Exercise: 1.6

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Question 1.

Solution :
We have to find the smallest set A such that $A \cup \{1,2\} = \{1,2,3,5,9\}$. The union of the two sets $A \& B$ is the set of all those elements that belong to A or to B or to both $A \& B$.
Thus, A must be $\{3,5,9\}$.

Question 2.
$(i)\ A \cup B \cap C = A \cup B \cap A \cup C$
$(ii)\ A \cap B \cup C = A \cap B \cup A \cap C$
(iii) $A \cap B - C = A \cap B - A \cap C$
$(iv)\ A - B \cup C = A - B \cap A - C$
$(v)\ A - B \cap C = A - B \cup A - C$
$(vi)\ A \cap B \Delta C = A \cap B \Delta A \cap C$

Solution :
Given:
$A = \{1,2,4,5\}, B = \{2,3,5,6\}$ *and* $C = \{4,5,6,7\}$ We have to verify the following identities:
$(i)\ A \cup B \cap C = A \cup B \cap A \cup C$
LHS
$(B \cap C) = \{5,6\}\ A \cup (B \cap C) = \{1,2,4,5,6\}$
RHS
$A \cup B = \{1,2,3,4,5,6\}\ A \cup C = \{1,2,4,5,6,7\}\ A \cup B \cap A \cup C = \{1,2,4,5,6\}$
LHS = RHS
$\therefore A \cup B \cap C = A \cup B \cap A \cup C$
$(ii)\ A \cap B \cup C = A \cap B \cup A \cap C$
LHS
$(B \cup C) = \{2,3,4,5,6,7\}\ A \cap (B \cup C) = \{2,4,5\}$
RHS
$A \cap B = \{2,5\}\ A \cap C = \{4,5\}\ A \cap B \cup A \cap C = \{2,4,5\}$
LHS = RHS
$\therefore A \cap B \cup C = A \cap B \cup A \cap C$
(iii) $A \cap B - C = A \cap B - A \cap C$
LHS
$(B - C) = \{2,3\}\ A \cap (B - C) = \{2\}$
RHS
$(A \cap B) = \{2,5\} (A \cap C) = \{4,5\} (A \cap B) - (A \cap C) = \{2\}$

LHS = RHS

$\therefore A \cap B - C = A \cap B - A \cap C$

$(iv)\ A - B \cup C = A - B \cap A - C$

LHS

$(B \cup C) = \{2,3,4,5,6,7\}\ A - (B \cup C) = \{1\}$

RHS

$(A - B) = \{1,4\}(A - C) = \{1,2\}(A - B) \cap (A - C) = \{1\}$

LHS = RHS

$\therefore A - B \cup C = A - B \cap A - C$

$(v)\ A - B \cap C = A - B \cup A - C$

LHS

$(B \cap C) = \{5,6\}\ A - (B \cap C) = \{1,2,4\}$

RHS

$(A - B) = \{1,4\}(A - C) = \{1,2\}(A - B) \cup (A - C) = \{1,2,4\}$

LHS = RHS

$\therefore A - B \cap C = A - B \cup A - C$

$(vi)\ A \cap B \Delta C = A \cap B \Delta A \cap C$

LHS

$(B \Delta C) = (B - C) \cup (C - B)(B - C) = \{2,3\}(C - B) = \{4,7\}(B - C) \cup (C - B) = \{2,3,4,7\}$

$\Rightarrow (B \Delta C) = \{2,3,4,7\}\ A \cap (B \Delta C) = \{2,4\}$

RHS

$(A \cap B) = \{2,5\}(A \cap C) = \{4,5\}(A \cap B) \Delta (A \cap C) = \{(A \cap B) - (A \cap C)\} \cup \{(A \cap C) - (A \cap B)\}(A \cap B)$

$-(A \cap C) = \{2\}(A \cap C) - (A \cap B) = \{4\}\{(A \cap B) - (A \cap C)\} \cup \{(A \cap C) - (A \cap B)\} = \{2,4\}$

$\Rightarrow (A \cap B) \Delta (A \cap C) = \{2,4\}$

LHS = RHS

$\therefore A \cap B \Delta C = A \cap B \Delta A \cap C$

Question 3.

Solution :

Given:

U = {2, 3, 5, 7, 9}

A = {3, 7}

B = {2, 5, 7, 9}

To prove :

(i) $(A \cup B)' = A' \cap B'$

(ii) A∩B'=A'∪B'

Proof :

(i) LHS:

(A∪B)={2,3,5,7,9}(A∪B)'=ϕ

RHS:

A'={2,5,9}B'={3}A'∩B'=ϕLHS=RHS

∴ A∪B'=A'∩B'

(ii) LHS:

(A∩B)={7}(A∩B)'={2,3,5,9}

RHS:

A'={2,5,9}B'={3}A'∪B'={2,3,5,9}

LHS = RHS

∴ A∩B'=A'∪B'

Question 4.

Solution:

(i) *For all* $x \in B$

$\Rightarrow x \in A \, or \, x \in B$

$\Rightarrow x \in A \cup B$ (Difinition of union of sets)

$\Rightarrow B \subset A \cup B$

(ii) *For all* $x \in A \cap B$

$\Rightarrow x \in A \, and \, x \in B$ (Difinition of union of sets)

$\Rightarrow x \in A$

Question 5.

Solution :

We have that the following statements are equivalent:

(i) A⊂B

(ii) A-B=ϕ

(iii) A∪B=B

(iv) A∩B=A

Proof:

Let A⊂BLet x be an arbitrary element of (A-B).

Now, $x \in (A - B) \Rightarrow x \in A \,\&\, x \notin B$ (Which is contradictory) Also,∵A⊂B⇒A-B⊆ϕ …(1) We know that null sets are the subsets of every set.

∴ϕ⊆ A-B …(2)From (1) & (2), we get,(A-B)=ϕ∴(i)=(ii)

Now, we have,(A-B)=ϕ

That means that there is no element in A that does not belong to B.

Now, A∪B=B∴(ii)=(iii) We haveA∪B=B⇒A⊂B⇒A∩B=A∴(iii)=(iv)We have, A∩B=A

It should be possible if A⊂B.

Now,A⊂B∴ (iv)=(i)We have,(i)=(ii)=(iii)=(iv)

Therefore, we can say that all statements are equivalent.

Question 6.

Solution :

(i) Let $A = \{2, 4, 5, 6\}$, $B = \{6, 7, 8, 9\}$ and $C = \{6, 10, 11, 12, 13\}$

So, A∩B=6 and A∩C=6

Hence, $A \cap B = A \cap C$ but $B \neq C$
(ii) Let $z \in C - B \ldots(1)$
$\Rightarrow z \in C \text{ and } z \notin B \Rightarrow z \in C \text{ and } z \notin A \because A \subset B \Rightarrow z \in C - A \ldots$ (2)From (1) and (2), we get $C - B \subset C - A$

Question 7.

Solution :
(i)

$$LHS = A \cup A \cap B$$
$$\Rightarrow LHS = A \cup A \cap A \cup B$$
$$\Rightarrow LHS = A \cap A \cup B$$
$$\because A \subset A \cup B$$
$$\Rightarrow LHS = A = RHS$$

(ii)

$$LHS = A \cap A \cup B$$
$$\Rightarrow LHS = A \cap A \cup A \cap B$$
$$\Rightarrow LHS = A \cup A \cap B$$
$$\Rightarrow LHS = A = RHS$$

Question 8.

Solution :
Let us consider the following sets,
A = {5, 6,10 }
B = {6,8,9}
C = {9,10,11}
Clearly, A∩B=6B∩C=9, A∩C=10 and A∩B∩C = ϕIt means that, A∩B,B∩C and A∩C are non emptysetsand A∩B∩C = ϕ

Question 9.
Solution :

Let $a \in A \Rightarrow a \notin B \because A \cap B = \phi.$
$\Rightarrow a \in B'$
Thus, $a \in A$ and $a \in B' \Rightarrow A \subseteq B'$

Question 10.

Solution :
(i) A-B and A∩BLet $a \in A - B \Rightarrow a \in A$ and $a \notin B \Rightarrow a \notin A \cap B$
Hence, A-B and A∩B are disjoint sets.

(ii) B-A and A∩BLet $a \in B - A \Rightarrow a \in B$ and $a \notin A \Rightarrow a \notin A \cap B$

Hence, B-A and A∩B are disjoint sets.

(iii) A-B and B-A A-B=x:x∈A and x∉B B-A=x:x∈B and x∉A
Hence, A-B and B-A are disjoint sets.

Question 11.

Solution:

$LHS = A \cup B \cup A \cap B'$
$\Rightarrow LHS = A \cup B \cap A \cup A \cup B \cap B'$
$\Rightarrow LHS = A \cup B \cap A \cup A \cup B \cap B'$
$\Rightarrow LHS = A \cup A \cup B \cap B'$
$\Rightarrow LHS = A \cup A \cap B' \cup B \cap B' \because B \cap B = \phi$
$\Rightarrow LHS = A \cup A \cap B'$
$\Rightarrow LHS = A = RHS$

Question 12.

Solution :

(i)

Let $a \in A$
$\Rightarrow a \in U$
$\Rightarrow a \in A' \cup B \because U = A' \cup B$
$\Rightarrow a \in B \because a \notin A'$
Hence, $A \subset B$.

(ii)

Let $a \in A$
$\Rightarrow a \notin A'$
$\Rightarrow a \notin B' \because B' \subset A'$
$\Rightarrow a \in B$
Hence, $A \subset B$.

Question 13.

Solution :

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False,

Let $X \in P(A) \cup P(B)$

$\Rightarrow X \in P(A) \ or \ X \in P(B)$

$\Rightarrow X \subset A \ or \ X \subset B$

$\Rightarrow X \subset A \cup B$

$\Rightarrow X \in P(A \cap B)$

$\therefore P(A) \cup P(B) \subset P(A \cup B)$

Again, let $X \in P(A \cup B)$ But $X \notin P(A) \ or \ x \notin P(B)$

For example let $A = 2,5$ and $B = 1,3,4$ and take $X = 1,2,3,4$

$So, \ X \notin P(A) \cup P(B)$

Thus, $P(A \cup B)$ is not necessarily a subset of $P(A) \cup P(B)$.

Question 14.

Solution :

(i)

$RHS = (A \cap B) \cup (A - B)$

$\Rightarrow RHS = (A \cap B) \cup (A \cap B)'$

$\Rightarrow RHS = (A \cap B) \cup (A \cap A) \cap (B \cup B)'$

$\Rightarrow RHS = A \cap (A \cup B)' \cap (B \cup B)'$

$\Rightarrow RHS = A \cap (A \cup B)' \cap U$

$\Rightarrow RHS = A \cap (A \cup B)'$

$\Rightarrow RHS = A = LHS$

(ii)

$LHS = A \cup (B - A)$

$\Rightarrow LHS = A \cup (B \cap A)'$

$\Rightarrow LHS = (A \cup B) \cap (A \cup A)'$

$\Rightarrow LHS = (A \cup B) \cap U$

$\Rightarrow LHS = A \cup B = RHS$

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Question 15.

It is given that each set X contains 5 elements and $\bigcup_{r=1}^{20} X_r = S$.

$\therefore n(S) = 20 \times 5 = 100$

But, it is given that each element of S belong to exactly 10 of the X_r's.

Number of distinct elements in $S = \frac{100}{10} = 10......(1)$

It is also given that each set Ycontains 2 elements and

$\bigcup_{r=1}^{n} Y_r =$ S.

$\therefore n(S) = n \times 2 = 2n$

Also, each element of S belong to eactly 4 of Y_r's.

$\frac{2n}{4}$.

Number of distinct elements in $S =$ 4

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