

## Exercise 4.6

## Q1

Find the principal value of  $\cot^{-1}(-\sqrt{3})$

## Solution

We know that, for any  $x \in \mathbb{R}$ ,  $\cot^{-1}x$  represents an angle in  $(0, \pi)$

$$\begin{aligned}\cot^{-1}(-\sqrt{3}) &= \text{An angle in } (0, \pi) \text{ whose cotangent is } (-\sqrt{3}) \\ &= \pi - \frac{\pi}{6} \\ &= \frac{5\pi}{6}\end{aligned}$$

$$\therefore \cot^{-1}(-\sqrt{3}) = \frac{5\pi}{6}$$

## Q2

Find the principal value of each of the following:

$$\cot^{-1}(\sqrt{3})$$

## Solution

$\cot^{-1}x$  represents an angle in  $(0, \pi)$  whose cotangent is  $x$ .

$$\text{Let } x = \cot^{-1}(\sqrt{3})$$

$$\Rightarrow \cot x = \sqrt{3} = \cot\left(\frac{\pi}{6}\right)$$

$$\Rightarrow x = \frac{\pi}{6}$$

$\therefore$  Principal value of  $\cot^{-1}(\sqrt{3})$  is  $\frac{\pi}{6}$ .

## Q3

Find the principal value of each of the following:

$$\cot^{-1}\left(-\frac{1}{\sqrt{3}}\right)$$

## Solution

$\cot^{-1}x$  represents an angle in  $(0, \pi)$  whose cotangent is  $x$ .

$$\text{Let } x = \cot^{-1}\left(-\frac{1}{\sqrt{3}}\right)$$

$$\Rightarrow \cot x = -\frac{1}{\sqrt{3}} = \cot\left(\frac{2\pi}{3}\right)$$

$$\Rightarrow x = \frac{2\pi}{3}$$

$\therefore$  Principal value of  $\cot^{-1}(\sqrt{3})$  is  $\frac{2\pi}{3}$ .

#### Q4

Find the principal value of each of the following:

$$\cot^{-1}\left(\tan\frac{3\pi}{4}\right)$$

#### Solution

$$\cot^{-1}\left(\tan\frac{3\pi}{4}\right) = \cot^{-1}(-1)$$

$\cot^{-1}x$  represents an angle in  $(0, \pi)$  whose cotangent is  $x$ .

$$\text{Let } x = \cot^{-1}(-1)$$

$$\Rightarrow \cot x = -1 = \cot\left(\frac{3\pi}{4}\right)$$

$$\Rightarrow x = \frac{3\pi}{4}$$

$\therefore$  Principal value of  $\cot^{-1}(-1)$  is  $\frac{3\pi}{4}$ .

#### Q5

Find the domain of  $f(x) = \cot x + \cot^{-1} x$ .

#### Solution

Domain of  $\cot x$  is  $(0, \pi)$ .

Domain of  $\cot^{-1}x$  is  $\mathbb{R}$ .

So domain of  $\cot x + \cot^{-1}x$  is  $\mathbb{R}$ .

#### Q6

Evaluate each of the following:

$$\cot^{-1}\frac{1}{\sqrt{3}} - \operatorname{cosec}^{-1}(-2) + \sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$$

**Solution**

$$\begin{aligned}
 & \cot^{-1} \frac{1}{\sqrt{3}} - \operatorname{cosec}^{-1}(-2) + \sec^{-1} \left( \frac{2}{\sqrt{3}} \right) \\
 &= \frac{\pi}{6} - \left( -\frac{\pi}{6} \right) + \frac{\pi}{3} \\
 &= \frac{\pi}{6} + \frac{\pi}{6} + \frac{\pi}{3} \\
 &= \frac{4\pi}{6} \\
 &= \frac{2\pi}{3}
 \end{aligned}$$

**Q7**

Evaluate each of the following

$$\cot^{-1} \left\{ 2 \cos \left( \sin^{-1} \frac{\sqrt{3}}{2} \right) \right\}$$

**Solution**

$$\begin{aligned}
 & \cot^{-1} \left\{ 2 \cos \left( \sin^{-1} \frac{\sqrt{3}}{2} \right) \right\} \\
 &= \cot^{-1} \left\{ 2 \cos \left( \frac{\pi}{3} \right) \right\} \\
 &= \cot^{-1} \left\{ 2 \times \frac{1}{2} \right\} \\
 &= \cot^{-1} (1) \\
 &= \frac{\pi}{4}
 \end{aligned}$$

**Q8**

Evaluate each of the following:

$$\operatorname{cosec}^{-1} \left( -\frac{2}{\sqrt{3}} \right) + 2 \cot^{-1} (-1)$$

**Solution**

$$\begin{aligned} & \cos \sec^{-1} \left( -\frac{2}{\sqrt{3}} \right) + 2 \cot^{-1} (-1) \\ &= -\frac{\pi}{3} + 2 \times \left( \frac{3\pi}{4} \right) \\ &= -\frac{\pi}{3} + \frac{3\pi}{2} \\ &= \frac{7\pi}{6} \end{aligned}$$

**Q9**

Evaluate each of the following

$$\tan^{-1} \left( -\frac{1}{\sqrt{3}} \right) + \cot^{-1} \left( \frac{1}{\sqrt{3}} \right) + \tan^{-1} \left( \sin \left[ -\frac{\pi}{2} \right] \right)$$

**Solution**

$$\begin{aligned} & \tan^{-1} \left( -\frac{1}{\sqrt{3}} \right) + \cot^{-1} \left( \frac{1}{\sqrt{3}} \right) + \tan^{-1} \left( \sin \left[ -\frac{\pi}{2} \right] \right) \\ &= -\frac{\pi}{6} + \frac{\pi}{3} + \tan^{-1} (-1) \\ &= -\frac{\pi}{6} + \frac{\pi}{3} - \frac{\pi}{4} \\ &= -\frac{\pi}{12} \end{aligned}$$

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