Exercise: 1.4

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Question 1.

Solution:

(i) **False.**
It is not necessary that for any two sets A & B, either A⊆B or B⊆A.It is not satisfactory always.
Let: $A=\{1,2\}$ & $B=\{a,\ b,\ c\}$
Here, neither A⊆B nor B⊆A.

(ii) **False**.

$A=\{-1,0,1,2,3\}$ is a finite set that is a subset of infinite set Z.

(iii) **True.**

Every subset of a finite set is a finite set.

(iv) **False.**
.
ϕ does not have a proper subset.

(v) **False.**
$\{a,b,a,b,a,b,...\}$ will be equal to $\{a,b\}$ which is a finite set.

(vi) **True**.
$\{a,b,c\}$ and $\{1,2,3\}$ are equivalent sets because the number of elements in both the sets are same.

(vii) **False.**
In the set $A=\{1,2\}$, subsets can be {ϕ}, {1} and {2}, which are finite

Question 2.

Solution:

(i) **True**

(ii) **False**
It should be written as $\{a\}\subset\{b,c,a\}$ or $a\in\{b,c,a\}$.

(iii) **False**
It should be written as $\{a\} \subset \{b, c, a\}$ *or* $a \in \{b, c, a\}$

.

(iv) **True**

(v) **False**
The element of the set $\{x : x + 8 = 8\}$ is $\{0\}$. Therefore, it is not an empty or null set.

Question 3.

Solution:
We have:

$A = \{x : x \text{ satisfies } x2 - 8x + 12 = 0.\} = \{2, 6\}$

$B = \{2, 4, 6\}$

$C = \{2, 4, 6, 8, \ldots\}$

$D = \{6\}$

Therefore, we can say that $D \subset A \subset B \subset C$.

Question 4.

Solution:

(i) **True**

A rational number is any $\frac{m}{n}$, where m and n are any integers ($n \neq 0$). Any integer can be put into that form by setting n = 1. Therefore, the set of all integers is contained in the set of all rational numbers.

(ii) **True**
All crows are birds. Therefore, the set of all crows is contained in the set of all birds.

(iii) **False**
Every square can be a rectangle, but every rectangle cannot be a square.

(iv) **True**
Every real number can be written in the (a + bi) form. Thus, we can say that the set of all real numbers is contained in the set of all complex numbers.

(v) **False**
P = {a}
B = {{a}} = {P}
P≠{P}

(vi) **True**
We have:
A = {x:x is a letter of the word LITTLE} = {L, I, T, E}
B = {x:x is a letter of the word TITLE} = {T, I, L, E}

Sets A & B are equal because every element of A is a member of B & every element of B is a member of A.

Question 5.

Solution:

Here, (viii) is correct.
The correct forms of each of the incorrect statements are:
(i) $a \in a, b, c$
(ii) $a \subset a, b, c$
(iii) $\{a\} \in \{a, b\}$
(iv) $\{a\} \subset a, b$
(v) $b, c \in a, b, c$
(vi) $\{a, b\} \not\subset \{a, \{b, c\}\}$
(vii) $\phi \subset a, b$
(ix) $x : x+3=3 \neq \phi$

Question 6.

Solution:
$A = \{a, b, \{c, d\}, e\}$
(i) **False**
The correct statement would be $\{c, d\} \subset A$.
(ii) **True**
(iii) **True**
(iv) **True**
(v) **False**
The correct statement would be $\{a\} \subset A$ or $a \in A$.
(vi) **True**
(vii) **False**
The correct statement would be $a, b, e \subset A$.
(viii) **False**
The correct statement would be $\{a, b, c\} \not\subset A$.
(ix) **False**
A null set is a subset of every set. Therefore, the correct statement would be $\phi \subset A$.
(x) **False**
ϕ is an empty set; in other words, this set has no element. It is denoted by ϕ. Therefore, the correct statement would be $\phi \subset A$.

Question 7.

Solution:
(i) **False**
If it could be $1 \notin A$, then it would be true .
(ii) **False**
The correct form would be $1, 2, 3 \in A$ or $\{1, 2, 3\} \subset A$.
(iii) **True**
(iv) **True**

(v) **False**
A null set is a subset of every set. Therefore, the correct form would be $\phi \subset A$.
(vi) **True**

Question 8.

Solution :
(i) **True**
(ii) **True**
(iii) **False**
The correct form would be $1 \subset A$.
(iv) **True**
(v) **False**
The correct form would be $1 \in A$.
(vi) **True**
(vii) **True**
(viii) **True**
(ix) **True**

Question 9.

Solution:
(i) ϕ ,{a}
(ii) (ii) ϕ,{0},{1},{0,1}
(iii) (iii) ϕ,{a},{b},{c},{a,b},{b,c},{a,c},{a,b,c}
(iv) (iv) ϕ,{1},{{1}},{1,{1}}
(v) (v) ϕ,{ϕ}

Question 10.

Solution:
(i) {1}, {2}
(ii) {1}, {2}, {3}, {1, 2}, {2, 3}, {1, 3}
(iii) No proper subsets are there in this set.

Question 11.

Solution:
We know that the total number of subsets of a finite set consisting of n elements is 2^n.
Therefore, the total number of proper subsets of a set consisting of n elements is 2^n-1.

Question 12.

Solution:
To prove: $A \subseteq \phi \Leftrightarrow A = \phi$
Proof:
Let: $A \subseteq \phi$
If A is a subset of an empty set, then A is the empty set.
$\therefore$ A=ϕ

Now, let A=ϕ.
This means that A is an empty set.
We know that every set is a subset of itself.
$\therefore A \subseteq \phi$
Thus, we have:
$A \subseteq \phi \Leftrightarrow A = \phi$

Question 13.

Solution:
Let $x \in A \Rightarrow x \in B \because A \subseteq B \Rightarrow x \in C \because B \subseteq C \therefore x \in A \Rightarrow x \in C \Rightarrow A \subseteq C$ …(1)
It is given that,C⊆A …(2)
From 1 and 2, we have $A = C$

Question 14.

Solution:
Given: $A = \phi$ This means $P(A) = \{\phi\}$.
Hence, $P(A)$ would have one element.

Question 15.

Solution:
(i) The set of all triangles in a plane.
(ii) The set of all triangles in a plane

Question 16.

Solution:

Given:
$X = 8n - 7n - 1 : n \in N$ and $Y = 49(n-1) : n \in N$
To prove:
X⊆Y
Let: $x_n = 8n - 7n - 1,\ n \in N$
$\Rightarrow x_1 = 8 - 7 - 1 = 0$
For any $n \geqslant 2$, we have:
$x_n = 8^n - 7n - 1 = (1+7)^n - 7n - 1$
$\Rightarrow x_n = {}^nC_0 + {}^nC_1(7) + {}^nC_2(7)^2 + {}^nC_3(7)^3 + \ldots + {}^nC_n 7^n - 7n - 1$
$\Rightarrow x_n = 1 + 7n + {}^nC_0 7^2 + {}^nC_0 7^3 + \ldots + 7^n - 7n - 1$
$[\because {}^nC_0 = 1 \ and \ {}^nC_1 = n]$
$\Rightarrow x_n = 7^2\left\{{}^nC_2 + {}^nC_3(7) + {}^nC_4(7)^2 + \ldots + {}^nC_n 7^{n-2}\right\}$
$\Rightarrow x_n = 49\left\{{}^nC_2 + {}^nC_3(7) + {}^nC_4(7)^2 + \ldots + {}^nC_n 7^{n-2}\right\}$

Thus, x_n is some positive integral multiple of 49 for all n⩾2. X consists of all those positive integral multiples of 49 that are of the form $49\left\{{}^{n}C_{2}+{}^{n}C_{3}\left(7\right)+{}^{n}C_{4}\left(7\right)^{2}+\ldots+{}^{n}C_{n}\left(7\right)^{n-2}\right\}$ along with zero. $Y=\{49\left(n-1\right):n\in N\}$ implies that it consists of all integral multiples of 49