Exercise 4.4

Q1

Find the principal value of $Sec^{-1}\left(-\sqrt{2}\right)$

Solution

We know that, for $x \in R$, $sec^{-1}x$ represents an angle in $[0, \pi] - \left\{\frac{\pi}{2}\right\}$.

$\sec^{-1}\left(-\sqrt{2}\right)$ = An angle in $[0, \pi] - \left\{\frac{\pi}{2}\right\}$ whose secant is $\left(-\sqrt{2}\right)$

$$= \pi - \frac{\pi}{4}$$

$$= \frac{3\pi}{4}$$

$\sec^{-1}\left(-\sqrt{2}\right) = \frac{3\pi}{4}$.

Q2

Find the principal value of $\sec^{-1}(2)$

Solution

We know that, for any $x \in R$, $\sec^{-1}x$ represents an angle in $[0, \pi] - \left\{\frac{\pi}{2}\right\}$.

$\sec^{-1}(2)$ = An angle is $[0, \pi] - \left\{\frac{\pi}{2}\right\}$ whose secant is 2

$$= \frac{\pi}{3}$$

$\therefore \sec^{-1}(2) = \frac{\pi}{3}$.

Q3

Find the principal value of each of the following:

$$\sec^{-1}\left(2\sin\frac{3\pi}{4}\right)$$

Solution

$\sec^{-1}\left(2\sin\frac{3\pi}{4}\right) = \sec^{-1}\left(2\times\left(\frac{1}{\sqrt{2}}\right)\right) = \sec^{-1}\left(\sqrt{2}\right)$

We know that for any $x \in R-(-1, 1)$, $\sec^{-1}x$ represents an angle in $[0,\pi]-\left\{\frac{\pi}{2}\right\}$ whose secant is x.

$\therefore \sec^{-1}\left(\sqrt{2}\right) = \frac{\pi}{4}$

$\therefore$ Principle value of $\sec^{-1}\left(2\sin\frac{3\pi}{4}\right)$ is $\frac{\pi}{4}$.

Q4

Find the principal value of each of the following:

$\sec^{-1}\left(2\tan\frac{3\pi}{4}\right)$

Solution

$\sec^{-1}\left(2\tan\frac{3\pi}{4}\right) = \sec^{-1}\left(2\times(-1)\right) = \sec^{-1}(-2)$

We know that for any $x \in R-(-1, 1)$, $\sec^{-1}x$ represents an angle in $[0,\pi]-\left\{\frac{\pi}{2}\right\}$ whose secant is x.

$\therefore \sec^{-1}(-2) = \frac{2\pi}{3}$

$\therefore$ Principle value of $\sec^{-1}\left(2\tan\frac{3\pi}{4}\right)$ is $\frac{2\pi}{3}$.

Q5

For the principal values, evaluate $\tan^{-1}\left(\sqrt{3}\right) - \sec^{-1}(-2)$

Solution

Let $\tan^{-1}\left(\sqrt{3}\right) = x$. Then, $\tan x = \sqrt{3} = \tan\left(\frac{\pi}{3}\right)$

$\therefore \tan^{-1}\left(\sqrt{3}\right) = \frac{\pi}{3}$

Let $\sec^{-1}(-2) = y$. Then, $\sec y = -2 = \sec\left(\pi - \frac{\pi}{3}\right)$

$\therefore \sec^{-1}(-2) = \frac{2\pi}{3}$

$\therefore \tan^{-1}\left(\sqrt{3}\right) - \sec^{-1}(-2) = \frac{\pi}{3} - \frac{2\pi}{3} = \frac{\pi - 2\pi}{3} = -\frac{\pi}{3}$

Q6

Find the principal value of each of the following:

$$\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)-2\sec^{-1}\left(2\tan\frac{\pi}{6}\right)$$

Solution

$$\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)-2\sec^{-1}\left(2\tan\frac{\pi}{6}\right)=\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)-2\sec^{-1}\left(2\times\frac{1}{\sqrt{3}}\right)=\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)-2\sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$$

We know that for any $x\in[-1,1]$, $\sin^{-1}x$ represents an angle in $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ whose secant is x.

$$\therefore\ \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)=-\frac{\pi}{3}$$

We know that for any $x\in R-(-1,\ 1)$, $\sec^{-1}x$ represents an angle in $[0,\pi]-\left\{\frac{\pi}{2}\right\}$ whose secant is x.

$$\therefore\ \sec^{-1}\left(\frac{2}{\sqrt{3}}\right)=\frac{\pi}{6}$$

$$\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)-2\sec^{-1}\left(\frac{2}{\sqrt{3}}\right)=-\frac{\pi}{3}-2\times\frac{\pi}{6}=-\frac{2\pi}{3}$$

$\therefore$ Principle value of $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)-2\sec^{-1}\left(2\tan\frac{\pi}{6}\right)$ is $-\frac{2\pi}{3}$

Q7

Find the domain of

$\sec^{-1}(3x-1)$

Solution

Domain of $\sec^{-1}x$ lies in the interval $(-\infty,-1]\cup[1,\infty)$.

$\therefore$ Domain of $\sec^{-1}(3x-1)$ lies in the interval $(-\infty,-1]\cup[1,\infty)$

$\Rightarrow -\infty\le 3x-1\le -1$ and $1\le 3x-1\le\infty$

$\Rightarrow -\infty\le 3x\le 0$ and $2\le 3x\le\infty$

$\Rightarrow -\infty\le x\le 0$ and $\frac{2}{3}\le x\le\infty$

Domain of $\sec^{-1}x$ lies in the interval $(-\infty,0]\cup\left[\frac{2}{3},\infty\right)$.

Q8

Find the domain of

$\sec^{-1}x-\tan^{-1}x$

Solution

Domain of $\sec^{-1}x$ lies in the interval $(-\infty, -1] \cup [1, \infty)$.
Domain of $\tan^{-1}x$ lies is R

Domain of $\sec^{-1}x - \tan^{-1}x\left(x^2 - 4\right)$ is $(-\infty, -1] \cup [1, \infty)$.