# Heron's Formula – 12.1

**1.**

**Sol:**

The triangle whose sides are

a = 150 cm

b = 120 cm

c = 200 cm

The area of a triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

Here 1s = semi perimeter of triangle

2s = a + b + c

S = $\frac{a+b+c}{2} = \frac{150+200+120}{2} = 2.35cm$

∴ area of triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

= $\sqrt{235(235-150)(235-200)(235-120)}$

= $\sqrt{235(85)(35)(115)}cm^2$

= $8966.56\ cm^2$

**2.**

**Sol:**

The triangle whose sides are a = 9cm, b = 12 cm and c = 15 cm

The area of a triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

Here 1s = semi-perimeter of a triangle

2s = a + b + c

S = $\frac{a+b+c}{2} = \frac{9+12+15}{2} = \frac{36}{2} = 18\ cm$

∴ area of a triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

= $\sqrt{18(18-9)(18-12)(18-15)} = \sqrt{18(9)(6)(3)}$

= $\sqrt{18\ cm\ \times 3\ cm\ \times 54cm^2}$ = 54 $cm^2$.

**3.**

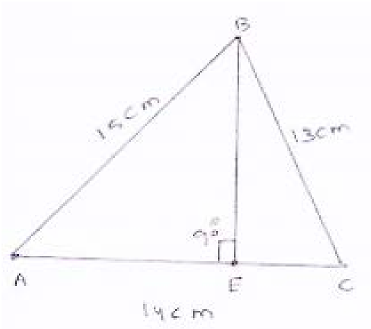
**Sol:**

$21\sqrt{11}cm^2$

**4.**

**Sol:**

The triangle sides are




Let a = AB = 15 cm, BC = 13cm = b C = AC = 14 cm say.
Now,
2s = a + b + c
⇒ S = $\frac{1}{2}(a+b+c)$
⇒ s = $\left(\frac{15+13+14}{2}\right) cm$
⇒ s = 21 cm
∴ area of a triangle = $\sqrt{s(s-a)(s-b)(s-c)}$
= $\sqrt{21(21-15)(21-14)(21-13)}\ cm^2$
= $\sqrt{21 \times 6 \times 8 \times 7}$ $cm^2$
= 84 $cm^2$
Let BE be perpendicular ($\perp^{er}$) to AC
Now, area of triangle = 84 $cm^2$
⇒ $\frac{1}{2} \times BE \times AC = 84$
⇒ BE = $\frac{84 \times 2}{AC}$
⇒ BE = $\frac{168}{14} = 12cm$
∴ Length of altitude on AC is 12 cm.

**5.**

**Sol:**
The sides of a triangle are in the ratio 25 : 17 : 12
Let the sides of a triangle are a = 25x, b = 17 x and c = 12x say.
Perimeter = 25 = a + b + c = 540 cm
⇒ 25x + 17x + 12x = 540 cm
⇒ 54x = 540cm
⇒ $x = \frac{540}{54}$
⇒ $x$ = 10 $cm$
∴ The sides of a triangle are a = 250 cm, b = 170 cm and c = 120 cm
Now, Semi perimeter s = $\frac{a+b+c}{2}$

$= \frac{540}{2} = 270\ cm$

∴ The area of the triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

= $\sqrt{270(270-250)(270-170)(270-120)}$

= $\sqrt{270(20)(100)(150)}$

= $\sqrt{(9000)(9000)}$

= $9000\ cm^2$

∴ The area of triangle = 900 $cm^2$.

**6.**

**Sol:**

Given that

The perimeter of a triangle = 300 m

The sides of a triangle in the ratio 3 : 5 : 7

Let 3x, 5x, 7x be the sides of the triangle

Perimeter ⇒ 2s = a + b + c

$\Rightarrow 3x + 5x + 17x = 300$

$\Rightarrow 15x = 300$

$\Rightarrow x = 20m$

The triangle sides are $a = 3x$

$= 3\ (20)m = 60\ m$

$b = 5x = (20)\ m = 100m$

$c = 7x = 140\ m$

Semi perimeter $s = \frac{a+b+c}{2}$

$= \frac{300}{2} m$

$= 150m$

∴ The area of the triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{150(150-60)(150-100)(150-140)}$

$= \sqrt{150 \times 10 \times 90 \times 50}$

$= \sqrt{1500 \times 1500 \times 3}\ cm^2$

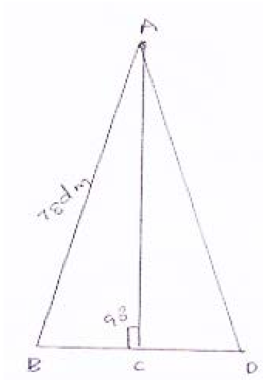
∴ $\Delta le$ Area $= 1500\sqrt{3}\ cm^2$

**7.**

**Sol:**

ABC be the triangle, Here a = 78 dm = AB,

BC = b = 50 dm




Now, perimeter = 240 dm

⇒ AB + BC + CA = 240 dm

⇒ AC = 240 – BC – AB

⇒ AC = 112 dm

Now, 2s = AB + BC + CA

⇒ 2s = 240

⇒ s = 120 dm

∴ Area of ΔABC = $\sqrt{s(s-a)(s-b)(s-c)}$ by heron's formula

= $\sqrt{120(120-78)(120-50)(120-112)}$

= $\sqrt{120 \times 42 \times 70 \times 8}$

$= 1680\ dm^2$

$Let\ AD\ be\ perpendicualar\ on\ BC$

Area of ΔABC = $\frac{1}{2} \times AD\ \times BC$ (area of triangle = $\frac{1}{2} \times b \times h$)

= $\frac{1}{2} \times AD \times BC = 1680$

⇒ AD = $\frac{3\times1680}{150} = 67.2\ dm$

**8.**

**Sol:**

The sides of a triangle are a = 35 cm, b = 54 cm and c = 61 cm

Now, perimeter a + b + c = 25

⇒ S = $\frac{1}{2}(35 + 54 + 61)$

⇒ s = 75 cm

By using heron's formula

∴Area of triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

= $\sqrt{75(75-35)(75-54)(75-61)}$

= $\sqrt{75(40)(21)(14)}$ = 939.14cm$^2$

∴ The altitude will be a smallest when the side corresponding to it is longest Here, longest side is 61 cm

[∴ Area of Δle = $\frac{1}{2} \times b \times h] = \frac{1}{2} \times base \times height$

∴ $\frac{1}{2} \times h \times 61 = 939.14$

⇒ $h = \frac{939.14 \times 2}{61} = 30.79\ cm$

**9.**

**Sol:**

Let the sides of a triangle are 3x, 4x and 5x.

Now, a = 3x, b = 4x and c = 5x

The perimeter 2s = 144

⇒ 3x + 4x + 5x = 144 [∵ a + b + c = 2s]

⇒ 12x = 144

⇒ x = 12

∴ sides of triangle are a = 3(x) = 36cm

b = 4(x) = 48 cm

c = 5(x) = 60 cm

Now semi perimeter s = $\frac{1}{2}(a + b + c) = \frac{1}{2}(144) = 12cm$

By heron's formulas ∴ Area of Δle = $\sqrt{s(s-a)(s-b)(s-c)}$

= $\sqrt{72(72-36)(72-48)(72-60)}$

= $864cm^2$

Let $l$ be the altitude corresponding to longest side, ∴ $\frac{1}{2} \times 60 \times l = 864$

⇒ $l = \frac{864 \times 2}{60}$

⇒ $l = 28.8cm$

Hence the altitude one corresponding long side = 28.8 cm

**10.**

**Sol:**

Let 'x' be the measure of each equal sides

∴ Base = $\frac{3}{2}x$

∴ x + x + $\frac{3}{2}x = 42$ [∵ Perimeter = a + b + c = 42 cm]

⇒ $\frac{7}{2}x = 42$

⇒ x = 12 cm

∴ Sides are a = x = 12 cm

B = x = 12 cm

C = x = $\frac{3}{2}$(12) cm = 18 cm

By heron's formulae

∴ Area of triangle = $\sqrt{s(s-a)(s-b)(s-c)}cm^2$

$= \sqrt{21(9)(9)(21-18)}\ cm^2$

$= \sqrt{(21)(9)(3)}cm^2$

$= 71.42cm^2$

∴ Area of triangle = 71.42 $cm^2$

**11.**

**Sol:**

Area of shaded region = Area of ΔABC – Area of ΔADB

Now in ΔADB

⇒ $AB^2 = AD^2 + BD^2$ --(i)

⇒ Given that AD = 12 cm BD = 16 cm

Substituting the values of AD and BD in the equation (i), we get

$AB^2 = 12^2 + 16^2$

$AB^2 = 144 + 256$

AB = $\sqrt{400}$

AB = 20 cm

∴ Area of triangle = $\frac{1}{2} \times AD \times BD$

$= \frac{1}{2} \times 12 \times 16$

$= 96\ cm^2$

Now

In ΔABC, $S = \frac{1}{2}(AB + BC + CA)$

$= \frac{1}{2} \times (52 + 48 + 20)$

$= \frac{1}{2}(120)$

= 60 cm

By using heron's formula

We know that, Area of Δle ABC = $\sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{60(60-20)(60-48)(60-52)}$

$= \sqrt{60(40)(12)(8)}$

= 480 $cm^2$

= Area of shaded region = Area of ΔABC – Area of ΔADB

$= (480 - 96)cm^2$

$= 384\ cm^2$

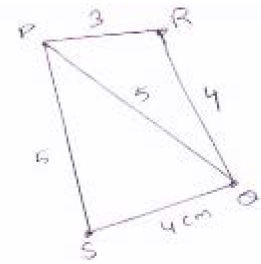
∴ Area of shaded region = 384 $cm^2$

## Heron's Formula – 12.2

**1.**

**Sol:**

For ΔPQR




$PQ^2 = QR^2 + RP^2$

$(5^2) = (3)^2 + (4)^2$ [$\because PR = 3\ QR = 4\ and\ PQ = 5$]

So, ΔPQR is a right angled triangle. Right angle at point R.

Area of ΔABC = $\frac{1}{2} \times QR \times RP$

= $\frac{1}{2} \times 3 \times 4$

= $6cm^2$

For ΔQPS

Perimeter = 2s = AC + CD + DA = (5 + 4 + 5)cm = 14 cm

S = 7 cm

By heron's formulae

Area of Δle $\sqrt{s(s-a)(s-b)(s-c)}\ cm^2$

Area of Δle PQS = $\sqrt{7(7-5)(7-4)(7-3)}cm^2$

= $2\sqrt{21}cm^2$

= $(2 \times 4.583)cm^2$

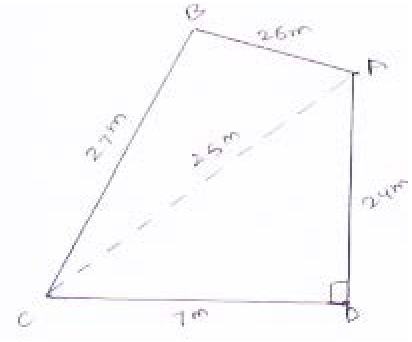
= $9.166\ cm^2$

Area of PQRS = Area of PQR + Area of ΔPQS = $(6 + 9.166)cm^2 = 15.166cm^2$

**2.**

**Sol:**

The sides of a quadrilateral field taken order as AB = 26m




BC = 27 m

CD = 7m and DA = 24 m

Diagonal AC is joined

Now ΔADC

By applying Pythagoras theorem

$\Rightarrow AC^2 = AD^2 + CD^2$

$\Rightarrow$ AC = $\sqrt{AD^2 + CD^2}$

$\Rightarrow$ AC = $\sqrt{24^2 + 7^2}$

$\Rightarrow$ AC = $\sqrt{625} = 25\ m$

Now area of ΔABC

S = $\frac{1}{2}(AB + BC + CA) = \frac{1}{2}(26 + 27 + 25)$

By using heron's formula

Area (ΔABC) = $\sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{39(39-26)(39-21)(39-25)}$

= $\sqrt{39 \times 14 \times 13 \times 12 \times 1}$

= 291.849 cm$^2$

Now for area of ΔADC

S = $\frac{1}{2}(AD + CD + AC)$

= $\frac{1}{2}(25 + 24 + 7) = 28m$

By using heron's formula

∴ Area of ΔADC = $\sqrt{s(s-a)(s-b)(s-c)}$

= $\sqrt{28(28-24)(28-7)(28-25)}$

= 84m$^2$

∴ Area of rectangular field ABCD = area of ΔABC + area of ΔADC

= 291.849 + 84

= 375.8m$^2$

**3.**

**Sol:**

Given that sides of equilateral are AB = 5 m, BC = 12 m, CD = 14 m and DA = 15 m

AB = 5m, BC = 12m, CD = 14 m and DA = 15 m

Join AC

Area of ΔABC = $\frac{1}{2} \times AB \times BC$ $\left[\because Area\ of\ \Delta le = \frac{1}{2}(3x+1)\right]$

= $\frac{1}{2} \times 5 \times 12$

= 3 $cm^2$

In ΔABC By applying Pythagoras theorem.

$AC^2 = AB^2 + BC^2$

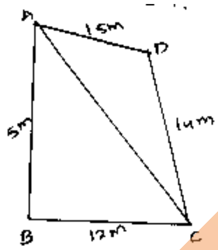
$\Rightarrow AC = \sqrt{5^2 + 12^2}$

$= \sqrt{25 + 44}$

$= \sqrt{169}$ = 13 m

*Now in ΔADC*

Let 2s be the perimeter




∴ 2s = (AD + DC + AC)

⇒ S = $\frac{1}{2}(15 + 14 + 13) = \frac{1}{2} \times 42 = 21m$

By using Heron's formula

∴ Area of ΔADC = $\sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{21(21-15)(21-14)(21-13)}$

$= \sqrt{21 \times 6 \times 6 \times 7 \times 8}$

$= 84m^2$

∴ $Area\ of\ quadrilateral\ ABCD = area\ of\ (\Delta ABC) + Area\ of\ (\Delta ADC)$ = 30 + 84 = 114 $m^2$

**4.**

**Sol:**

Given sides of a quadrilaterals are AB = 9, BC = 12, CD = 05, DA = 08

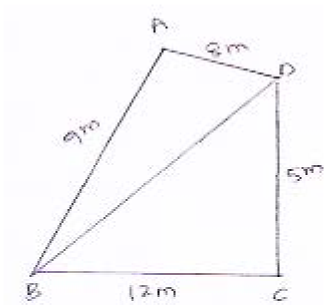
Let us joint BD

In ΔBCD applying Pythagoras theorem.

$BD^2 = BC^2 + CD^2$

$= (12)^2 + (5)^2$

$= 144 + 25$
$= 169$
$BD = 13m$




Area of ΔBCD = $\frac{1}{2} \times BC \times CD = \left[\frac{1}{2} \times 12 \times 5\right] m^2 = 30\ m^2$

For ΔABD

S = $\frac{perimeter}{2} = \frac{(9+8+13)}{2} = 15cm$

By heron's formula = $\sqrt{s(s-a)(s-b)(s-c)}$

Area of the triangle = $\sqrt{15(15-9)(15-8)(15-13)}m^2$

$= \sqrt{15(6)(7)(2)}m^2 = 6\sqrt{35}\ m^2 = 35.496m^2$

Area of park = Area of ΔABD + ΔABD + Area of BCD

= 35.496 + 30 $m^2$

= 65.5 $m^2$ (approximately)

**5.**

**Sol:**

Given that two parallel sides of trapezium are AB = 77 and CD = 60 cm

Other sides are BC = 26 m and AD = 25 cm.

Join AE and CF

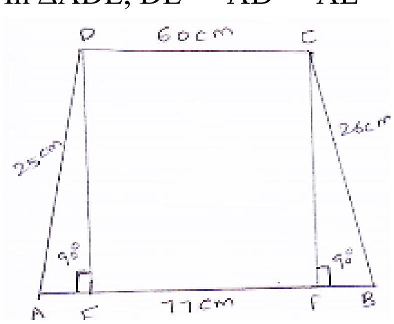
Now, DE ⊥ AB and CF ⊥ AB

∴ DC = EF = 60 cm

Let AE = x

⇒ BF = 77 – 60 – x = 17 – x

In ΔADE, $DE^2 = AD^2 - AE^2 = 25^2 - x^2$ [∵ Pythagoras theorem]




And in ΔBCF, $CF^2 = BC^2 - BF^2$ [∴ By Pythagoras theorem]

$\Rightarrow$ CF = $\sqrt{26^2 - (17 - x)^2}$

But DE = CF $\Rightarrow$ $DE^2 = CF^2$

$\Rightarrow 25^2 - x^2 = 26^2 - (17 - x)^2$

$\Rightarrow 25^2x^2 = 25^2 - (289 + x^2 - 34x)$ $\quad [\because (a - b)^2 = a^2 - 2ab + b^2]$

$\Rightarrow 625 - x^2 = 676 - 289 - x^2 + 34x$

$\Rightarrow$ 34x = 238

$\Rightarrow$ x = 7

$\therefore$ DE = $\sqrt{25^2 - x^2} = \sqrt{625 - 7^2} = \sqrt{516} = 24cm$

$\therefore$ Area of trapezium = $\frac{1}{2}(sum\ of\ parallel\ sides) \times height = \frac{1}{2}(60 \times 77) \times 24 =$ $1644cm^2$

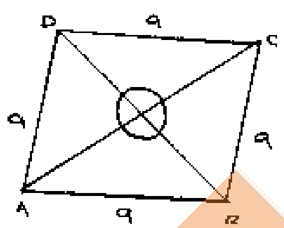
**6.**

**Sol:**

Given that,

Perimeter of rhombus = 80m

Perimeter of rhombus = 7 × side




$\Rightarrow$ 4a = 80

$\Rightarrow$ a = 20m

Let AC = 24 m

$\therefore$ OA = $\frac{1}{2}AC = \frac{1}{2} \times 24 = 12m$

In ΔAOB

$OB^2 = AB^2 - OA^2$ $\quad$ [By using Pythagoras theorem]

$\Rightarrow OB = \sqrt{20^2 - 12^2}$

$= \sqrt{400 - 144}$

$= \sqrt{256} = 16\ m$

Also BO = OD $\quad$ [Diagonal of rhombus bisect each other at 90°]

$\therefore$ BD = 20B = 2 ×16 = 32 m

$\therefore$Area of rhombus = $\frac{1}{2} \times 32 \times 24 = 384m^2$ $\quad [\because$Area of rhombus = $\frac{1}{2} \times BD \times AC]$

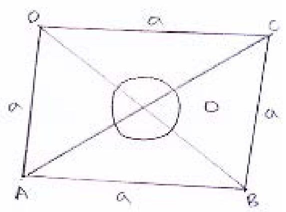
**7.**

**Sol:**

Given that,

Perimeter of a rhombus = 32 m

We know that,
Perimeter of rhombus = 4 × side
⇒ 49 = 32m
⇒ a = 8 m
Let AC = 10 = OA = $\frac{1}{2}AC$
= $\frac{1}{2} \times 10$
= $5m$




By using Pythagoras theorem:
$\therefore OB^2 = AB^2 - OA^2$
⇒ OB = $\sqrt{AB^2 - OA^2}$
⇒ OB = $\sqrt{8^2 - 5^2}$
⇒ OB = $\sqrt{64 - 25}$
⇒ OB = $2\sqrt{39}\ m$
Now, BD = 2OB = $2\sqrt{39}m$
∴ Area of sheet = $\frac{1}{2} \times BD \times AC = \frac{1}{2} \times 2\sqrt{39} \times 10 = 10\sqrt{39}m^2$
∴ Cost of printing on both sides at the rate of Rs 5 per $m^2$ = Rs $2 \times 10\sqrt{39} \times 5$ = Rs. 625.00

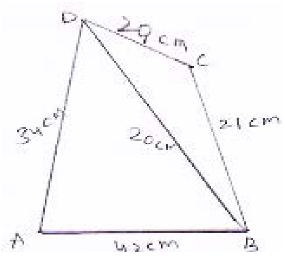
**8.**

**Sol:**
Given that, a quadrilateral ABCD in which AD = 24 cm, ∠B = AD = 90°
BCD is equilateral triangle and sides BC = CD = BD = 26 cm
In ΔBAD By using Pythagoras theorem
$BA^2 = BD^2 - AD^2$
$\Rightarrow BA = \sqrt{BD^2 - AD^2}$
$= \sqrt{676 - 576}$
$= \sqrt{100} = 10\ cm$
$Area\ of\ \Delta BAD = \frac{1}{2} \times BA \times AD$
$= \frac{1}{2} \times 10 \times 24$
$= 120cm^2$
$Area\ of\ \Delta BCD = \frac{\sqrt{3}}{4} \times (26)^2 = 292.37cm^2$
$\therefore Area\ of\ quadrilateral$

ABCD = Area of ΔBAD + area of ΔBCD
= 120 + 292.37
= 412.37 $cm^2$

**9.**

**Sol:**
Given that
Sides of a quadrilateral are AB = 42 cm, BC = 21 cm, CD = 29 cm
DA = 34 cm and diagonal = BD = 20 cm
Area of quadrilateral = area of ΔADB + area of ΔBCD.
Now, area of ΔABD
Perimeter of ΔABD
We know that




2s = AB + BD + DA
$\Rightarrow S = \frac{1}{2}(AB + BD + DA)$
$= \frac{1}{2}(34 + 42 - 120) = 38cm = 48\ cm$
Area of ΔABD = $\sqrt{S(s-a)(s-b)(s-c)}$
$= \sqrt{48(48-42)(48-20)(48-34)}$
$= \sqrt{48(14)(6)(28)}$
$= 336\ cm^2$
Also for area of ΔBCD,
Perimeter of ΔBCD
2s = BC + CD + BD
$\Rightarrow S = \frac{1}{2}(29 + 21 + 20) = 35\ cm$
By using heron's formulae
Area of ΔBCD $= \sqrt{s(s-bc)(s-cd)(s-db)}$
$= \sqrt{35(35-21)(35-29)(35-20)}$
$= \sqrt{210 \times 210} cm^2$
$= 210 cm^2$
∴ Area of quadrilateral ABCD = 336 + 210 = 546 $cm^2$

**10.**

**Sol:**

The sides of a quadrilateral ABCD in which AB = 17 cm, AD = 9 cm, CD = 12 cm, ∠ACB = 90° and AC = 15 cm

Here, By using Pythagoras theorem

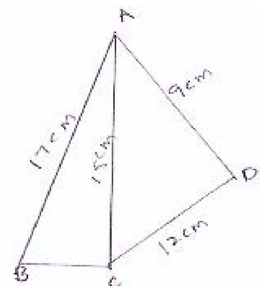
BC = $\sqrt{17^2 - 15^2} = \sqrt{289 - 225} = \sqrt{64} = 8cm$

Now, area of ΔABC = $\frac{1}{2} \times 8 \times 15 = 60\ cm^2$

For area of Δle ACD,

Let a = 15 cm, b = 12 cm and c = 9 cm

Therefore, S = $\frac{15+12+9}{2} = \frac{36}{2} = 18\ cm$




Area of ACD = $\sqrt{s(s-a)(s-b)(s-c)}$

= $\sqrt{18(18-15)(18-12)(18-9)}$

= $\sqrt{18 \times 18 \times 3 \times 3}$

= $(18 \times 3)^2$

∴ Thus, the area of quadrilateral ABCD = 60 + 54 = 114 cm$^2$

**11.**

**Sol:**

Given that adjacent sides of a parallelogram ABCD measure 34 cm and 20 cm, and the diagonal AC measures 42 cm.

Area of parallelogram = Area of ΔADC + area of ΔABC

[∵ Diagonal of a parallelogram divides into two congruent triangles]

= 2 × $[Area\ of\ \Delta ABC]$

Now for Area of ΔABC

Let 2s = AB + BC + CA [∴ Perimeter of ΔABC]

⇒ S = $\frac{1}{2}(AB + BC + CA)$

⇒ S = $\frac{1}{2}(34 + 20 + 42)$

= $\frac{1}{2}(96) = 48\ cm$

∴ Area of ΔABC = $\sqrt{s(s-ab)(s-bc)(s-ca)}$ $[heron's formula]$

= $\sqrt{48(48-34)(48-20)(48-42)}$

$= \sqrt{48(14)(28)(6)} = 336cm^2$

$\therefore Area\ of\ parallelogram\ ABCD = 2[Area\ of\ \Delta ABC] = 2 \times 336 = 672\ cm^2$

**12.**

**Sol:**

Area of the blades of magnetic compass = Area of ΔADB + Area of ΔCDB

Now, for area of ΔADB

Let, 2s = AD + DB + BA (Perimeter of ΔADB)

Semi perimeter (S) = $\frac{1}{2}(5 + 1 + 5) = \frac{11}{2} cm$

By using heron's formulae

Now, area of ΔADB = $\sqrt{s(s - ad)(s - bd)(s - ba)}$

= $\sqrt{\frac{11}{2}\left(\frac{11}{2} - 5\right)\left(\frac{11}{2} - 1\right)\left(\frac{11}{2} - 5\right)}$

= 2.49 $cm^2$

= Also, area of triangle ADB = $Area\ of\ \Delta le\ CDB$

∴ Area of the blades of magnetic compass

= 2 × $(area\ of\ \Delta ADB)$

= 2 × 2.49

= 4.98 $m^2$

**13.**

**Sol:**

Given that the sides of ΔAOB are

AD = 24 cm

OB = 25 cm

BA = 14 cm

Area of each equal strips = Area of Δle AOB

Now, for area of ΔAOB

Perimeter of ΔAOB

Let 2s = AD + OB + BA

$\Rightarrow s = \frac{1}{2}(AD + OB + BA)$

$= \frac{1}{2}(25 + 25 + 14) = 32\ cm$

$\therefore By\ using\ Heron's formulae$

$Area\ of\ (\Delta AOB) = \sqrt{s(s - ao)(s - ob)(s - ba)}$

= $\sqrt{32(32 - 25)(32 - 25)(32 - 14)}$

= $\sqrt{32(7)(4)(18)}$

$= 168\ cm^2$

∴ Area of each type of paper needed to make the hand fan = $5 \times (area\ of\ \Delta AOB)$

$= 5 \times 168$

$= 840\ cm^2$

**14.**

**Sol:**

The sides of a triangle DCE are

DC = 15 cm, CE = 13 cm, ED = 14 cm

Let h be the height of parallelogram ABCD

Given,

Perimeter of ΔDCE

2s = DC + CE + ED

$\Rightarrow S = \frac{1}{2}(15 + 13 + 4)$

$\Rightarrow S = \frac{1}{2}(42)$

$\Rightarrow S = 21\ cm$

$Area\ of\ \Delta DCE = \sqrt{s(s-dc)(s-ce)(s-ed)}$ [By heron's formula]

$= \sqrt{21(21-15)(21-13)(21-14)}$

$= \sqrt{21 \times 7 \times 8 \times 6}$

$= \sqrt{84 \times 84}$

$= 84cm^2$

$\Rightarrow 24 \times h = 84$ [∴ Area of parallelogram = base × height]

$\Rightarrow$ h = 6 cm