Exercise 4.9

Q1

Evaluate:

$$\cos\left\{\sin^{-1}\left(-\frac{7}{25}\right)\right\}$$

Solution

$$\cos\left\{\sin^{-1}\left(-\frac{7}{25}\right)\right\}$$

$$= \cos\left\{-\sin^{-1}\left(\frac{7}{25}\right)\right\}\ldots\ldots\left(\sin^{-1}(-x) = -\sin^{-1}(x) \text{ for all } x \in [-1, 1]\right)$$

$$= \cos\left\{-\cos^{-1}\left(\frac{24}{25}\right)\right\}\ldots\ldots\left(\sin^{-1}\left(\frac{p}{h}\right) = \cos^{-1}\left(\frac{b}{h}\right)\right)$$

$$= \cos\left\{\cos^{-1}\left(\frac{24}{25}\right)\right\}\ldots\ldots\left(\cos(-x) = \cos x\right)$$

$$= \frac{24}{25}$$

Q2

Evaluate:

$$\sec\left\{\cot^{-1}\left(-\frac{5}{12}\right)\right\}$$

Solution

$$\sec\left\{\cot^{-1}\left(-\frac{5}{12}\right)\right\}$$

$$= \sec\left\{-\cot^{-1}\left(\frac{5}{12}\right)\right\}\ldots\ldots\left(\cot^{-1}(-x) = -\cot^{-1}(x) \text{ for all } x \in (-1, 1)\right)$$

$$= \sec\left\{-\sec^{-1}\left(\frac{13}{5}\right)\right\}\ldots\ldots\left(\cot^{-1}\left(\frac{b}{p}\right) = \sec^{-1}\left(\frac{h}{b}\right)\right)$$

$$= \sec\left\{\sec^{-1}\left(\frac{13}{5}\right)\right\}\ldots\ldots\left(\sec(-x) = \sec x\right)$$

$$= \frac{13}{5}$$

Q3

Evaluate:

$$\cot\left\{\sec^{-1}\left(-\frac{13}{25}\right)\right\}$$

Solution

$$\cot\left\{\sec^{-1}\left(-\frac{13}{25}\right)\right\}$$
$$= \cot\left\{-\sec^{-1}\left(\frac{13}{25}\right)\right\} \ldots\ldots \left\{\sec^{-1}(-x) = -\sec^{-1}(x)\right\}$$
$$= \cot\left\{-\cot^{-1}\left(\frac{5}{12}\right)\right\} \ldots\ldots \left\{\cot^{-1}\left(\frac{b}{p}\right) = \sec^{-1}\left(\frac{h}{b}\right)\right\}$$
$$= -\cot\left\{\cot^{-1}\left(\frac{5}{12}\right)\right\} \ldots\ldots \left(\cot(-x) = \cot x\right)$$
$$= \frac{5}{12}$$

Q4

Evaluate:

$$\tan\left\{\cos^{-1}\left(-\frac{7}{25}\right)\right\}$$

Solution

$$\tan\left\{\cos^{-1}\left(-\frac{7}{25}\right)\right\}$$
$$= \tan\left\{\cos^{-1}\left(\frac{7}{25}\right)\right\} \ldots\ldots \left\{\cos^{-1}(-x) = \cos^{-1}(x)\right\}$$
$$= \tan\left\{\tan^{-1}\left(\frac{24}{7}\right)\right\} \ldots\ldots \left\{\tan^{-1}\left(\frac{p}{b}\right) = \cos^{-1}\left(\frac{b}{h}\right)\right\}$$
$$= \frac{24}{7}$$

Q5

Evaluate:

$$\operatorname{cosec}\left\{\cot^{-1}\left(-\frac{12}{5}\right)\right\}$$

Solution

$$\begin{aligned}
&\operatorname{cosec}\left\{\cot^{-1}\left(-\frac{12}{5}\right)\right\} \\
&= \operatorname{cosec}\left\{-\cot^{-1}\left(\frac{12}{5}\right)\right\} \ldots\ldots\ldots \left(\cot^{-1}(-x) = -\cot^{-1}(x)\right) \\
&= \operatorname{cosec}\left\{-\operatorname{cosec}^{-1}\left(\frac{13}{12}\right)\right\} \ldots\ldots\ldots \left(\cot^{-1}\left(\frac{b}{p}\right) = \operatorname{cosec}^{-1}\left(\frac{h}{p}\right)\right) \\
&= -\operatorname{cosec}\left\{\operatorname{cosec}^{-1}\left(\frac{13}{12}\right)\right\} \ldots\ldots\ldots \left(\operatorname{cosec}(-x) = -\operatorname{cosec} x\right) \\
&= -\frac{13}{12}
\end{aligned}$$

Q6

Evaluate:

$$\cos\left\{\tan^{-1}\left(-\frac{3}{4}\right)\right\}$$

Solution

$$\begin{aligned}
&\cos\left\{\tan^{-1}\left(-\frac{3}{4}\right)\right\} \\
&= \cos\left\{-\tan^{-1}\left(\frac{3}{4}\right)\right\} \ldots\ldots\ldots \left(\tan^{-1}(-x) = -\tan^{-1}(x)\right) \\
&= \cos\left\{-\cos^{-1}\left(\frac{4}{5}\right)\right\} \ldots\ldots\ldots \left(\tan^{-1}\left(\frac{p}{b}\right) = \cos^{-1}\left(\frac{b}{h}\right)\right) \\
&= \cos\left\{\cos^{-1}\left(\frac{4}{5}\right)\right\} \ldots\ldots\ldots \left(\cos(-x) = \cos x\right) \\
&= \frac{4}{5}
\end{aligned}$$

Q7

Evaluate:

$$\sin\left\{\cos^{-1}\left(-\frac{3}{5}\right) + \cot^{-1}\left(-\frac{5}{12}\right)\right\}.$$

Solution

$$\sin\left\{\cos^{-1}\left(-\frac{3}{5}\right)+\cot^{-1}\left(-\frac{5}{12}\right)\right\}$$

$$=\sin\left\{\cos^{-1}\left(\frac{3}{5}\right)-\cot^{-1}\left(\frac{5}{12}\right)\right\}\ldots\ldots\begin{pmatrix}\cos^{-1}(-x)=\cos^{-1}(x)\\ \cot^{-1}(-x)=-\cot^{-1}(x)\end{pmatrix}$$

$$=\sin\left\{\tan^{-1}\left(\frac{4}{3}\right)-\tan^{-1}\left(\frac{12}{5}\right)\right\}\ldots\ldots\begin{pmatrix}\cot^{-1}\left(\frac{b}{p}\right)=\tan^{-1}\left(\frac{p}{b}\right)\\ \tan^{-1}\left(\frac{p}{b}\right)=\cos^{-1}\left(\frac{b}{h}\right)\end{pmatrix}$$

$$=\sin\left\{\tan^{-1}\left(\frac{\frac{4}{3}-\frac{12}{5}}{1+\frac{4}{3}\times\frac{12}{5}}\right)\right\}\ldots\ldots\left(\tan^{-1}(x)-\tan^{-1}(y)=\tan^{-1}\left(\frac{x-y}{1+xy}\right)\right)$$

$$=\sin\left\{\tan^{-1}\left(-\frac{16}{63}\right)\right\}$$

$$=\sin\left\{-\tan^{-1}\left(-\frac{16}{63}\right)\right\}\ldots\ldots\left(\tan^{-1}(-x)=-\tan^{-1}(x)\right)$$

$$=\sin\left\{-\sin^{-1}\left(\frac{16}{65}\right)\right\}\ldots\ldots\left(\tan^{-1}\left(\frac{b}{p}\right)=\sin^{-1}\left(\frac{b}{h}\right)\right)$$

$$=-\sin\left\{\sin^{-1}\left(\frac{16}{65}\right)\right\}\ldots\ldots(\sin(-x)=-\sin(x))$$

$$=-\frac{16}{65}$$