QR$ and $SP = QR$

As in quadrilateral SPQR one pair of opposites are equal and parallel to each other

So, SPQR is parallelogram

Since, diagonals of a parallelogram bisect each other

Hence PR and QS bisect each other.

20.

Sol:

(i) Isosceles

(ii) Right triangle

(iii) parallelogram