Exercise 4.5

Q1

Find the principal value of $\text{cosec}^{-1}\left(-\sqrt{2}\right)$

Solution

Let $\text{cosec}^{-1}\left(-\sqrt{2}\right)=y$. Then, $\text{cosec}\, y=-\sqrt{2}=-\text{cosec}\left(\frac{\pi}{4}\right)=\text{cosec}\left(-\frac{\pi}{4}\right)$.

We know that the range of the principal value branch of cosec^{-1} is $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]-\{0\}$ and $\text{cosec}\left(-\frac{\pi}{4}\right)=-\sqrt{2}$.

Therefore, the principal value of $\text{cosec}^{-1}\left(-\sqrt{2}\right)$ is $-\frac{\pi}{4}$.

Q2

Find the principal value of each of the following:

$\text{cosec}^{-1}(-2)$

Solution

$\text{cosec}^{-1}x$ represents an angle in $\left[-\frac{\pi}{2},0\right)\cup\left(0,\frac{\pi}{2}\right]$ whose cosecant is x.

Let $x=\text{cosec}^{-1}(-2)$

$\Rightarrow \text{cosec}\, x=-2=\text{cosec}\left(-\frac{\pi}{6}\right)$

$\Rightarrow x=-\frac{\pi}{6}$

$\therefore$ Principal value of $\text{cosec}^{-1}(-2)$ is $-\frac{\pi}{6}$.

Q3

Find the principal value of $\text{cosec}^{-1}\left(\frac{2}{\sqrt{3}}\right)$

Solution

We know that, for any $x \in R$, $\text{cosec}^{-1}x$ is an angle in $\left[\frac{-\pi}{2}, 0\right) \cup \left(0, \frac{\pi}{2}\right]$

$\text{cosec}^{-1}\left(\frac{2}{\sqrt{3}}\right)$ = An angle is $\left[\frac{-\pi}{2}, 0\right) \cup \left(0, \frac{\pi}{2}\right]$ whose cosecant is $\left(\frac{2}{\sqrt{3}}\right)$

$= \frac{\pi}{3}$

$\therefore \text{cosec}^{-1}\left(\frac{2}{\sqrt{3}}\right) = \frac{\pi}{3}$.

Q4

Find the principal value of each of the following:

$\text{cosec}^{-1}\left(2\cos\frac{2\pi}{3}\right)$

Solution

$\text{cosec}^{-1}x$ represents an angle in $\left[-\frac{\pi}{2}, 0\right) \cup \left(0, \frac{\pi}{2}\right]$ whose cosecant is x.

Let $x = \text{cosec}^{-1}\left(2\cos\frac{2\pi}{3}\right)$

$\Rightarrow \text{cosec}\, x = 2\cos\frac{2\pi}{3} = 2\times\left(-\frac{1}{2}\right) = -1 = \text{cosec}\left(-\frac{\pi}{2}\right)$

$\Rightarrow x = -\frac{\pi}{2}$

$\therefore$ Principal value of $\text{cosec}^{-1}\left(2\cos\frac{2\pi}{3}\right)$ is $-\frac{\pi}{2}$.

Q5

Find the set of values of

$\text{cosec}^{-1}\left(\frac{\sqrt{3}}{2}\right)$

Solution

$\text{cosec}^{-1}x$ represents an angle in $\left[-\frac{\pi}{2}, 0\right) \cup \left(0, \frac{\pi}{2}\right]$ whose cosecant is x.

Domain of $\text{cosec}^{-1}x$ is $(-\infty, -1] \cup [1, \infty)$.

$\frac{\sqrt{3}}{2} \notin (-\infty, -1] \cup [1, \infty)$

Hence $\text{cosec}^{-1}\left(\frac{\sqrt{3}}{2}\right)$ does not exist or its ϕ.

Q6

For the principal value evaluate the following:

$$\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)+\operatorname{cosec}^{-1}\left(-\frac{2}{\sqrt{3}}\right)$$

Solution

$\sin^{-1}x$ represents an angle in $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ whose sine is x.

Let $x=\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

$\Rightarrow \sin x=-\frac{\sqrt{3}}{2}=\sin\left(-\frac{\pi}{3}\right)$

$\operatorname{cosec}^{-1}x$ represents an angle in $\left[-\frac{\pi}{2},0\right)\cup\left(0,\frac{\pi}{2}\right]$ whose cosecant is x.

Let $x=\operatorname{cosec}^{-1}\left(-\frac{2}{\sqrt{3}}\right)$

$\Rightarrow \operatorname{cosec}x=-\frac{2}{\sqrt{3}}=\operatorname{cosec}\left(-\frac{\pi}{3}\right)$

$\therefore \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)+\operatorname{cosec}^{-1}\left(-\frac{2}{\sqrt{3}}\right)$

$=-\frac{\pi}{3}--\frac{\pi}{3}$

$=-\frac{2\pi}{3}$

$\therefore$ Principal value of $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)+\operatorname{cosec}^{-1}\left(-\frac{2}{\sqrt{3}}\right)$ is $-\frac{2\pi}{3}$.

Q7

For the principal value evaluate the following:

$$\sec^{-1}\left(\sqrt{2}\right)+2\operatorname{cosec}^{-1}\left(-\sqrt{2}\right)$$

Solution

$\sec^{-1}x$ represents an angle in $[0, \pi]-\left\{\frac{\pi}{2}\right\}$ whose secant is x.

Let $x = \sec^{-1}\left(\sqrt{2}\right)$

$\Rightarrow \sec x = \sqrt{2} = \sec\left(\frac{\pi}{4}\right)$

$\text{cosec}^{-1}x$ represents an angle in $\left[-\frac{\pi}{2}, 0\right)\cup\left(0, \frac{\pi}{2}\right]$ whose cosecant is x.

Let $x = \text{cosec}^{-1}\left(-\sqrt{2}\right)$

$\Rightarrow \text{cosec}\, x = -\sqrt{2} = \text{cosec}\left(-\frac{\pi}{4}\right)$

$\therefore \sec^{-1}\left(\sqrt{2}\right)+2\text{cosec}^{-1}\left(-\sqrt{2}\right)$

$= \frac{\pi}{4}+2\times\left(-\frac{\pi}{4}\right)$

$= -\frac{\pi}{4}$

$\therefore$ Principal value of $\sec^{-1}\left(\sqrt{2}\right)+2\text{cosec}^{-1}\left(-\sqrt{2}\right)$ is $-\frac{\pi}{4}$.

Q8

For the principal value evaluate the following:

$\sin^{-1}\left[\cos\left\{2\,\text{cosec}^{-1}\left(-2\right)\right\}\right]$

Solution

$\text{cosec}^{-1}x$ represents an angle in $\left[-\frac{\pi}{2},0\right)\cup\left(0,\frac{\pi}{2}\right]$ whose cosecant is x.

Let $x = \text{cosec}^{-1}(-2)$

$\Rightarrow \text{cosec}\,x = -2 = \text{cosec}\left(-\frac{\pi}{6}\right)$

$\Rightarrow x = -\frac{\pi}{6}$

$\sin^{-1}\left[\cos\left\{2\,\text{cosec}^{-1}(-2)\right\}\right] = \sin^{-1}\left[\cos\left\{2\times\left(-\frac{\pi}{6}\right)\right\}\right] = \sin^{-1}\left[\cos\left(-\frac{\pi}{3}\right)\right] = \sin^{-1}\left[\frac{1}{2}\right]$

$\sin^{-1}x$ represents an angle in $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ whose sine is x.

Let $x = \sin^{-1}\left[\frac{1}{2}\right]$

$\Rightarrow \sin x = \frac{1}{2} = \sin\left(\frac{\pi}{6}\right)$

$\Rightarrow x = \frac{\pi}{6}$

$\therefore$ Principal value of $\sin^{-1}\left[\cos\left\{2\,\text{cosec}^{-1}(-2)\right\}\right]$ is $\frac{\pi}{6}$.

Q9

For the principal value evaluate the following:

$\text{cosec}^{-1}\left(2\tan\frac{11\pi}{6}\right)$

Solution

$\text{cosec}^{-1}\left(2\tan\frac{11\pi}{6}\right) = \text{cosec}^{-1}\left(2\tan\left(\frac{11\pi}{6}\right)\right) = \text{cosec}^{-1}\left(2\tan\left(\frac{3\pi}{2}+\frac{\pi}{3}\right)\right) = \text{cosec}^{-1}\left(2\times\left(-\frac{1}{\sqrt{3}}\right)\right)$

$\text{cosec}^{-1}x$ represents an angle in $\left[-\frac{\pi}{2},0\right)\cup\left(0,\frac{\pi}{2}\right]$ whose cosecant is x.

Let $x = \text{cosec}^{-1}\left(-\frac{2}{\sqrt{3}}\right)$

$\Rightarrow \text{cosec}\,x = -\frac{2}{\sqrt{3}} = \text{cosec}\left(-\frac{\pi}{3}\right)$

$\Rightarrow x = -\frac{\pi}{3}$

$\therefore$ Principal value of $\text{cosec}^{-1}\left(2\tan\frac{11\pi}{6}\right)$ is $-\frac{\pi}{3}$.