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Question 1.

Solution:

$LHS = A' - B' = \left(A' \cap B'\right)'$

$\because C - D = \left(C \cap D\right)' = A' \cap B = \left(B \cap A\right)' = B - A$

$\because \left(C \cap D\right)' = C - D$

$RHS = B - A$

$So, LHS = RHS$

Question 2.

Solution:

(i)

$LHS = A \cap \left(A' \cup B\right)$

$= \left(A \cap A'\right) \cup \left(A \cap B\right)$

$= \phi \cup \left(A \cap B\right)$

$= A \cap B = RHS$

Hence proved.

(ii)

$LHS = A - \left(A - B\right)$

$= A - \left(A \cap B'\right)$

$= A \cap \left(A \cap B'\right)'$

$= A \cap \left\{A' \cup \left(B'\right)'\right\}$

$= A \cap \left(A' \cup B\right)$

$= \left(A \cap A'\right) \cup \left(A \cup B\right)$

$= \varnothing \cup \left(A \cup B\right)$

$= \left(A \cup B\right)$

$= RHS$

Hence proved

(iii)

$LHS = A \cap (A \cup B)'$

$= A \cap (A' \cap B')$

$= (A \cap A') \cap (A \cap B')$

$= \phi \cap (A \cap B')$

$= \phi = RHS$

Hence proved.

(iv)

$LHS = A \Delta (A \cap B)$

$= \{A - (A \cap B)'\} \cup \{(A \cup B) - A\}$

$= \{A \cap (A \cap B)'\} \cup \{(A \cup B) \cap A'\}$

$= \{A \cap (A' \cap B')\} \cup \{(A \cup B) \cap A'\}$

$= \{(A \cap A') \cup (A \cap B')\} \cup \{(A \cap A') \cap (B \cap A')\}$

$= (A \cap B') \cup (\phi)$

$= (A \cap B')$

$= A - B$

$= RHS$

Question 3.

Solution:

Let $a \in C - B$

$\Rightarrow a \in C$ and $a \notin B$

$\Rightarrow a \in C$ and $a \notin A \because A \subset B$

$\Rightarrow a \in C - A$

Hence, $C - B \subset C - A$

Question 4.

Solution:

(i)

$$(A\cap B)-B=(A\cup B)\cap B' \qquad (X-Y=X\cap Y')$$

$$=(A\cap B')\cap(B\cap B')$$

$$=(A\cap B')\cup\varphi$$

$$=A\cap B'$$

$$=A-B$$

(ii)

$$A-(A\cap B)=A\cap(A\cap B)' \qquad (X-Y=X\cap Y')$$

$$=A\cap(A'\cup B')$$

$$=(A\cap A')\cup(A\cap B')$$

$$=\varphi\cup(A\cap B')$$

$$=A\cap B'$$

$$=A-B$$

(iii)

$$A\cup(B-A)=A\cup(B\cap A') \qquad (X-Y=X\cap Y')$$

$$=(A\cup B)\cap(A\cup A') \qquad (\textit{Distributive law})$$

$$=(A\cup B)\cap U \qquad (\cup \textit{ is the universal set})$$

(iv)

$$(A-B)\cup(A\cap B)=(A\cap B')\cup(A\cap B) \qquad (X-Y=X\cap Y')$$

. $$=A\cap(B'\cup B) \qquad (\textit{Distributive law})$$

$$=A\cap U$$

$$=A$$