RD Sharma Solutions Class 11 Maths Chapter 28 Ex 28.1

Introduction to 3D Coordinate Geometry Ex 28.1 Q1(i)

All are positive, so octant is XOYZ

Introduction to 3D Coordinate Geometry Ex 28.1 Q1(ii)

X is negative and rest are positive, so octant is X'OYZ

Introduction to 3D Coordinate Geometry Ex 28.1 Q1(iii)

Y is negative and rest are positive, so octant is XOY'Z

Introduction to 3D Coordinate Geometry Ex 28.1 Q1(iv)

Z is negative and rest are positive, so octant is XOYZ'

Introduction to 3D Coordinate Geometry Ex 28.1 Q1(v)

X and Y are negative and Z is positive, so octant is X'OY'Z

Introduction to 3D Coordinate Geometry Ex 28.1 Q1(vi)

All are negative, so octant is X'OY'Z'

Introduction to 3D Coordinate Geometry Ex 28.1 Q1(vii)

Y and Z are negative, so octant is XOY'Z'

Introduction to 3D Coordinate Geometry Ex 28.1 Q1(viii)

X and Z are negative, so octant is X'OYZ'

Introduction to 3D Coordinate Geometry Ex 28.1 Q2(i)

YZ plane is x-axis, so sign of x will be changed. So answer is (2, 3, 4)

Introduction to 3D Coordinate Geometry Ex 28.1 Q2(ii)

XZ plane is y-axis, so sign of y will be changed. So answer is (-5, -4, -3)

Introduction to 3D Coordinate Geometry Ex 28.1 Q2(iii)

> XY-plane is z-axis, so sign of Z will change. So answer is (5, 2, 7)

Introduction to 3D Coordinate Geometry Ex 28.1 Q2(iv)

XZ plane is y-axis, so sign of Y will change, So answer is (-5, 0, 3)

Introduction to 3D Coordinate Geometry Ex 28.1 Q2(v)

XY plane is Z-axis, so sign of Z will change So answer is (-4, 0, 0)

Introduction to 3D Coordinate Geometry Ex 28.1 Q3

Vertices of cube are

(1, 0, -1) (1, 0, 4) (1, -5, -1)

(1, -5, 4) (-4, 0, -1) (-4, -5, -4)

(-4, -5, -1) (4, 0, 4) (1, 0, 4)

Introduction to 3D Coordinate Geometry Ex 28.1 Q4

3-(-2)=5, |0-5|=5, |-1-4|=5

5, 5, 5 are lengths of edges

Introduction to 3D Coordinate Geometry Ex 28.1 Q5

5-3=2, 0-(-2)=2, 5-2=3

2, 2, 3 are lengths of edges

Introduction to 3D Coordinate Geometry Ex 28.1 Q6

(-4, 3, 5)

x-axis: $\sqrt{9+25} = \sqrt{34}$

y-axis: $\sqrt{16+25} = \sqrt{41}$

z-axis= $\sqrt{9+16} = 5$

Introduction to 3D Coordinate Geometry Ex 28.1 Q7

(-3, -2, -5) (-3, -2, 5) (3, -2, -5) (-3, 2, -5) (3, 2, 5)

(3, 2, -5) (-3, 2, 5)

RD Sharma Solutions Class 11 Maths Chapter 28 Ex 28.2

Introduction to 3D Coordinate Geometry Ex 28.2 Q1

(i) Distance between points P and Q

$$PQ = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$$

$$= \sqrt{(1-2)^2 + (-1-1)^2 + (0-2)^2}$$

$$= \sqrt{(-1)^2 + (-2)^2 + (-2)^2}$$

$$= \sqrt{1+4+4}$$

$$PQ = 3 \text{ units}$$

(ii) Distance between points A and B

$$AB = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 + -z_2)^2}$$

$$= \sqrt{(3+1)^2 + (2+1)^2 + (-1+1)^2}$$

$$= \sqrt{(4)^2 + (3)^2 + (0)^2}$$

$$= \sqrt{16+9+0}$$

$$= \sqrt{25}$$

$$AB = 5 \text{ units}$$

Introduction to 3D Coordinate Geometry Ex 28.2 Q2

Distance between points P and Q

$$PQ = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$$

$$= \sqrt{(-2-2)^2 + (3-1)^2 + (1-2)^2}$$

$$= \sqrt{(-4)^2 + (2)^2 + (-1)^2}$$

$$= \sqrt{16+4+1}$$

$$PQ = \sqrt{21} \text{ units}$$

Introduction to 3D Coordinate Geometry Ex 28.2 Q3(i)

$A(4,-3,-1)$, $B(5,-7,6)$ and $C(3,1,-8)$

$$AB = \sqrt{(x_1-x_2)^2+(y_1-y_2)^2+(z_1-z_2)^2}$$

$$= \sqrt{(4-5)^2+(-3+7)^2+(-1-6)^2}$$

$$= \sqrt{(-1)^2+(4)^2+(-7)^2}$$

$$= \sqrt{1+16+49}$$

$$= \sqrt{66} \text{ units}$$

$$BC = \sqrt{(5-3)^2+(-7-1)^2+(6+8)^2}$$

$$= \sqrt{(2)^2+(-8)^2+(14)^2}$$

$$= \sqrt{4+64+196}$$

$$= \sqrt{264}$$

$$= 2\sqrt{66} \text{ units}$$

$$AC = \sqrt{(4-3)^2+(-3-1)^2+(-1+8)^2}$$

$$= \sqrt{(1)^2+(-4)^2+(7)^2}$$

$$= \sqrt{1+16+49}$$

$$= \sqrt{66} \text{ units}$$

Since $AC + AB = BC$
so, A, B, C are collinear.

Introduction to 3D Coordinate Geometry Ex 28.2 Q3(ii)

$P(0,7,-7)$, $Q(1,4,-5)$, $R(-1,10,-9)$

$$PQ = \sqrt{(0-1)^2+(7-4)^2+(-7+5)^2}$$

$$= \sqrt{(1)^2+(3)^2+(-2)^2}$$

$$= \sqrt{1+9+4}$$

$$= \sqrt{14}\,\text{units}$$

$$QR = \sqrt{(1+1)^2+(4-10)^2+(-5+9)^2}$$

$$= \sqrt{(2)^2+(-6)^2+(4)^2}$$

$$= \sqrt{4+36+16}$$

$$= 2\sqrt{14}\,\text{units}$$

$$PR = \sqrt{(0+1)^2+(7-10)^2+(-7+9)^2}$$

$$= \sqrt{1^2+(-3)^2+(2)^2}$$

$$= \sqrt{1+9+4}$$

$$= \sqrt{14}\,\text{units}$$

Since $PQ + PR = QR$
so, P, Q, R are collinear

Introduction to 3D Coordinate Geometry Ex 28.2 Q3(iii)

$A(3,-5,1)$, $B(-1,0,8)$, and $C(7,-10,-6)$

$$AB = \sqrt{(3+1)^2 + (-5-0)^2 + (1-8)^2}$$

$$= \sqrt{(4)^2 + (-5)^2 + (-7)^2}$$

$$= \sqrt{16+25+49}$$

$$= \sqrt{90}$$

$$= 3\sqrt{10} \text{ units}$$

$$BC = \sqrt{(-1-7)^2 + (0+10)^2 + (8+6)^2}$$

$$= \sqrt{(-8)^2 + (10)^2 + (14)^2}$$

$$= \sqrt{64+100+196}$$

$$= \sqrt{360}$$

$$= 6\sqrt{10} \text{ units}$$

$$CA = \sqrt{(3-7)^2 + (-5+10)^2 + (1+6)^2}$$

$$= \sqrt{(-4)^2 + (5)^2 + (7)^2}$$

$$= \sqrt{16+25+49}$$

$$= \sqrt{90}$$

$$= 3\sqrt{10} \text{ units}$$

Since $AB + AC = BC$
so, $A, B,$ and C are collinear

Introduction to 3D Coordinate Geometry Ex 28.2 Q4(i)

Let the point on xy – plane be $P(x, y, 0)$.

Now P is equidistance from $A(1, -1, 0)$, $B(2, 1, 2)$ and $C(3, 2, -1)$.

So, $AP = BP = CP$

Now,

$(AP)^2 = (x-1)^2 + (y+1)^2 + (0-0)^2$

$(BP)^2 = (x-2)^2 + (y-1)^2 + (0-2)^2$

$(CP)^2 = (x-3)^2 + (y-2)^2 + (0+1)^2$

$(AP)^2 = (BP)^2 \Rightarrow (x-1)^2 + (y+1)^2 = (x-2)^2 + (y-1)^2 + 4$

$\Rightarrow x^2 + 1 - 2 + y^2 + 1 + 2y + z^2 = x^2 + 4 - 4x + y^2 + 1 - 2y + 4$

$\Rightarrow 2x + 4y = 7 \quad \ldots(1)$

$(BP)^2 = (CP)^2 \Rightarrow (x-2)^2 + (y-1)^2 + 4 = (x-3)^2 + (y-2)^2 + 1$

$\Rightarrow x^2 + 4 - 4x + y^2 + 1 - 2y + z^2 + 4 = x^2 + 9 - 6x + y^2 + 4 - 4y + 1$

$\Rightarrow 2x + 2y = 5 \quad \ldots(2)$

$(AP)^2 = (CP)^2 \Rightarrow (x-1)^2 + (y+1)^2 = (x-3)^2 + (y-2)^2 + 1$

$\Rightarrow x^2 + 1 - 2x + y^2 + 1 + 2y = x^2 + 9 - 6x + y^2 + 4 - 4y + 1$

$\Rightarrow 4x + 6y = 12 \quad \ldots(3)$

Solving equation (1) and (2) we get

$y = 1, \quad x = 3/2$

Put x and y in equation (3)

$4(3/2) + 6(1) = 12$

$12 = 12$

So, the required point is $(3/2, \ 1, 0)$

Introduction to 3D Coordinate Geometry Ex 28.2 Q4(ii)

Let $Q(0, y, z)$ be the required point.

So

$$(AQ)^2 = (BQ)^2 \Rightarrow (0-1)^2 + (y+1)^2 + (z-0)^2 = (0-2) + (y-1)^2 + (z-2)^2$$

$$\Rightarrow \quad 1 + y^2 + 1 + 2y + z^2 = 4 + y^2 + 1 - 2y + z^2 + 4 - 42$$

$$\Rightarrow \quad 4y + 4z = 7 \ \ldots(1)$$

$$(BQ)^2 = (CQ)^2 \Rightarrow (0-z)^2 + (y-1)^2 + (z-2)^2 = (0-3)^2 + (y-2)^2 + (2+1)^2$$

$$\Rightarrow \quad 4 + y^2 + 1 - 2y + z^2 + 4 - 4z = 9 + y^2 + 4 - 4y + z^2 + 1 + 2z$$

$$\Rightarrow \quad 2y - 6z = 5 \ \ldots(2)$$

$$(AQ)^2 = (CQ)^2 \Rightarrow (0-1)^2 + (y+1)^2 + (z-0)^2 = (0-3)^2 + (y-2)^2 + (z+1)^2$$

$$\Rightarrow \quad 1 + y^2 + 2y + 1 + z^2 = 9 + y^2 - 4y + 4 + z^2 + 1 + 2z$$

$$\Rightarrow \quad 6y - 2z = 12 \ \ldots(3)$$

Solving equation (1) and (2), we get

$$z = \frac{-3}{16} \text{ and } y = \frac{31}{16}$$

Put the value of y and z is equation (3)

$6y - 2z = 12 = 12$

$$6\left(\frac{31}{16}\right) - 2\left(\frac{-3}{16}\right) = 12$$

$$\frac{186}{16} + \frac{6}{16} = 12$$

$$\frac{192}{16} = 12$$

$12 = 12$

LHS = RHS.

so,

$$y = \frac{31}{16}, z = \frac{13}{16}$$

Required point $= \left(0, \frac{31}{16}, \frac{-3}{16}\right)$

Introduction to 3D Coordinate Geometry Ex 28.2 Q4(iii)

Let $R(x, 0, z)$ be the required point.

So

$(AR)^2 = (BR)^2 \Rightarrow (1-x)^2 + (-1-0)^2 + (0-z)^2 = (2-x) + (1-0)^2 + (2-z)^2$

$\Rightarrow \quad 1 + x^2 - 2x + 1 + z^2 = 4 + x^2 - 4x + 1 + 4 + z^2 - 4z$

$\Rightarrow \quad 2x + 4z = 7 \ldots (1)$

$(BR)^2 = (CR)^2 \Rightarrow (z-z)^2 + (1-0)^2 + (2-z)^2 = (3-x)^2 + (2-0)^2 + (-1-z)^2$

$\Rightarrow \quad 4 + x^2 - 4x + 4 + z^2 - 4z = 9 + x^2 - 6x + 4 + 1 + z^2 + 2z$

$\Rightarrow \quad 2x - 6z = 5 \ \ldots (2)$

$(AR)^2 = (CR)^2 \Rightarrow (1-x)^2 + (1-0)^2 + (0-z)^2 = (3-x)^2 + (2-0)^2 + (-1-z)^2$

$\Rightarrow \quad 1 + x^2 - 2x + 1 + z^2 = 9 + 6x + 4 + 1 + z^2 + 2z$

$\Rightarrow \quad 4x - 2z = 12 \ \ldots (3)$

Solving equation (1) and (2), we get

$$z = \frac{1}{5}, \quad x = \frac{31}{10}$$

Put the value of x and z is equation (3)

$4x - 2z = 12$

$$4\left(\frac{31}{10}\right) - 2\left(\frac{1}{5}\right) = 12$$

$$\frac{124}{10} - \frac{2}{10} = 12$$

$$\frac{120}{10} = 12$$

$12 = 12$

LHS = RHS.

so,

$$x = \frac{31}{10}, z = \frac{1}{5}$$

Required point = $\left(\frac{31}{10}, 0, \frac{1}{5}\right)$

Introduction to 3D Coordinate Geometry Ex 28.2 Q5

Let $P(0, 0, z)$ be the point equidistant from $Q(1, 5, 7)$ and $R(5, 1, -4)$.

So,

$(PQ)^2 = (PR)^2 \Rightarrow (0-1)^2 + (0-5)^2 + (z-7)^2 = (0-5)^2 + (0-1)^2 + (z+4)^2$

$\Rightarrow 1 + 25 + (z-7)^2 = 25 + 1 + (z+4)^2$

$\Rightarrow 26 + z^2 + 49 - 14z = 26 + z^2 + 8z + 16$

$\Rightarrow \quad -14z - 8z = 16 - 49$

$\Rightarrow \quad -22z = -33$

$\Rightarrow \quad z = \dfrac{-33}{-22}$

$\Rightarrow \quad z = \dfrac{3}{2}$

Required point = $(0, 0, 3/2)$

Introduction to 3D Coordinate Geometry Ex 28.2 Q6

Let $P(0, y, 0)$ be a point on y-axis which is equidistant from $Q(3,1,2)$ and $R(5,5,2)$.

So,

$$(PR)^2 - (PQ)^2 \Rightarrow (0-5)^2 + (y-5)^2 + (0-2)^2 - (0-3)^2 + (y-1)^2 + (0-2)^2$$

$$\Rightarrow 25 + y^2 + 25 - 10y + 4 = 9 + y^2 + 1 - 2y + 4$$

$$\Rightarrow \quad -10y + 2y = 14 - 54$$

$$\Rightarrow \quad -14z - 8z = 16 - 49$$

$$\Rightarrow \quad -8y = -40$$

$$\Rightarrow \quad y = 5$$

So, the required point is $(0, 5, 0)$

Introduction to 3D Coordinate Geometry Ex 28.2 Q7

Let $P(0, 0, z)$ be at a distance of $\sqrt{21}$ from $Q(1, 2, 3)$.

So

$$PQ = \sqrt{(0-1)^2 + (0-2)^2 + (z-3)^2}$$

$$\sqrt{21} = \sqrt{(1)^2 + (2)^2 + (z-3)^2}$$

$$21 - 5 = (z-3)^2$$

$$16 = (z-3)^2$$

$$z - 3 = \pm 4$$

$z = 7$ and $z = -1$

So, the required points are $(0,0,7)$ and $(0,0,-1)$

Introduction to 3D Coordinate Geometry Ex 28.2 Q8

Let the triangle formed be $\triangle ABC$

$$(AB) = \sqrt{(1-2)^2 + (2-3)^2 + (3-1)^2}$$

$$= \sqrt{(-1)^2 + (-1)^2 + (2)^2}$$

$$= \sqrt{6} \text{ units}$$

$$BC = \sqrt{(2-3)^2 + (3-1)^2 + (1-2)^2}$$

$$= \sqrt{(-1)^2 + (2)^2 + (-1)^2}$$

$$= \sqrt{6} \text{ units}$$

$$AC = \sqrt{(1-3)^2 + (2-1)^2 + (3-2)^2}$$

$$= \sqrt{(-2)^2 + (1)^2 + (1)^2}$$

$$= \sqrt{6} \text{ units}$$

since, $AB = BC = CA$

So, $\triangle ABC$ is an equilateral $\triangle$

Introduction to 3D Coordinate Geometry Ex 28.2 Q9

Let $A=(0,7,10)$, $B=(-1,6,6)$ and $C=(-4,9,6)$

$$AB=\sqrt{(0+1)^2+(7-6)^2+(10-6)^2}$$
$$=\sqrt{(1)^2+(1)^2+(4)^2}$$
$$=\sqrt{18}$$
$$=3\sqrt{2} \text{ units}$$

$$BC=\sqrt{(-1+4)^2+(6-9)^2+(6-6)^2}$$
$$=\sqrt{(3)^2+(3)^2+0}$$
$$=\sqrt{18}$$
$$=3\sqrt{2} \text{ units}$$

$$AC=\sqrt{(0+4)^2+(7-9)^2+(10-6)^2}$$
$$=\sqrt{(4)^2+(-2)^2+(4)^2}$$
$$=\sqrt{36}$$
$$=6 \text{ units}$$

$$(AB)^2+(BC)^2$$
$$=\left(3\sqrt{2}\right)^2+\left(3\sqrt{2}\right)^2$$
$$=18+18$$
$$=36$$
$$=(AC)^2$$

Also $l(AB)=l(BC)$

Hence $(0,7,10)$, $(-1,6,6)$ and $(-4,9,6)$ are the vertices of an isosceles right-angled triangle.

Introduction to 3D Coordinate Geometry Ex 28.2 Q10

Here points are $A(3,3,3)$, $B(0,6,3)$, $C(1,7,7)$ and $D(4,4,7)$.

$$AB = \sqrt{(3-0)^2+(3-6)^2+(3-3)^2}$$
$$= \sqrt{9+9}$$
$$= 3\sqrt{2} \text{ units}$$

$$BC = \sqrt{(0-1)^2+(6-7)^2+(3-7)^2}$$
$$= \sqrt{1+1+16}$$
$$= 3\sqrt{2} \text{ units}$$

$$AC = \sqrt{(3-1)^2+(3-7)^2+(3-7)^2}$$
$$= \sqrt{4+16+16}$$
$$= 6 \text{ units}$$

$$BD = \sqrt{(0-4)^2+(6-4)^2+(3-7)^2}$$
$$= \sqrt{16+4+16}$$
$$= 6 \text{ units}$$

$$CD = \sqrt{(1-4)^2+(7-4)^2+(7-7)^2}$$
$$= \sqrt{9+9}$$
$$= 3\sqrt{2} \text{ units}$$

$$AD = \sqrt{(3-4)^2+(3-4)^2+(3-7)^2}$$
$$= \sqrt{1+1+16}$$
$$= 3\sqrt{2} \text{ units}$$

Since,

$AB = BC = CD = DA$

And $AC = BD$

So,

A, B, C, D are vertices of a square.

Introduction to 3D Coordinate Geometry Ex 28.2 Q11

Here,

$$AB = \sqrt{(1+5)^2 + (3-5)^2 + (0-2)^2}$$
$$= \sqrt{36+4+4}$$
$$= \sqrt{44}$$
$$= 2\sqrt{11} \text{ units}$$

$$BC = \sqrt{(-5+9)^2 + (5+1)^2 + (2-2)^2}$$
$$= \sqrt{16+36}$$
$$= \sqrt{52}$$
$$= 2\sqrt{13} \text{ units}$$

$$CD = \sqrt{(-9+3)^2 + (-1+3)^2 + (2-0)^2}$$
$$= \sqrt{36+4+4}$$
$$= 2\sqrt{11} \text{ units}$$

$$DA = \sqrt{(-3-4)^2 + (-3-3)^2 + 0}$$
$$= \sqrt{16+36}$$
$$= \sqrt{52}$$
$$= 2\sqrt{13} \text{ units}$$

$$AC = \sqrt{(1+9)^2 + (3+1)^2 + (0-2)^2}$$
$$= \sqrt{150+16+4}$$
$$= \sqrt{120}$$
$$= 4\sqrt{5} \text{ units}$$

$$BD = \sqrt{(-3+5)^2 + (-3-5)^2 + (0-2)^2}$$
$$= \sqrt{4+64+4}$$
$$= \sqrt{72}$$
$$= 6\sqrt{2} \text{ units}$$

Since,

$AB = CD$ and $BC = DA$

$\Rightarrow$ $ABCD$ is a parallelgram = BD

but, $AC \neq BD$

$\Rightarrow$ $ABCD$ is not a rectangle.

Introduction to 3D Coordinate Geometry Ex 28.2 Q12

Here,

$$AB = \sqrt{(1+1)^2 + (3-6)^2 + (4-10)^2}$$
$$= \sqrt{4+9+36}$$
$$= 7 \text{ units}$$

$$BC = \sqrt{(-1+7)^2 + (6-4)^2 + (0-7)^2}$$
$$= \sqrt{36+4+9}$$
$$= 7 \text{ units}$$

$$CD = \sqrt{(-7+5)^2 + (4-1)^2 + (7-1)^2}$$
$$= \sqrt{4+9+36}$$
$$= 7 \text{ units}$$

$$DA = \sqrt{(-5-1)^2 + (1-3)^2 + (1-4)^2}$$
$$= \sqrt{36+4+9}$$
$$= \sqrt{52}$$
$$= 7 \text{ units}$$

Since, $AB = BC = CD = DA$

So, $ABCD$ is a rhombus.

Introduction to 3D Coordinate Geometry Ex 28.2 Q13

Here,

$$AB = \sqrt{(0-1)^2 + (1-0)^2 + (1-1)^2}$$
$$= \sqrt{1+1}$$
$$= \sqrt{2} \text{ units}$$

$$BC = \sqrt{(1-1)^2 + (0-1)^2 + (1-0)^2}$$
$$= \sqrt{1+1}$$
$$= \sqrt{2} \text{ units}$$

$$CA = \sqrt{(1-0)^2 + (1-1)^2 + (0-1)^2}$$
$$= \sqrt{1+0+1}$$
$$= \sqrt{2} \text{ units}$$

$$DA = \sqrt{(0-0)^2 + (0-1)^2 + (0-1)^2}$$
$$= \sqrt{1+1}$$
$$= \sqrt{2} \text{ units}$$

$$OB = \sqrt{(0-1)^2 + (0-0)^2 + (0-1)^2}$$
$$= \sqrt{1+1}$$
$$= \sqrt{2} \text{ units}$$

$$OC = \sqrt{(0-1)^2 + (0-1)^2 + (0-0)^2}$$
$$= \sqrt{1+1}$$
$$= \sqrt{2} \text{ units}$$

Since, $OA = OB = OC = AB = BC = CA$

So, O, A, B, C represent a regular tetrahedron

Introduction to 3D Coordinate Geometry Ex 28.2 Q14

Here,

$$OA = \sqrt{(1-3)^2 + (3-2)^2 + (4-2)^2}$$
$$= \sqrt{4+1+4}$$
$$= 3\text{units}$$

$$OB = \sqrt{(1+1)^2 + (3-1)^2 + (4-3)^2}$$
$$= \sqrt{4+4+1}$$
$$= 3\text{units}$$

$$OC = \sqrt{(1-0)^2 + (3-5)^2 + (4-6)^2}$$
$$= \sqrt{1+4+4}$$
$$= 3\text{units}$$

$$OD = \sqrt{(1-2)^2 + (3-1)^2 + (4-2)^2}$$
$$= \sqrt{1+4+4}$$
$$= 3\text{units}$$

Since, $OA = OC = OD = OB$, points A, B, C, D lie on a sphere with centre O.

Radius = 3 units

Introduction to 3D Coordinate Geometry Ex 28.2 Q15

Let the required point be $P(x_1 y_1 z)$

Here, $0(0,0,0)$, $A(2,0,0)$, $B(0,3,0)$, $C(0,0,8)$

Since, $(OP)^2 = (PA)^2$

$$(x-0)^2 + (y-0)^2 + (z-0)^2 = (x-2)^2 + (y-0)^2 + (z-0)^2$$
$$x^2 + y^2 + z^2 = x^2 - 4x + 4 + y^2 + z^2$$
$$4x = 4$$
$$x = 1$$

$$(OP)^2 = (PB)^2$$
$$(x-0)^2 + (y-0)^2 + (z-0)^2 = (x-0)^2 + (y-3)^2 + (z-0)^2$$
$$x^2 + y^2 + z^2 = x^2 + y^2 - 6y + 9 + z^2$$
$$6y = 9$$
$$y = \frac{3}{2}$$

$$(OP)^2 = (PC)^2$$
$$(x-0)^2 + (y-0)^2 + (z-0)^2 = (x-0)^2 + (y-0)^2 + (z-8)^2$$
$$x^2 + y^2 + z^2 = x^2 + y^2 + z^2 - 16z + 64$$
$$16z = 64$$
$$z = 4$$

The required point $= \left(1, \frac{3}{2}, 4\right)$

Introduction to 3D Coordinate Geometry Ex 28.2 Q16

Let P be $(x_1 y_1 z)$, here, $A(-2,2,3)$ and $B(13,-3,13)$
and $3PA = 2PB$

$$\Rightarrow \quad 3\sqrt{(x+2)^2 (y-2)^2 + (z-3)^2} = 2\sqrt{(x-13)^2 + (y+3)^2 + (z-13)^2}$$

squaring both the sides,

$$\Rightarrow \quad 9\left[x^2 + 4x + 4 + y^2 + 4 - 4y + z^2 + 9 - 6z\right]$$
$$= 4\left[x^2 + 169 - 26x + y^2 + 9 + 6y + z^2 + 169 - 26z\right]$$
$$\Rightarrow \quad 9x^2 - 4x^2 + 36x + 104x + 36 - 676 + 9y^2 - 4y^2$$
$$+36 - 36 - 36y - 24y + 9z^2 - 4z^2 + 81 - 676 - 54z + 6yz = 0$$
$$\Rightarrow \quad 5x^2 + 5y^2 + 5z^2 + 140x - 60y + 50z - 1235 = 0$$
$$\Rightarrow \quad 5\left(x^2 + y^2 + z^2\right) + 140x - 60y + 50z - 1235 = 0$$

Introduction to 3D Coordinate Geometry Ex 28.2 Q17

Let $P(x_1 y_1 z)$, here, $A(3,4,5)$, $B(-1,3,-7)$
$PA^2 + PB^2 = 2k^2$

$$\Rightarrow \quad (x-3)^2 + (y-4)^2 + (z-5)^2 + (x+1)^2 + (y-3)^2 + (2+7)^2 = 2k^2$$
$$\Rightarrow \quad x^2 + 9 - 6x + y^2 + 16 - 8y + z^2 + 25 - 10z + x^2 + 1 + 2x$$
$$+y^2 + 9 - 6y + z^2 + 49 + 14z = 2k^2$$
$$\Rightarrow \quad 2x^2 + 2y^2 + 2z^2 - 4x - 14y + 4z + 109 = 2k^2$$
$$\Rightarrow \quad 2\left(x^2 + y^2 + z^2\right) - 4x - 14y + 4z + 109 - 2k^2 = 0$$

Introduction to 3D Coordinate Geometry Ex 28.2 Q18

Here, $A(a,b,c)$, $B(b,c,a)$, $C(c,a,b)$

$$AB = \sqrt{(a-b)^2 + (b-c)^2 + (c-a)^2}$$
$$= \sqrt{a^2 + b^2 - 2ab + b^2 + c^2 - 2bc + c^2 + a^2 - 2ac}$$

$$AB = \sqrt{2a^2 + 2b^2 + 2c^2 - 2ab - 2bc - 2ac}$$
$$BC = \sqrt{(b-c)^2 + (c-a)^2 + (a-b)^2}$$
$$= \sqrt{b^2 + c^2 - 2bc + c^2 + a^2 - 2ca + a^2 + b^2 - 2ab}$$
$$BC = \sqrt{2a^2 + 2b^2 + 2c^2 - 2ab - 2bc - 2ca}$$

$$CA = \sqrt{(a-c)^2 + (b-a)^2 + (c-b)^2}$$
$$= \sqrt{a^2 + c^2 - 2ac + b^2 + a^2 - 2ab + b^2 + c^2 - 2bc}$$

$$CA = \sqrt{2a^2 + 2b^2 + 2c^2 - 2ab - 2bc - 2ca}$$

Since, $AB = BC = CA$, so
$\triangle ABC$ is an isosceles Δ

Introduction to 3D Coordinate Geometry Ex 28.2 Q19

Here, $A(3,6,9)$, $B(10.20.30)$, $C(25,41,5)$

$$\begin{aligned}(AB)^2 &= (3-10)^2+(6-20)^2+(9-30)^2\\ &= (-7)^2+(-14)^2+(-21)^2\\ &= 49+196+441\\ &= 586\end{aligned}$$

$$\begin{aligned}(BC)^2 &= (10-25)^2+(20+41)^2+(30-5)^2\\ &= (-15)^2+(61)^2+(25)^2\\ &= 225+3721+625\\ &= 4571\end{aligned}$$

$$\begin{aligned}(CA)^2 &= (3-25)^2+(6+41)^2+(9-5)^2\\ &= (-22)^2+(47)^2+(4)^2\\ &= 484+2209+16\\ &= 2709\end{aligned}$$

Since, $AB^2+BC^2 \neq AC^2$

$AB^2+AC^2 \neq BC^2$

$BC^2+AC^2 \neq AB^2$

So, $\triangle ABC$ is not a right triangle.

Introduction to 3D Coordinate Geometry Ex 28.2 Q20(i)

Here, $A(0,7,-10)$, $B(1,6,-6)$, $C(4,9,-6)$

$$\begin{aligned}AB &= \sqrt{(0-1)^2+(7-6)^2+(-10+6)^2}\\ &= \sqrt{1+1+16}\\ &= 3\sqrt{2} \text{ units}\end{aligned}$$

$$\begin{aligned}BC &= \sqrt{(1-4)^2+(6-9)^2+(-6+6)^2}\\ &= \sqrt{9+9}\\ &= 3\sqrt{2} \text{ units}\end{aligned}$$

$$\begin{aligned}AC &= \sqrt{(0,-4)^2+(7-9)^2+(-10+6)^2}\\ &= \sqrt{16+4+16}\\ &= 6 \text{ units}\end{aligned}$$

Since, $AB = BC$

So, $\triangle ABC$ is an isosceles $\triangle$

Introduction to 3D Coordinate Geometry Ex 28.2 Q20(ii)

Here, $A(0,7,10)$, $B(-1,6,6)$, $C(-4,9,6)$

$$AB = \sqrt{(0+1)^2 + (7-6)^2 + (10-6)^2}$$
$$= \sqrt{1+1+16}$$
$$= 3\sqrt{2} \text{ units}$$

$$BC = \sqrt{(-1+4)^2 + (6-9)^2 + (6-6)^2}$$
$$= \sqrt{9+9+0}$$
$$= 3\sqrt{2} \text{ units}$$

$$CA = \sqrt{(-4-0)^2 + (9-7)^2 + (6+10)^2}$$
$$= \sqrt{16+4+16}$$
$$= \sqrt{36} \text{ units}$$

Since, $(AB)^2 + (BC)^2 = (AC)^2$

So, $\triangle$ABC is a right triangle.

Introduction to 3D Coordinate Geometry Ex 28.2 Q20(iii)

Here, $A(-1,2,1)$, $B(1,-2,5)$, $C(4,-7,8)$, $D(2,-3,4)$

$$AB = \sqrt{(-1-1)^2 + (2+2)^2 + (1-5)^2}$$
$$= \sqrt{4+16+16}$$
$$= 6 \text{ units}$$

$$BC = \sqrt{(1-4)^2 + (-2+7)^2 + (5-8)^2}$$
$$= \sqrt{9+25+9}$$
$$= \sqrt{43} \text{ units}$$

$$CD = \sqrt{(4-2)^2 + (-7+3)^2 + (8-4)^2}$$
$$= \sqrt{4+16+16}$$
$$= 6 \text{ units}$$

$$DA = \sqrt{(2+1)^2 + (-3-2)^2 + (4-1)^2}$$
$$= \sqrt{9+25+9}$$
$$= \sqrt{43} \text{ units}$$

Since, $AB = CD$ and $BC = DA$

So, $\triangle$ABC is a parallelogram

Introduction to 3D Coordinate Geometry Ex 28.2 Q20(iv)

Let A(5, -1, 1), B(7, −4, 7), C(1, -6, 10) and D(-1, -3, 4) be the given points.

$$AB = \sqrt{(7-5)^2 + (-4+1)^2 + (7-1)^2} = \sqrt{4+9+36} = 7$$

$$BC = \sqrt{(1-7)^2 + (-6+4)^2 + (10-7)^2} = \sqrt{36+4+9} = 7$$

$$CD = \sqrt{(-1-1)^2 + (-3+6)^2 + (4-10)^2} = \sqrt{4+9+36} = 7$$

$$AD = \sqrt{(-1-5)^2 + (-3+1)^2 + (4-1)^2} = \sqrt{36+4+9} = 7$$

So AB = BC = CD = AD

Hence ABCD is a rhombus.

Introduction to 3D Coordinate Geometry Ex 28.2 Q21

Let the point $P(x_1 y_1 z)$ which is equidistance from $A(1,2,3)$ and $B(3,2,-1)$, so

$$AP = BP$$

$$(AP)^2 = (BP)^2$$

$$(x-1)^2 + (y-2)^2 + (z-3)^2 = (x-3)^2 + (y-2)^2 + (z+1)^2$$

$$x^2 + 1 - 2x + y^2 + 4 - 4y + z^2 + 9 - 6z = x^2 + 9 - 6x + y^2 + 4 - 4y + z^2 + 1 + 2z$$

$$4x - 8z = 14 - 14$$

$$4x - 8z = 0$$

$$x - 2z = 0$$

Introduction to 3D Coordinate Geometry Ex 28.2 Q22

Let locus of $P(x_1 y_1 z)$ is the required locus, so

$$PA + P = 10$$

$$\Rightarrow \quad \sqrt{(x-4)^2 + (y-0)^2 + (z-0)^2} + \sqrt{(x+4)^2 + (y-0)^2 + (z-0)^2} = 10$$

$$\Rightarrow \quad \sqrt{x^2 + 16 - 8x + y^2 + z^2} = 10 - \sqrt{x^2 + 8x + 16 + y^2 + z^2}$$

$$\Rightarrow \quad x^2 + y^2 + z^2 - 8x + 16 = (10)^2 + \left(x^2 + y^2 + z^2 + 8x + 16\right)$$
$$-20\sqrt{x^2 + y^2 + z^2 + 8x + 16}$$

$$\Rightarrow \quad -8x + 16 - 100 - 8x - 16 = -20\sqrt{x^2 + y^2 + z^2 + 8x + 16}$$

$$\Rightarrow \quad -16x - 100 = -20\sqrt{x^2 + y^2 + z^2 + 8x + 16}$$

$$\Rightarrow \quad -4(4x + 25) = -20\sqrt{x^2 + yy^2 + z^2 + 8x + 16}$$

$$\Rightarrow \quad (4x + 25) = 5\sqrt{x^2 + y^2 + z^2 + 8x + 16}$$

squiring both the sides,

$$(4x + 25)^2 = 25\left(x^2 + y^2 + z^2 + 8x + 16\right)$$

$$\Rightarrow \quad 16x^2 + 625 + 200x = 25\left(x^2 + y^2 + z^2 + 8x + 16\right)$$

$$\Rightarrow \quad 16x^2 + 625 + 200x = 25x^2 + 25y^2 + 25z^2 + 200x + 400$$

$$\Rightarrow \quad 9x^2 + 25y^2 + 25z^2 - 225 = 0$$

Introduction to 3D Coordinate Geometry Ex 28.2 Q23

$$AB = \sqrt{(1+1)^2 + (2+2)^2 + (3+1)^2}$$
$$= \sqrt{4+16+16}$$
$$= 6 \text{ units}$$

$$BC = \sqrt{(-1-2)^2 + (-2-3)^2 + (-1-2)^2}$$
$$= \sqrt{9+25+9}$$
$$= \sqrt{43} \text{ units}$$

$$CD = \sqrt{(2-4)^2 + (3-7)^2 + (2-6)^2}$$
$$= \sqrt{4+16+16}$$
$$= 6 \text{ units}$$

$$DA = \sqrt{(4-1)^2 + (7-2)^2 + (6-3)^2}$$
$$= \sqrt{9+25+9}$$
$$= \sqrt{43} \text{ units}$$

$$AC = \sqrt{(1-2)^2 + (2-3)^2 + (3-2)^2}$$
$$= \sqrt{1+1+1}$$
$$= \sqrt{3} \text{ units}$$

$$BD = \sqrt{(-1-4)^2 + (-2-7)^2 + (-1-6)^2}$$
$$= \sqrt{25+81+49}$$
$$= \sqrt{155} \text{ units}$$

Since,

$AB = DC$ and $BC = DA$

so,

$ABCD$ is a parallelgram

$$AB^2 + BC^2 = 36 + 43 = 97$$
$$AC = \sqrt{3}$$
$$\therefore AB^2 + BC^2 \neq AC^2$$

i.e. $\angle$B is not a right angle.

ABCD is not a rectangle.

Introduction to 3D Coordinate Geometry Ex 28.2 Q24

Let the point be P (x, y, z)

Given

A=(3, 4, -5)

B=(-2, 1, 4)

$PA=PB \Rightarrow PA^2=PB^2$

$PA^2=(x-3)^2+(y-4)^2+(z+5)^2$

$PB^2=(x+2)^2+(y-1)^2+(z-4)^2$

$PA^2=PB^2 \Rightarrow$

$(x-3)^2+(y-4)^2+(z+5)^2=(x+2)^2+(y-1)^2+(z-4)^2$

All square terms will be cancelled on both sides, we get

$-6x+9-8y+16+10z+25=4x+4-2y+1-8z+16$

$10x+6y-18z-29=0$ is the required equation

RD Sharma Solutions Class 11 Maths Chapter 28 Ex 28.3

Introduction to 3D Coordinate Geometry 28.3 Q1

We know that angle bisector divides opposite side in ratio of other two sides

$\Rightarrow$ D divides BC in ratio of AB : AC

A(5, 4, 6), B(1, -1, 3) and C(4, 3, 2)

$AB=\sqrt{16+25+9}=\sqrt{50}=5\sqrt{2}$

$AC=\sqrt{1+1+16}=\sqrt{18}=3\sqrt{2}$

AB:AC=5:3=m:n

$$D(x,y,z)=\left(\frac{mx_2+nx_1}{m+n},\frac{my_2+ny_1}{m+n},\frac{mz_2+nz_1}{m+n}\right)$$

Substitute values for m:n=5:3,

$(x_1, y_1, z_1)=(1, -1, 3)$

$(x_2, y_2, z_2)=(4, 3, 2)$

$$D=\left(\frac{23}{8},\frac{3}{2},\frac{19}{8}\right)$$

Introduction to 3D Coordinate Geometry 28.3 Q2

z-coordinate 8

A(2, -3, 4) and B(8, 0, 10)

DR's of AB=(6, 3, 6)

DR's of BC= (x-8, y-0, 8-10)

Given A, B, C lie on same line

So values of DR's should be proportional

$$\frac{x-8}{6}=\frac{y}{3}=\frac{8-10}{6}$$

$So\ x=6, y=-1$

point is (6, -1, 8)

Introduction to 3D Coordinate Geometry 28.3 Q3

If points are collinear then all points lie on same line

and DR's should be proportional

A(2, 3, 4), B(-1, 2, -3) and C(-4, 1, -10)

DR's of AB=(3, 1, 7)

DR's of BC=(3, 1, 7)

So A, B , C are collinear

Length of $AC=\sqrt{36+4+196}=\sqrt{236}$

Length of $AB=\sqrt{9+1+49}=\sqrt{59}$

Ratio is AC:AB=2:1

So C divides AB in ratio 2:1 externally

Introduction to 3D Coordinate Geometry 28.3 Q4

yz plane means x=0

Given (2, 4, 5) and (3, 5, 4)

assume ratio to be m:n

lets equate x-term

lets equate x term

$$0=\frac{3m+2n}{m+n}$$

$$3m=-2n$$

$$m:n=-2:3$$

which means yz plane divides the line in 2:3 ratio externally

Introduction to 3D Coordinate Geometry 28.3 Q5

(2, -1, 3) and (-1, 2, 1)

$$x+y+z=5$$

Assume plane divides line in ratio $\lambda:1$

so point P which is diving line in $\lambda:1$ ratio is

$$P=\left(\frac{-\lambda+2}{\lambda+1},\frac{2\lambda-1}{\lambda+1},\frac{\lambda+3}{\lambda+1}\right)$$

P lies on plane x+y+z=5

$$-\lambda+2+2\lambda-1+\lambda+3=5\lambda+5$$

$$3\lambda=-1\Rightarrow\lambda=-1:3$$

So plane diving line in 1:3 ratio externally

Introduction to 3D Coordinate Geometry 28.3 Q6

A(3, 2, -4), B(9, 8, -10) and C(5, 4, -6)

$$AC=\sqrt{4+4+4}=2\sqrt{3}$$

$$AB=\sqrt{36+36+36}=6\sqrt{3}$$

$$BC=\sqrt{16+16+16}=4\sqrt{3}$$

$$AC:BC=1:2$$

Introduction to 3D Coordinate Geometry 28.3 Q7

Given midpoints D(-2, 3, 5), E(4, -1, 7) and F(6, 5, 3)

Assume D is midpoint of AB, E is midpoint of BC

F is midpoint of CA

$A(x_1, y_1, z_1)$ $B(x_2, y_2, z_2)$ $C(x_3, y_3, z_3)$

From midpoint formula, we get following equations

$x_1+x_2=-4;\ x_2+x_3=8;\ x_3+x_1=12$

$y_1+y_2=6;\ y_2+y_3=-2;\ y_3+y_1=10$

$z_1+z_2=10;\ z_2+z_3=14;\ z_3+z_1=6$

Solving above set of equations we get

A=(0, 9, 1)

B=(-4, -3, 9)

C=(12, 1, 5)

Introduction to 3D Coordinate Geometry 28.3 Q8

A(1, 2, 3), B(0, 4, 1), C(-1, -1, -3)

Angle bisctor at A divides BC in ratio of AB:AC

$AB=\sqrt{1+4+4} = 3$

$AC = \sqrt{4+9+36} = 7$

Assume D divides BC

m:n=3:7

so $D=\left(\frac{-3}{10}, \frac{25}{10}, \frac{-2}{10}\right)$

Introduction to 3D Coordinate Geometry 28.3 Q9

(12, -4, 8) and (27, -9, 18)

Assume point P is dividing line in λ:1 ratio, we get

$$P=\left(\frac{27\lambda+12}{\lambda+1},\frac{-9\lambda-4}{\lambda+1},\frac{18\lambda+8}{\lambda+1}\right)$$

P lies on Sphere, so substitute in Sphere equation

$$x^2+y^2+z^2=504$$

$$9(9\lambda+4)^2+(9\lambda+4)^2+4(9\lambda+4)^2=504(\lambda+1)^2$$

$$729\lambda^2+81\lambda^2+324\lambda^2+648\lambda+72\lambda+288\lambda+144+16+64=504\lambda^2+1008\lambda+504$$

$$(1134-504)\lambda^2+(1008-1008)\lambda+224-504=0$$

$$630\lambda^2=280$$

$$\lambda^2=\frac{4}{9}$$

$$\lambda=2:3$$

Introduction to 3D Coordinate Geometry 28.3 Q10

Assume ratio is $\lambda:1$

Plane is $ax+by+cz+d=0$

points (x_1,y_1,z_1) and (x_2,y_2,z_2)

Assume point of intersection of line and plane is D

$$D=\left(\frac{\lambda x_2+x_1}{\lambda+1},\frac{\lambda y_2+y_1}{\lambda+1},\frac{\lambda z_2+z_1}{\lambda+1}\right)$$

As D lies on plane, substitute D in plane equation, we get

$$\lambda(ax_2+by_2+cz_2+d)+ax_1+by_1+cz_1+d=0$$

$$\Rightarrow\lambda=-\frac{ax_1+by_1+cz_1+d}{ax_2+by_2+cz_2+d}$$

Introduction to 3D Coordinate Geometry 28.3 Q11

(1, 2, -3), (3, 0, 1) and (-1, 1, -4)

Centroid of Traingle is given by

$$\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}, \frac{z_1+z_2+z_3}{3}\right)$$

We know that

$x_1+x_2=2$

$x_2+x_3=6$

$x_1+x_3=-2$

Adding all gives $\Rightarrow 2(x_1+x_2+x_3)=6$

so $x_1+x_2+x_3=3$

similarly, $y_1+y_2+y_3=3; z_1+z_2+z_3=-6$

Centroid $=(1,1,-2)$

Introduction to 3D Coordinate Geometry 28.3 Q12

Given Centroid (1, 1, 1)

A(3, -5, 7) and B(-1, 7, -6)

Equating terms, we get

$1=\dfrac{3\text{-}1+x_3}{3}$

$1=\dfrac{-5+7+y_3}{3}$

$1=\dfrac{7-6+z_3}{3}$

$(x_3,y_3,z_3)=(1,1,2)$

Introduction to 3D Coordinate Geometry 28.3 Q13

Trisection points are those which divide line in ratio 1:2 or 2:1

P(4, 2, -6) and Q(10, -16, 6)

Consider 1:2 case, we get

$$\left(\frac{10+8}{3}, \frac{-16+4}{3}, \frac{6-12}{3}\right) = (6, -4, -2)$$

Consider 2:1 case, we get

$$\left(\frac{20+4}{3}, \frac{-32+2}{3}, \frac{12-6}{3}\right) = (8, -10, 2)$$

$(6, -4, -2)$ and $(8, -10, 2)$ are trisection points

Introduction to 3D Coordinate Geometry 28.3 Q14

A(2, -3, 4), B(-1, 2, 1) and C(0, 1/3, 2)

DR's of AB are (3, -5, 3)

DR's of BC are $\left(-1, \frac{5}{3}, -1\right)$

DR's of AC are $\left(2, \frac{-10}{3}, 2\right)$

Its clear that all DR's are proportional

Introduction to 3D Coordinate Geometry 28.3 Q15

P(3, 2, -4), Q(5, 4, -6) and R(9, 8, -10)

$PQ = \sqrt{4+4+4} = 2\sqrt{3}$

$QR = \sqrt{16+16+16} = 4\sqrt{3}$

$PQ : QR = 1 : 2$