Exercise 4.14

Q1

Evaluate $\tan\left\{2\tan^{-1}\frac{1}{5}-\frac{\pi}{4}\right\}$

Solution

$$\tan\left\{2\tan^{-1}\frac{1}{5}-\frac{\pi}{4}\right\}$$

$$=\tan\left\{\tan^{-1}\frac{2\times\left(\frac{1}{5}\right)}{1-\left(\frac{1}{5}\right)^2}-\tan^{-1}(1)\right\} \qquad \left\{\text{Sicne } 2\tan^{-1}=\tan^{-1}\left(\frac{2x}{1-x^2}\right)\right\}$$

$$=\tan\left\{\tan^{-1}\frac{\frac{2}{5}}{\frac{24}{25}}-\tan^{-1}(1)\right\}$$

$$=\tan\left\{\tan^{-1}\frac{5}{12}-\tan^{-1}1\right\}$$

$$=\tan\left\{\tan^{-1}\left(\frac{\frac{5}{12}-1}{1+\frac{5}{12}\times 1}\right)\right\} \qquad \left\{\text{Since } \tan^{-1}x-\tan^{-1}y=\tan^{-1}\left(\frac{x-y}{1+xy}\right)\right\}$$

$$=\tan\left\{\tan^{-1}\left(\frac{\frac{-7}{12}}{\frac{17}{12}}\right)\right\}$$

$$=\tan\left\{\tan^{-1}\left(-\frac{7}{17}\right)\right\}$$

$$=-\frac{7}{17} \qquad \left\{\text{Since } \tan\left(\tan^{-1}x\right)=x \text{ if } x\in R\right\}$$

Hence,

$$\tan\left\{2\tan^{-1}\frac{1}{5}-\frac{\pi}{4}\right\}=-\frac{7}{17}$$

Q2

Evaluate the following

$\tan\left(\frac{1}{2}\sin^{-1}\frac{3}{4}\right)$

Solution

$Let\ \frac{1}{2}\sin^{-1}\frac{3}{4} = x$

$\sin^{-1}\frac{3}{4} = 2x$

$\sin 2x = \frac{3}{4}$

$\cos 2x = \frac{\sqrt{7}}{4}$

$\tan\left(\frac{1}{2}\sin^{-1}\frac{3}{4}\right)$

$= \tan x$

$= \sqrt{\frac{1-\cos 2x}{1+\cos 2x}}$

$= \sqrt{\frac{1-\frac{\sqrt{7}}{4}}{1+\frac{\sqrt{7}}{4}}}$

$= \sqrt{\frac{4-\sqrt{7}}{4+\sqrt{7}}}$

$= \sqrt{\frac{(4-\sqrt{7})(4-\sqrt{7})}{(4+\sqrt{7})(4-\sqrt{7})}}$

$= \sqrt{\frac{(4-\sqrt{7})^2}{9}}$

$= \frac{4-\sqrt{7}}{3}$

Q3

Evaluate $\sin\left(\frac{1}{2}\cos^{-1}\frac{4}{5}\right)$

Solution

$\sin\left(\frac{1}{2}\cos^{-1}\frac{4}{5}\right)$

$= \sin\left\{\frac{1}{2}2\sin^{-1}\left(\pm\sqrt{\frac{1-\frac{4}{5}}{2}}\right)\right\}$ $\left\{\text{Since } \cos^{-1}x = 2\sin^{-1}\left(\pm\sqrt{\frac{1-x}{2}}\right)\right\}$

$= \sin\left\{\sin^{-1}\left(\pm\frac{1}{\sqrt{10}}\right)\right\}$

$= \pm\frac{1}{\sqrt{10}}$ $\{\text{Since } \sin(\sin^{-1}x) = x \text{ as } n \in [-1, 1]\}$

Hence,

$\sin\left(\frac{1}{2}\cos^{-1}\frac{4}{5}\right) = \pm\frac{1}{\sqrt{10}}$

Q4

Evaluate the following:

$$\sin\left(2\tan^{-1}\frac{2}{3}\right)+\cos\left(\tan^{-1}\sqrt{3}\right)$$

Solution

$$\sin\left(2\tan^{-1}\frac{2}{3}\right)+\cos\left(\tan^{-1}\sqrt{3}\right)$$

$$=\sin\left(\sin^{-1}\left(\frac{\frac{4}{3}}{1+\frac{4}{9}}\right)\right)+\cos\left(\cos^{-1}\left(\frac{1}{\sqrt{1+3}}\right)\right)$$

$$=\sin\left(\sin^{-1}\left(\frac{12}{13}\right)\right)+\cos\left(\cos^{-1}\left(\frac{1}{2}\right)\right)$$

$$=\frac{12}{13}+\frac{1}{2}$$

$$=\frac{37}{26}$$

Q5

Prove that $2\sin^{-1}\frac{3}{5}=\tan^{-1}\left(\frac{24}{7}\right)$

Solution

$2\sin^{-1}\frac{3}{5} = \tan^{-1}\left(\frac{24}{7}\right)$

LHS $= 2\sin^{-1}\frac{3}{5}$

$= 2\times\tan^{-1}\left(\frac{\frac{3}{5}}{\sqrt{1-\left(\frac{3}{5}\right)^2}}\right)$ $\left\{\text{Since } \sin^{-1}x = \tan^{-1}\frac{x}{\sqrt{1-x^2}}\right\}$

$= 2\tan^{-1}\left(\frac{\frac{3}{5}}{\frac{4}{5}}\right)$

$= 2\tan^{-1}\left(\frac{3}{4}\right)$

$= \tan^{-1}\left(\frac{2\times\frac{3}{4}}{1-\left(\frac{3}{4}\right)^2}\right)$ $\left\{\text{Since } 2\tan^{-1}x = \tan^{-1}\frac{2x}{1-x^2}\right\}$

$= \tan^{-1}\left(\frac{\frac{3}{2}}{\frac{7}{16}}\right)$

$= \tan^{-1}\left(\frac{24}{7}\right)$

= RHS

So,

$2\sin^{-1}\left(\frac{3}{5}\right) = \tan^{-1}\left(\frac{24}{7}\right)$

Q6

Prove that $\tan^{-1}\left(\frac{1}{4}\right) + \tan^{-1}\left(\frac{2}{9}\right) = \frac{1}{2}\cos^{-1}\left(\frac{3}{5}\right) = \frac{1}{2}\sin^{-1}\left(\frac{4}{5}\right)$

Solution

$$\tan^{-1}\left(\frac{1}{4}\right)+\tan^{-1}\left(\frac{2}{9}\right)=\frac{1}{2}\cos^{-1}\left(\frac{3}{5}\right)=\frac{1}{2}\sin^{-1}\left(\frac{4}{5}\right)$$

LHS $=\tan^{-1}\left(\frac{1}{4}\right)+\tan^{-1}\left(\frac{2}{9}\right)$

$=\tan^{-1}\left(\frac{\frac{1}{4}+\frac{2}{9}}{1-\frac{1}{4}\times\frac{2}{9}}\right)$ $\left\{\text{Since } \tan^{-1}x+\tan^{-1}y=\tan^{-1}\left(\frac{x+y}{1-xy}\right)\right\}$

$=\tan^{-1}\left(\frac{\frac{9+8}{36}}{\frac{36-2}{36}}\right)$

$=\tan^{-1}\left(\frac{17}{36}\times\frac{36}{34}\right)$

$=\tan^{-1}\left(\frac{1}{2}\right)$

Multiplying and dividing by 2,

$=\frac{1}{2}\left\{2\tan^{-1}\left(\frac{1}{2}\right)\right\}$

$=\frac{1}{2}\left\{\cos^{-1}\left(\frac{1-\left(\frac{1}{2}\right)^2}{1+\left(\frac{1}{2}\right)^2}\right)\right\}$ $\left\{\text{Since } 2\tan^{-1}x=\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)\right\}$

$=\frac{1}{2}\left\{\cos^{-1}\left(\frac{1-\frac{1}{4}}{1+\frac{1}{4}}\right)\right\}$

$=\frac{1}{2}\left\{\cos^{-1}\left(\frac{3}{4}\times\frac{4}{5}\right)\right\}$

$=\frac{1}{2}\cos^{-1}\left(\frac{3}{5}\right)$

So,

$\tan^{-1}\left(\frac{1}{4}\right)+\tan^{-1}\left(\frac{2}{9}\right)=\frac{1}{2}\cos^{-1}\left(\frac{3}{5}\right)$

$=\frac{1}{2}\cos^{-1}\left(\frac{3}{5}\right)$

$=\frac{1}{2}\sin^{-1}\left(\frac{4}{5}\right)$ $\left\{\text{Since } \cos^{-1}x=\sin^{-1}\left(\sqrt{1-x^2}\right)\right\}$

So,

$\tan^{-1}\left(\frac{1}{4}\right)+\tan^{-1}\left(\frac{2}{9}\right)=\frac{1}{2}\sin^{-1}\left(\frac{4}{5}\right)$

Q7

Prove that $\tan^{-1}\left(\frac{2}{3}\right)=\frac{1}{2}\tan^{-1}\left(\frac{12}{5}\right)$

Solution

$\tan^{-1}\left(\frac{2}{3}\right) = \frac{1}{2}\tan^{-1}\left(\frac{12}{5}\right)$

LHS $= \tan^{-1}\left(\frac{2}{3}\right)$

Dividing and multiplying by 2,

$= \frac{1}{2}\left\{2\tan^{-1}\left(\frac{2}{3}\right)\right\}$

$= \frac{1}{2}\left\{\tan^{-1}\left(\frac{2\left(\frac{2}{3}\right)}{1-\left(\frac{2}{3}\right)^2}\right)\right\}$ $\left\{\text{Since } 2\tan^{-1}x = \tan^{-1}\left(\frac{2x}{1-x^2}\right)\right\}$

$= \frac{1}{2}\tan^{-1}\left(\frac{\frac{4}{3}}{\frac{5}{9}}\right)$

$= \frac{1}{2}\tan^{-1}\left(\frac{4}{3}\times\frac{9}{5}\right)$

$= \frac{1}{2}\tan^{-1}\left(\frac{12}{5}\right)$

$\tan^{-1}\left(\frac{2}{3}\right) = \frac{1}{2}\tan^{-1}\left(\frac{12}{5}\right)$

Q8

Prove that $\tan^{-1}\left(\frac{1}{7}\right) + 2\tan^{-1}\left(\frac{1}{3}\right) = \frac{\pi}{4}$

Solution

$\tan^{-1}\left(\frac{1}{7}\right)+2\tan^{-1}\left(\frac{1}{3}\right)=\frac{\pi}{4}$

LHS $=\tan^{-1}\left(\frac{1}{7}\right)+2\tan^{-1}\left(\frac{1}{3}\right)$

$=\tan^{-1}\left(\frac{1}{7}\right)+\tan^{-1}\left(\frac{2\left(\frac{1}{3}\right)}{1-\left(\frac{1}{3}\right)^2}\right)$ $\left\{\text{Since } 2\tan^{-1}x=\tan^{-1}\left(\frac{2x}{1-x^2}\right)\right\}$

$=\tan^{-1}\left(\frac{1}{7}\right)+\tan^{-1}\left(\frac{2}{3}\times\frac{9}{8}\right)$

$=\tan^{-1}\left(\frac{1}{7}\right)+\tan^{-1}\left(\frac{3}{4}\right)$

$=\tan^{-1}\left(\frac{\frac{1}{7}+\frac{3}{4}}{1-\frac{1}{7}\times\frac{3}{4}}\right)$ $\left\{\text{Since } \tan^{-1}x+\tan^{-1}y=\tan^{-1}\left(\frac{x+y}{1-xy}\right)\right\}$

$=\tan^{-1}\left(\frac{\frac{25}{28}}{\frac{25}{28}}\right)$

$=\tan^{-1}(1)$

$=\frac{\pi}{4}$

= RHS Hence, proved

$\tan^{-1}\left(\frac{1}{7}\right)+2\tan^{-1}\left(\frac{1}{3}\right)=\frac{\pi}{4}$

Q9

Prove that $\sin^{-1}\frac{4}{5}+2\tan^{-1}\frac{1}{3}=\frac{\pi}{2}$

Solution

$\sin^{-1}\frac{4}{5}+2\tan^{-1}\frac{1}{3}=\frac{\pi}{2}$

LHS $=\sin^{-1}\frac{4}{5}+2\tan^{-1}\left(\frac{1}{3}\right)$

$=\tan^{-1}\left(\frac{\frac{4}{5}}{\sqrt{1-\left(\frac{4}{5}\right)^2}}\right)+\tan^{-1}\left(\frac{2\left(\frac{1}{3}\right)}{1-\left(\frac{1}{3}\right)^2}\right)$

$\left\{\text{Since } \sin^{-1}x=\tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right) \text{ and } 2\tan^{-1}x=\tan^{-1}\left(\frac{2x}{1-x^2}\right)\right\}$

$=\tan^{-1}\left(\frac{\frac{4}{5}}{\frac{3}{5}9}\right)+\tan^{-1}\left(\frac{\frac{2}{3}}{\frac{8}{9}}\right)$

$=\tan^{-1}\left(\frac{4}{3}\right)+\tan^{-1}\left(\frac{3}{4}\right)$

$=\tan^{-1}\left(\frac{\frac{4}{3}+\frac{3}{4}}{1-\frac{4}{3}\times\frac{3}{4}}\right)$

$=\tan^{-1}\left(\frac{\frac{25}{12}}{0}\right)$

$=\tan^{-1}(\infty)$

$=\frac{\pi}{2}$

$=$ RHS

So,

$\sin^{-1}\frac{4}{5}+2\tan^{-1}\frac{1}{3}=\frac{\pi}{2}$

Q10

Prove that $2\sin^{-1}\frac{3}{5}-\tan^{-1}\frac{17}{31}=\frac{\pi}{4}$

Solution

$2\sin^{-1}\frac{3}{5}-\tan^{-1}\frac{17}{31}=\frac{\pi}{4}$

LHS $=2\sin^{-1}\left(\frac{3}{5}\right)-\tan^{-1}\left(\frac{17}{31}\right)$

$=2\tan^{-1}\left(\frac{\frac{3}{5}}{\sqrt{1-\left(\frac{3}{5}\right)^2}}\right)-\tan^{-1}\left(\frac{17}{31}\right)$

$=2\tan^{-1}\left(\frac{\frac{3}{5}}{\frac{4}{5}}\right)-\tan^{-1}\left(\frac{17}{31}\right)$ $\left\{\text{Since } \sin^{-1}x=\tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)\right\}$

$=2\tan^{-1}\left(\frac{3}{4}\right)-\tan^{-1}\left(\frac{17}{31}\right)$

$=\tan^{-1}\left(\frac{2\times\frac{3}{4}}{1-\left(\frac{3}{4}\right)^2}\right)-\tan^{-1}\left(\frac{17}{31}\right)$ $\left\{\text{Since } 2\tan^{-1}x=\tan^{-1}\left(\frac{2x}{1-x^2}\right)\right\}$

$=\tan^{-1}\left(\frac{\frac{3}{2}}{\frac{7}{16}}\right)-\tan^{-1}\left(\frac{17}{31}\right)$

$=\tan^{-1}\left(\frac{24}{7}\right)-\tan^{-1}\left(\frac{17}{31}\right)$

$=\tan^{-1}\left(\frac{\frac{24}{7}-\frac{17}{31}}{1+\frac{24}{7}y\times\frac{17}{31}}\right)$ $\left\{\text{Since } \tan^{-1}x-\tan^{-1}y=\tan^{-1}\left(\frac{x-y}{1+xy}\right)\right\}$

$=\tan^{-1}\left(\frac{\frac{744-119}{217}}{\frac{217+408}{217}}\right)$

$=\tan^{-1}\left(\frac{625}{625}\right)$

$=\tan^{-1}(1)$

$=\frac{\pi}{4}$

Hence,

$2\sin^{-1}\left(\frac{3}{5}\right)-\tan^{-1}\left(\frac{17}{31}\right)=\frac{\pi}{4}$

Q11

Prove the result $2\tan^{-1}\frac{1}{5}+\tan^{-1}\frac{1}{8}=\tan^{-1}\frac{4}{7}$

Solution

$$LHS = 2\tan^{-1}\frac{1}{5} + \tan^{-1}\frac{1}{8}$$

$$= \tan^{-1}\frac{2\cdot\frac{1}{5}}{1-\left(\frac{1}{5}\right)^2} + \tan^{-1}\frac{1}{8} \qquad \left[\text{Since } 2\tan^{-1}x = \tan^{-1}\frac{2x}{1-x^2}\right]$$

$$= \tan^{-1}\frac{\frac{2}{5}}{\frac{24}{25}} + \tan^{-1}\frac{1}{8}$$

$$= \tan^{-1}\frac{5}{12} + \tan^{-1}\frac{1}{8}$$

$$= \tan^{-1}\left(\frac{\frac{5}{12}+\frac{1}{8}}{1-\frac{5}{12}\cdot\frac{1}{8}}\right) \qquad \left[\text{Since } \tan^{-1}x + \tan^{-1}y = \tan^{-1}\frac{x+y}{1-xy}\right]$$

$$= \tan^{-1}\left(\frac{\frac{10+3}{24}}{\frac{96-5}{96}}\right)$$

$$= \tan^{-1}\left(\frac{13}{24}\cdot\frac{96}{91}\right)$$

$$= \tan^{-1}\left(\frac{4}{7}\right)$$

$$= RHS$$

Hence, $2\tan^{-1}\frac{1}{5} + \tan^{-1}\frac{1}{8} = \tan^{-1}\frac{4}{7}$

Q12

Prove that $2\tan^{-1}\frac{3}{4} - \tan^{-1}\frac{17}{31} = \frac{\pi}{4}$

Solution

$$\text{LHS} = 2\tan^{-1}\frac{3}{4} - \tan^{-1}\frac{17}{31}$$

$$= \tan^{-1}\frac{2\cdot\frac{3}{4}}{1-\left(\frac{3}{4}\right)^2} - \tan^{-1}\frac{17}{31} \qquad \left[\text{Since } 2\tan^{-1}x = \tan^{-1}\frac{2x}{1-x^2}\right]$$

$$= \tan^{-1}\left(\frac{3}{2}\cdot\frac{16}{7}\right) - \tan^{-1}\frac{17}{31}$$

$$= \tan^{-1}\frac{24}{7} - \tan^{-1}\frac{17}{31}$$

$$= \tan^{-1}\left(\frac{\frac{24}{7}-\frac{17}{31}}{1+\frac{24}{7}\cdot\frac{17}{31}}\right) \qquad \left[\text{Since } \tan^{-1}x - \tan^{-1}y = \tan^{-1}\frac{x-y}{1+xy}\right]$$

$$= \tan^{-1}\left(\frac{\frac{744-119}{217}}{\frac{217+408}{217}}\right)$$

$$= \tan^{-1}\left(\frac{625}{625}\right)$$

$$= \tan^{-1}1$$

$$= \frac{\pi}{4}$$

$$= \text{RHS}$$

Hence, $2\tan^{-1}\frac{3}{4} - \tan^{-1}\frac{17}{31} = \frac{\pi}{4}$

Q13

Prove that $2\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{7} = \tan^{-1}\frac{31}{17}$

Solution

$$\text{LHS}=2\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{1}{7}$$

$$=\tan^{-1}\frac{2\cdot\frac{1}{2}}{1-\left(\frac{1}{2}\right)^2}+\tan^{-1}\frac{1}{7} \qquad \left[\text{Since } 2\tan^{-1}x=\tan^{-1}\frac{2x}{1-x^2}\right]$$

$$=\tan^{-1}\frac{4}{3}+\tan^{-1}\frac{1}{7}$$

$$=\tan^{-1}\left(\frac{\frac{4}{3}+\frac{1}{7}}{1-\frac{4}{3}\cdot\frac{1}{7}}\right) \qquad \left[\text{Since } \tan^{-1}x+\tan^{-1}y=\tan^{-1}\frac{x+y}{1-xy}\right]$$

$$=\tan^{-1}\left(\frac{\frac{28+3}{21}}{\frac{21-4}{21}}\right)$$

$$=\tan^{-1}\left(\frac{31}{17}\right)$$

$$=\text{RHS}$$

Hence, $2\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{1}{7}=\tan^{-1}\frac{31}{17}$

Q14

Prove the following result :

$$4\tan^{-1}\frac{1}{5}-\tan^{-1}\frac{1}{239}=\frac{\pi}{4}$$

Solution

$$4\tan^{-1}\frac{1}{5}-\tan^{-1}\frac{1}{239}$$

$$=\tan^{-1}\left[\frac{4\left(\frac{1}{5}\right)-4\left(\frac{1}{5}\right)^3}{1-6\left(\frac{1}{5}\right)^2+\left(\frac{1}{5}\right)^4}\right]-\tan^{-1}\frac{1}{239}\ \ldots\ldots\left[4\tan^{-1}(x)=\tan^{-1}\left(\frac{4x-4x^3}{1-6x^2+x^4}\right)\right]$$

$$=\tan^{-1}\left[\frac{120}{119}\right]-\tan^{-1}\frac{1}{239}$$

$$=\tan^{-1}\left(\frac{120\times239-119}{119\times239+120}\right)\ \ldots\ldots\ldots\ldots\left[\tan^{-1}(x)-\tan^{-1}(y)=\tan^{-1}\left(\frac{x-y}{1+xy}\right)\right]$$

$$=\tan^{-1}\left(\frac{28561}{28561}\right)$$

$$=\tan^{-1}(1)$$

$$=\frac{\pi}{4}$$

Q15

If $\sin^{-1}\frac{2a}{1+a^2} - \cos^{-1}\frac{1-b^2}{1+b^2} = \tan^{-1}\frac{2x}{1-x^2}$, then prove that $x = \frac{a-b}{1+ab}$

Solution

Given

$$\sin^{-1}\left(\frac{2a}{1+a^2}\right) - \cos^{-1}\left(\frac{1-b^2}{1+b^2}\right) = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$$

$$\Rightarrow \quad 2\tan^{-1}a - 2\tan^{-1}b = 2\tan^{-1}x$$

$$\left\{\text{Since, } 2\tan^{-1}x = \sin^{-1}\left(\frac{2x}{1+x^2}\right) = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) = \tan^{-1}\left(\frac{2x}{1-x^2}\right)\right\}$$

$$\Rightarrow \quad 2\left\{\tan^{-1}a - \tan^{-1}b\right\} = 2\tan^{-1}x$$

$$\Rightarrow \quad \tan^{-1}a - \tan^{-1}b = \tan^{-1}x$$

$$\Rightarrow \quad \tan^{-1}\left(\frac{a-b}{1+ab}\right) = \tan^{-1}x$$

$$\left\{\text{Since } \tan^{-1}x - \tan^{-1}y = \tan^{-1}\left(\frac{x-y}{1+xy}\right)\right\}$$

$\Rightarrow$ On comparing, we get

$$\frac{a-b}{1+ab} = x$$

Q16

Prove that

$$\tan^{-1}\left(\frac{1-x^2}{2x}\right) + \cot^{-1}\left(\frac{1-x^2}{2x}\right) = \frac{\pi}{2}$$

Solution

$$\tan^{-1}\left(\frac{1-x^2}{2x}\right)+\cot^{-1}\left(\frac{1-x^2}{2x}\right)=\frac{\pi}{2}$$

LHS $=\tan^{-1}\left(\frac{1-x^2}{2x}\right)+\cot^{-1}\left(\frac{1-x^2}{2x}\right)$

$=\tan^{-1}\left(\frac{1-x^2}{2x}\right)+\tan^{-1}\left(\frac{2x}{1-x^2}\right)$ $\left\{\text{Since } \cot^{-1}x=\tan^{-1}\left(\frac{1}{x}\right)\right\}$

$=\tan^{-1}\left[\frac{\frac{(1-x^2)}{2x}+\left(\frac{2x}{1-x^2}\right)}{1-\left(\frac{1-x^2}{2x}\right)\left(\frac{2x}{1-x^2}\right)}\right]$ $\left\{\text{Since } \tan^{-1}x+\tan^{-1}y=\tan^{-1}\frac{x+y}{1-xy}\right\}$

$=\tan^{-1}\left[\frac{\frac{1+x^4-2x^2+4x^2}{2x(1-x^2)}}{\frac{2x(1-x^2)-2x(1-x^2)}{2x(1-x^2)}}\right]$

$=\tan^{-1}\left[\frac{1+x^4+2x^2}{0}\right]$

$=\tan^{-1}(\infty)$

$=\frac{\pi}{2}$

$=$ RHS

$$\tan^{-1}\left(\frac{1-x^2}{2x}\right)+\cot^{-1}\left(\frac{1-x^2}{2x}\right)=\frac{\pi}{2}$$

Q17

Prove that

$$\sin\left[\tan^{-1}\left(\frac{1-x^2}{2x}\right)+\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)\right]=1$$

Solution

$$\sin\left[\tan^{-1}\left(\frac{1-x^2}{2x}\right)+\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)\right]=1$$

LHS $=\sin\left[\tan^{-1}\left(\frac{1-x^2}{2x}\right)+\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)\right]$

$=\sin\left[\tan^{-1}\left(\frac{1-x^2}{2x}\right)+2\tan^{-1}x\right]$ $\left\{\text{Since } 2\tan^{-1}x=\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)\right\}$

$=\sin\left[\tan^{-1}\right]\left(\frac{1-x^2}{2x}\right)+\tan^{-1}\left(\frac{2x}{1-x^2}\right)$ $\left\{\text{Since } 2\tan^{-1}x=\tan\frac{2x}{1-x^2}\right\}$

$=\sin\left[\tan^{-1}\left(\frac{\frac{1-x^2}{2x}+\frac{2x}{1-x^2}}{1-\frac{1-x^2}{2x}\times\frac{2x}{1-x^2}}\right)\right]$ $\left\{\text{Since } \tan^{-1}x+\tan^{-1}y=\tan^{-1}\left(\frac{x+y}{1-xy}\right)\right\}$

$=\sin\left[\tan^{-1}\left(\frac{\frac{1+x^4-2x^2+4x^2}{2x\left(1-x^2\right)}}{0}\right)\right]$

$=\sin\left[\tan^{-1}(\infty)\right]$

$=\sin\left[\frac{\pi}{2}\right]$

$=1$

$=$ RHS

Hence,

$$\sin\left[\tan^{-1}\left(\frac{1-x^2}{2x}\right)+\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)\right]=1$$

Q18

If $\sin^{-1}\frac{2a}{1+a^2}+\sin^{-1}\frac{2b}{1+b^2}=2\tan^{-1}x$, prove that $x=\frac{a+b}{1-ab}$.

Solution

Given,

$$\sin^{-1}\left(\frac{2a}{1+a^2}\right)+\sin^{-1}\left(\frac{2b}{1+b^2}\right)=2\tan^{-1}x$$

$\Rightarrow\quad 2\tan^{-1}a+2\tan^{-1}b=2\tan^{-1}x$ $\left\{\text{Since, } 2\tan^{-1}x=\sin^{-1}\left(\frac{2x}{1+x}\right)\right\}$

$\Rightarrow\quad 2\left(\tan^{-1}a+\tan^{-1}b\right)=2\tan^{-1}x$

$\Rightarrow\quad \tan^{-1}\left(\frac{a+b}{1-ab}\right)=\tan^{-1}x$ $\left\{\text{Since, } \tan^{-1}x+\tan^{-1}y=\tan^{-1}\left(\frac{x+y}{1-xy}\right)\right\}$

$\Rightarrow\quad \frac{a+b}{1-ab}=x$

Q19

Show that $2\tan^{-1}x + \sin^{-1}\frac{2x}{1+x^2}$ is constant for $x \geq 1$, find that constant.

Solution

$$2\tan^{-1}x + \sin^{-1}\frac{2x}{1+x^2}$$
$$= 2\tan^{-1}x + 2\tan^{-1}x \qquad \left\{\text{Since, } 2\tan^{-1}x = \sin\frac{2x}{1+x^2}\right\}$$
$$= 4\tan^{-1}x$$

For $x \geq 1$
$$= 4 \times \tan^{-1}(1)$$
$$= 4 \times \frac{\pi}{4}$$
$$= \pi \quad \text{(Constant)}$$

Hence,
$$2\tan^{-1}x + \sin^{-1}\left(\frac{2x}{1+x^2}\right) = \pi$$

Q20

Find the value of $\tan^{-1}\left\{2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right\}$

Solution

$$\tan^{-1}\left\{2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right\}$$
$$= \tan^{-1}\left\{2\cos\left(2 \times \frac{\pi}{6}\right)\right\}$$
$$= \tan^{-1}\left\{2\cos\frac{\pi}{3}\right\}$$
$$= \tan^{-1}\left\{2 \times \frac{1}{2}\right\}$$
$$= \tan^{-1}(1)$$
$$= \frac{\pi}{4}$$

Hence,
$$\tan^{-1}\left\{2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right\} = \frac{\pi}{4}$$

Q21

Find the value of $\cos\left(\sec^{-1}x + \operatorname{cosec}^{-1}x\right)$, $|x| \geq 1$

Solution

$\cos\left(\sec^{-1}x + \operatorname{cosec}^{-1}x\right), \quad |x| \geq 1$

$= \cos\left(\frac{\pi}{2}\right) \quad \left\{\text{Since, } \sec^{-1}x + \operatorname{cosec}^{-1}x = \frac{\pi}{2}\right\}$

$= 0$

Hence,

$\cos\left(\sec^{-1}x + \operatorname{cosec}^{-1}x\right) = 0$

Q22

Solve the equation for x :

$\tan^{-1}\frac{1}{4} + 2\tan^{-1}\frac{1}{5} + \tan^{-1}\frac{1}{6} + \tan^{-1}\frac{1}{x} = \frac{\pi}{4}$

Solution

Given,

$$\tan^{-1}\frac{1}{4}+2\tan^{-1}\frac{1}{5}+\tan^{-1}\frac{1}{6}+\tan^{-1}\frac{1}{x}=\frac{\pi}{4}$$

$$\Rightarrow \tan^{-1}\frac{1}{4}+\tan^{-1}\left(\frac{2\times\frac{1}{5}}{1-\left(\frac{1}{5}\right)^2}\right)+\tan^{-1}\frac{1}{6}+\tan\frac{1}{x}=\frac{\pi}{4} \qquad \left\{\text{Since, } 2\tan^{-1}x=\tan^{-1}\left(\frac{2x}{1-x^2}\right)\right\}$$

$$\Rightarrow \tan^{-1}\frac{1}{4}+\tan^{-1}\left(\frac{\frac{2}{5}}{\frac{24}{25}}\right)+\tan^{-1}\frac{1}{6}+\tan^{-1}\frac{1}{x}=\frac{\pi}{4}$$

$$\Rightarrow \tan^{-1}\frac{1}{4}+\tan^{-1}\frac{5}{12}+\tan^{-1}\frac{1}{6}+\tan^{-1}\frac{1}{x}=\frac{\pi}{4}$$

$$\Rightarrow \tan^{-1}\left(\frac{\frac{1}{4}+\frac{5}{12}}{1-\frac{1}{4}\times\frac{5}{12}}\right)+\tan^{-1}\frac{1}{6}+\tan^{-1}\frac{1}{x}=\frac{\pi}{4}$$

$$\left\{\text{Since } \tan^{-1}x+\tan^{-1}y=\tan^{-1}\left(\frac{x+y}{1-xy}\right) \text{ if } xy<1\right\}$$

$$\Rightarrow \tan^{-1}\left(\frac{\frac{8}{12}}{\frac{43}{48}}\right)+\tan^{-1}\frac{1}{6}\tan^{-1}\frac{1}{x}=\frac{\pi}{4}$$

$$\Rightarrow \tan^{-1}\left(\frac{32}{43}\right)+\tan^{-1}\frac{1}{6}+\tan^{-1}\frac{1}{x}=\frac{\pi}{4}$$

$$\Rightarrow \tan^{-1}\left(\frac{\frac{32}{43}+\frac{1}{6}}{1-\frac{32}{43}\times\frac{1}{6}}\right)+\tan^{-1}\frac{1}{x}=\tan^{-1}1 \qquad \left\{\text{Since } \tan^{-1}x+\tan^{-1}y=\tan^{-1}\left(\frac{x+y}{1-xy}\right) \text{ if } xy<1\right\}$$

$$\Rightarrow \tan^{-1}\left(\frac{\frac{235}{258}}{\frac{226}{258}}\right)+\tan^{-1}\frac{1}{x}=\tan^{-1}1$$

$$\Rightarrow \tan^{-1}\left(\frac{235}{226}\right)+\tan^{-1}\frac{1}{x}=\tan^{-1}1$$

$$\Rightarrow \tan^{-1}\left(\frac{\frac{235}{226}+\frac{1}{x}}{1-\frac{235}{226x}}\right)=\tan^{-1}1, \quad \frac{235}{226x}<1 \qquad \left\{\text{Since } \tan^{-1}x+\tan^{-1}y=\tan^{-1}\left(\frac{x+y}{1-xy}\right) \text{ if } xy<1\right\}$$

$$\Rightarrow \frac{235x+226}{226x-235}=1 \qquad x>\frac{235}{226}$$

$$\Rightarrow 235x+226=226x-235 \qquad x>\frac{235}{226}$$

$$\Rightarrow 235x-226x=-235-226$$

$$\Rightarrow x=-\frac{461}{9}$$

Q23

Solve the equation for x :

$$3\sin^{-1}\frac{2x}{1+x^2}-4\cos^{-1}\frac{1-x^2}{1+x^2}+2\tan^{-1}\frac{2x}{1-x^2}=\frac{\pi}{3}$$

Solution

Given,

$$3\sin^{-1}\frac{2x}{1+x^2}-4\cos^{-1}\frac{1-x^2}{1+x^2}+2\tan^{-1}\frac{2x}{1-x^2}=\frac{\pi}{3}$$

$\Rightarrow \quad 3\left(2\tan^{-1}x\right)-4\left(2\tan^{-1}x\right)+2\left(2\tan^{-1}x\right)=\frac{\pi}{3}$

$\left\{\text{Since, } 2\tan^{-1}x=\tan^{-1}\frac{2x}{1-x^2}=\sin^{-1}\frac{2x}{1+x^2}=\cos^{-1}\frac{1-x^2}{1+x^2}\right\}$

$\Rightarrow \quad 6\tan^{-1}x-8\tan^{-1}x+4\tan^{-1}x=\frac{\pi}{3}$

$\Rightarrow \quad 2\tan^{-1}x=\frac{\pi}{3}$

$\Rightarrow \quad \tan^{-1}x=\frac{\pi}{6}$

$\Rightarrow \quad \tan^{-1}x=\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$

$\Rightarrow \quad x=\frac{1}{\sqrt{3}}$

Q24

Solve the equation for x :

$$\tan^{-1}\left(\frac{2x}{1-x^2}\right)+\cot^{-1}\left(\frac{1-x^2}{2x}\right)=\frac{2\pi}{3},\ x>0$$

Solution

Given,

$$\tan^{-1}\left(\frac{2x}{1-x^2}\right)+\cot^{-1}\left(\frac{1-x^2}{2x}\right)=\frac{2\pi}{3},\ x>0$$

$\Rightarrow \quad \tan^{-1}\left(\frac{2x}{1-x^2}\right)+\tan^{-1}\left(\frac{2x}{1-x^2}\right)=\frac{2\pi}{3},\ x>0 \quad \left\{\text{Since, } \cot^{-1}x=\tan^{-1}\frac{1}{x}\right\}$

$\Rightarrow \quad 2\tan^{-1}x+2\tan^{-1}x=\frac{2\pi}{3} \quad \left\{\text{Since, } 2\tan^{-1}=\tan^{-1}\frac{2x}{1-x^2}\right\}$

$\Rightarrow \quad 4\tan^{-1}x=\frac{2\pi}{3}$

$\Rightarrow \quad \tan^{-1}x=\frac{2\pi}{12}$

$\Rightarrow \quad \tan^{-1}x=\frac{\pi}{6}$

$\Rightarrow \quad \tan^{-1}x=\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$

$\Rightarrow \quad x=\frac{1}{\sqrt{3}}$

Q25

Evaluate the equation $2\tan^{-1}(\sin x)=\tan^{-1}(2\sec x),\ x\neq\frac{\pi}{2}$ for x

Solution

$2\tan^{-1}(\sin x) = \tan^{-1}(2\sec x)$

$\tan^{-1}\left(\frac{2\sin x}{1-\sin^2 x}\right) = \tan^{-1}(2\sec x)$ $\left[\text{Since } 2\tan^{-1} = \tan^{-1}\left(\frac{2x}{1-x^2}\right)\right]$

$\frac{2\sin x}{\cos^2 x} = 2\sec x$

$\frac{\sin x}{\cos x \cdot \cos x} = \sec x$

$\tan x \sec x = \sec x$

$\tan x = 1$

$x = \frac{\pi}{4}$

Hence, the value of x is $\frac{\pi}{4}$

Thus, the solution is $x = n\pi + \frac{\pi}{4}$

Q26

Prove that $2\tan^{-1}\left(\sqrt{\frac{a-b}{a+b}}\tan\frac{\theta}{2}\right) = \cos^{-1}\left(\frac{a\cos\theta + b}{a + b\cos\theta}\right)$

Solution

$$2\tan^{-1}\left(\sqrt{\frac{a-b}{a+b}}\tan\frac{\theta}{2}\right)=\cos^{-1}\left(\frac{a\cos\theta+b}{a+b\cos\theta}\right)$$

$$\text{LHS} = 2\tan^{-1}\left(\sqrt{\frac{a-b}{a+b}}\tan\frac{\theta}{2}\right)$$

$$=\cos^{-1}\left(\frac{1-\left(\sqrt{\frac{a-b}{a+b}}\tan\frac{\theta}{2}\right)^2}{1+\left(\sqrt{\frac{a-b}{a+b}}\tan\frac{\theta}{2}\right)^2}\right) \quad \left\{\text{Since } 2\tan^{-1}x=\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)\right\}$$

$$=\cos^{-1}\left(\frac{1-\left(\frac{a-b}{a+b}\right)\tan^2\frac{\theta}{2}}{1+\left(\frac{a-b}{a+b}\right)\tan^2\frac{\theta}{2}}\right)$$

$$=\cos^{-1}\left(\frac{a+b-(a-b)\tan^2\frac{\theta}{2}}{a+b+(a-b)\tan^2\frac{\theta}{2}}\right)$$

$$=\cos^{-1}\left(\frac{a\left(1-\tan^2\frac{\theta}{2}\right)+b\left(1+\tan^2\frac{\theta}{2}\right)}{a\left(1+\tan^2\frac{\theta}{2}\right)+b\left(1-\tan^2\frac{\theta}{2}\right)}\right)$$

Dividing numerator and denominator by $\left(1+\tan^2\frac{\theta}{2}\right)$, we get

$$=\cos^{-1}\left(\frac{a\left(\frac{1+\tan^2\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}}\right)+b}{a+b\left(\frac{1-\tan^2\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}}\right)}\right)$$

$$=\cos^{-1}\left(\frac{a\cos\theta+b}{a+b\cos\theta}\right) \quad \left\{\text{Since } \cos\theta=\frac{1-\tan^2\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}}\right\}$$

$$=\text{RHS}$$

Hence,

$$2\tan^{-1}\left(\sqrt{\frac{a-b}{a+b}}\tan\frac{\theta}{2}\right)=\cos^{-1}\left(\frac{a\cos\theta+b}{a+b\cos\theta}\right)$$

Q27

Prove that:

$$\tan^{-1}\left(\frac{2ab}{a^2-b^2}\right)+\tan^{-1}\left(\frac{2xy}{x^2-y^2}\right)=\tan^{-1}\left(\frac{2\alpha\beta}{\alpha^2-\beta^2}\right)$$

Solution

$\tan^{-1}\frac{2ab}{a^2-b^2}+\tan^{-1}\frac{2xy}{x^2-y^2}=\tan^{-1}\frac{2\alpha\beta}{\alpha^2-\beta^2}$ as $\alpha = ax-by$, $\beta = ay+bx$

$$\text{LHS} = \tan^{-1}\frac{2ab}{a^2-b^2}+\tan^{-1}\frac{2xy}{x^2-y^2}$$

$$= \tan^{-1}\left[\frac{\frac{2ab}{a^2-b^2}+\frac{2xy}{x^2-y^2}}{1-\left(\frac{2ab}{a^2-b^2}\right)\left(\frac{2xy}{x^2-y^2}\right)}\right] \quad \left\{\text{Since, } \tan^{-1}x+\tan^{-1}y=\tan^{-1}\frac{x+y}{1-xy}\right\}$$

$$= \tan^{-1}\left[\frac{\frac{2abx^2-2aby^2+2xya^2-2xyb^2}{(a^2-b^2)(x^2-y^2)}}{\frac{a^2x^2-a^2y^2-b^2x^2+b^2y^2-4abxy}{(a^2-b^2)(x^2-y^2)}}\right]$$

$$= \tan^{-1}\left[\frac{2(abx^2+xya^2-aby^2-xyb^2)}{a^2x^2+b^2y^2-2abxy-a^2y^2-b^2x^2-2abxy}\right]$$

$$= \tan^{-1}\left[\frac{2\{ax(bx+ay)-by(ay+bx)\}}{(ax-by)^2-(a^2y^2+b^2x^2+2abxy)}\right]$$

$$= \tan^{-1}\left[\frac{2(bx+ay)(ax-by)}{(ax-by)^2-(bx+ay)^2}\right]$$

$$= \tan^{-1}\left[\frac{2\alpha\beta}{\alpha^2-\beta^2}\right] \quad \{\text{Since, } bx+ay=\alpha, ax-by=\beta\}$$

$$= \text{RHS}$$

Hence,

$$\tan^{-1}\left(\frac{2ab}{a^2-b^2}\right)+\tan^{-1}\left(\frac{2xy}{x^2-y^2}\right)=\tan^{-1}\left(\frac{2\alpha\beta}{\alpha^2-\beta^2}\right)$$

Q28

For any $a,b,x,y>0$, prove that:

$$\frac{2}{3}\tan^{-1}\left(\frac{3ab^2-a^3}{b^3-3a^2b}\right)+\frac{2}{3}\tan^{-1}\left(\frac{3xy^2-x^3}{y^3-3x^2y}\right)=\tan^{-1}\left(\frac{2\alpha\beta}{\alpha^2-\beta^2}\right)$$

where $\alpha=-ax+by$, $\beta=bx+ay$

Solution

$$\frac{2}{3}\tan^{-1}\left(\frac{3ab^2-a^3}{b^3-3a^2b}\right)+\frac{2}{3}\tan^{-1}\left(\frac{3xy^2-x^3}{y^3-3x^2y}\right)=\tan^{-1}\left(\frac{2\alpha\beta}{\alpha^2-\beta^2}\right) \text{ as } \alpha=-ax+by,\ \beta=bx-ay$$

LHS

$$=\frac{2}{3}\tan^{-1}\left(\frac{3ab^2-a^3}{b^3-3a^2b}\right)+\frac{2}{3}\tan^{-1}\left(\frac{3xy^2-x^3}{y^3-3x^2y}\right)$$

$$=\frac{2}{3}\tan^{-1}\left(\frac{\frac{3ab^2-a^3}{b^3}}{\frac{b^3-3a^2b}{b^3}}\right)+\frac{2}{3}\tan^{-1}\left(\frac{\frac{3xy^2-x^3}{y^3}}{\frac{y^3-3x^2y}{y^3}}\right)$$

[Dividing Numerator and denominator of first function and second function by b^3 and y^3 respectively.]

$$=\frac{2}{3}\tan^{-1}\left(\frac{3\left(\frac{a}{b}\right)-\left(\frac{a}{b}\right)^3}{1-3\left(\frac{a}{b}\right)^2}\right)+\frac{2}{3}\tan^{-1}\left(\frac{3\left(\frac{x}{y}\right)-\left(\frac{x}{y}\right)^3}{1-3\left(\frac{x}{y}\right)^2}\right)$$

$$=\frac{2}{3}\left\{3\tan^{-1}\left(\frac{a}{b}\right)\right\}+\frac{2}{3}\left\{3\tan^{-1}\left(\frac{x}{y}\right)\right\}$$ $\left\{\text{Since, } 3\tan^{-1}x=\tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)\right\}$

$$=2\tan^{-1}\left(\frac{a}{b}\right)+2\tan^{-1}\left(\frac{x}{y}\right)$$

$$=2\left\{\tan^{-1}\left(\frac{a}{b}\right)+\tan^{-1}\left(\frac{x}{y}\right)\right\}$$

$$=2\tan^{-1}\left\{\frac{\frac{a}{b}+\frac{x}{y}}{1-\frac{a}{b}\times\frac{x}{y}}\right\}$$ $\left\{\text{Since, } \tan^{-1}x+\tan^{-1}y=\tan^{-1}\left(\frac{x+y}{1-xy}\right)\right\}$

$$=2\tan^{-1}\left\{\frac{\frac{ay+bx}{by}}{\frac{by-ax}{by}}\right\}$$

$$=2\tan^{-1}\left(\frac{ay+bx}{by-ax}\right)$$

$$=2\tan^{-1}\left(\frac{\beta}{\alpha}\right)$$ $\{\text{Since, } ay+bx=\beta,\ -ax+by=\alpha\}$

$$=\tan^{-1}\left[\frac{2\times\frac{\beta}{\alpha}}{1-\left(\frac{\beta}{\alpha}\right)^2}\right]$$ $\left\{\text{Since, } 2\tan^{-1}x=\tan^{-1}\left(\frac{2x}{1-x^2}\right)\right\}$

$$=\tan^{-1}\left[\frac{2\beta}{\alpha}\times\frac{\alpha^2}{\alpha^2-\beta^2}\right]$$

$$=\tan^{-1}\left[\frac{2\alpha\beta}{\alpha^2-\beta^2}\right]$$

= RHS

Hence,

$$\frac{2}{3}\tan^{-1}\left(\frac{3ab^2-a^3}{b^3-3a^2b}\right)+\frac{2}{3}\tan^{-1}\left(\frac{3xy^2-x^3}{y^3-3x^2y}\right)=\tan^{-1}\left(\frac{2\alpha\beta}{\alpha^2-\beta^2}\right)$$

as $\alpha=-ax+by,\ \beta=bx+ay.$

Exercise MCQ

Q1

If $\tan^{-1}\left\{\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right\}=\alpha$, then $x^2=$

a. sin 2α
b. sin α
c. cos 2α
d. cos α

Solution

Correct option: (a)

$$\tan^{-1}\left\{\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right\}=\alpha$$
$$\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}=\tan\alpha$$
$$\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\times\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}=\tan\alpha$$
$$\frac{1+x^2-2\sqrt{1-x^2}\sqrt{1+x^2}+1-x^2}{1+x^2-1+x^2}=\tan\alpha$$
$$\frac{1-\sqrt{1-x^4}}{x^2}=\tan\alpha$$
$$1-\sqrt{1-x^4}=x^2\tan\alpha$$
$$\left(1-x^2\tan\alpha\right)^2=1-x^4$$
$$1-2x^2\tan\alpha+x^4\tan^2\alpha=1-x^4$$
$$x^4-2x^2\tan\alpha+x^4\tan^2\alpha=0$$
$$x^2\left(x^2-2\tan\alpha+x^2\tan^2\alpha\right)=0$$
$$x^2=\frac{2\tan\alpha}{1+\tan^2\alpha}$$
$$x^2=\frac{2\tan\alpha}{\sec^2\alpha}$$
$$x^2=2\tan\alpha\cos^2\alpha$$
$$x^2=2\sin\alpha\cos\alpha=\sin2\alpha$$

Q2

The value of $\tan\left\{\cos^{-1}\frac{1}{5\sqrt{2}} - \sin^{-1}\frac{4}{\sqrt{17}}\right\}$ is

a. $\frac{\sqrt{29}}{3}$

b. $\frac{29}{3}$

c. $\frac{\sqrt{3}}{29}$

d. $\frac{3}{29}$

Solution

Correct option: (d)

Given that to find $\tan\left\{\cos^{-1}\frac{1}{5\sqrt{2}} - \sin^{-1}\frac{4}{\sqrt{17}}\right\}$

Put, $\cos^{-1}\frac{1}{5\sqrt{2}} = u$ and $\sin^{-1}\frac{4}{\sqrt{17}} = v$

$\frac{1}{5\sqrt{2}} = \cos u$ and $\frac{4}{\sqrt{17}} = \sin v$

$\Rightarrow \tan u = 7$ and $\tan z = 4$

Using properties of trigonometry,

$\tan\left\{\cos^{-1}\frac{1}{5\sqrt{2}} - \sin^{-1}\frac{4}{\sqrt{17}}\right\} = \tan(y - z)$

$\tan(y - z) = \frac{\tan y - \tan z}{1 + \tan y \tan z} = \frac{7 - 4}{1 + 28} = \frac{3}{29}$

Q3

$2\tan^{-1}\{\text{cosec}(\tan^{-1}x) - \tan(\cot^{-1}x)\}$ is equal to

a. $\cot^{-1}x$

b. $\cot^{-1}\frac{1}{x}$

c. $\tan^{-1}x$

d. None of these

Solution

Correct option: (c)

Put, $\tan^{-1}x = z$

$$2\tan^{-1}\left\{\operatorname{cosec}\left(\tan^{-1}x\right) - \tan\left(\cot^{-1}x\right)\right\}$$

$$= 2\tan^{-1}\left\{\operatorname{cosec}\left(\tan^{-1}x\right) - \tan\left(\tan^{-1}\frac{1}{x}\right)\right\}$$

$$= 2\tan^{-1}\left\{\operatorname{cosec}\left(\tan^{-1}x\right) - \frac{1}{x}\right\}$$

$$= 2\tan^{-1}\left\{\operatorname{cosec}z - \frac{1}{\tan z}\right\}$$

$$= 2\tan^{-1}\left\{\frac{1}{\sin z} - \frac{\cos z}{\sin z}\right\}$$

$$= 2\tan^{-1}\left\{\frac{1-\cos z}{\sin z}\right\}$$

$$= 2\tan^{-1}\left\{\frac{2\sin^2\frac{z}{2}}{2\sin\frac{z}{2}\cos\frac{z}{2}}\right\}$$

$$= 2\tan^{-1}\left(\tan\frac{z}{2}\right)$$

$$= 2\times\frac{z}{2}$$

$$= z$$

$$= \tan^{-1}x$$

Q4

If $\cos^{-1}\frac{x}{a} + \cos^{-1}\frac{y}{b} = \alpha$, then $\frac{x^2}{a^2} - \frac{2xy}{ab}\cos\alpha + \frac{y^2}{b^2} =$

a. $\sin^2\alpha$
b. $\cos^2\alpha$
c. $\tan^2\alpha$
d. $\cot^2\alpha$

Solution

Correct option: (a)

$$\cos^{-1} x + \cos^{-1} y = \cos^{-1}\left\{xy - \sqrt{1-x^2}\sqrt{1-y^2}\right\}$$

Consoder, $\cos^{-1}\dfrac{x}{a} + \cos^{-1}\dfrac{y}{b} = \alpha$

$$\Rightarrow \cos^{-1}\left(\frac{x}{a} \times \frac{y}{b} - \sqrt{1-\frac{x^2}{a^2}}\sqrt{1-\frac{y^2}{a^2}}\right) = \alpha$$

$$\Rightarrow \frac{x}{a} \times \frac{y}{b} - \sqrt{1-\frac{x^2}{a^2}}\sqrt{1-\frac{y^2}{a^2}} = \cos\alpha$$

$$\Rightarrow \frac{x}{a} \times \frac{y}{b} - \cos\alpha = \sqrt{1-\frac{x^2}{a^2}}\sqrt{1-\frac{y^2}{a^2}}$$

Squaring on both sides,

$$\Rightarrow \frac{x^2y^2}{a^2b^2} + \cos^2\alpha - \frac{2xy}{ab}\cos\alpha = \left(1-\frac{x^2}{a^2}\right)\left(1-\frac{y^2}{a^2}\right)$$

$$\Rightarrow \frac{x^2y^2}{a^2b^2} + \cos^2\alpha - \frac{2xy}{ab}\cos\alpha = 1 - \frac{x^2}{a^2} - \frac{y^2}{a^2} + \frac{x^2y^2}{a^2b^2}$$

$$\Rightarrow \frac{x^2}{a^2} + \frac{y^2}{a^2} - \frac{2xy}{ab}\cos\alpha = 1 - \cos^2\alpha$$

$$\Rightarrow \frac{x^2}{a^2} + \frac{y^2}{a^2} - \frac{2xy}{ab}\cos\alpha = \sin^2\alpha$$

Q5

The positive integral solution of the equation $\tan^{-1} x + \cos^{-1}\dfrac{y}{\sqrt{1+y^2}} = \sin^{-1}\dfrac{3}{\sqrt{10}}$ is

a. x = 1, y = 2
b. x = 2, y = 1
c. x = 3, y = 2
d. x = -2, y = -1

Solution

Correct option: (a)

$\tan^{-1} x + \cos^{-1} \frac{y}{\sqrt{1+y^2}} = \sin^{-1} \frac{3}{\sqrt{10}}$

Let, $\cos^{-1} \frac{y}{\sqrt{1+y^2}} = u \Rightarrow \cos u = \frac{y}{\sqrt{1+y^2}}$

Also, $\sin^{-1} \frac{3}{\sqrt{10}} = v \Rightarrow \sin v = \frac{3}{\sqrt{10}}$

Using trigonometric identities,

$\tan u = \frac{1}{y}$ and $\tan v = 3$

$\Rightarrow u = \tan^{-1} \frac{1}{y}$ and $v = \tan^{-1} 3$

Consider,

$\tan^{-1} x + \cos^{-1} \frac{y}{\sqrt{1+y^2}} = \sin^{-1} \frac{3}{\sqrt{10}}$

$\Rightarrow \tan^{-1} x + \tan^{-1} \frac{1}{y} = \tan^{-1} 3$

$\Rightarrow \tan^{-1} \left(\frac{x + \frac{1}{y}}{1 - \frac{x}{y}} \right) = \tan^{-1} 3$

$\Rightarrow \frac{xy+1}{y-x} = 3$

$\Rightarrow xy + 1 = 3y - 3x$

$\Rightarrow x = \frac{3y-1}{3+y}$

Put, $y = 1 \Rightarrow x = \frac{1}{2}$

Put, $y = 2 \Rightarrow x = 1$

Put, $y = 3 \Rightarrow x = \frac{4}{3}$

and so on..........

$\Rightarrow$ Integral soluions are: $x = 1$, $y = 2$

Q6

If $\sin^{-1} x - \cos^{-1} x = \frac{\pi}{6}$, then $x =$

a. $\frac{1}{2}$

b. $\frac{\sqrt{3}}{2}$

c. $-\frac{1}{2}$

d. None of these

Solution

Correct option: (b)

$\sin^{-1}x - \cos^{-1}x = \frac{\pi}{6}$

$\frac{\pi}{2} - \cos^{-1}x - \cos^{-1}x = \frac{\pi}{6}$

$\frac{\pi}{2} - 2\cos^{-1}x = \frac{\pi}{6}$

$\frac{\pi}{2} - \frac{\pi}{6} = 2\cos^{-1}x$

$\frac{2\pi}{6} = 2\cos^{-1}x$

$\frac{\pi}{3} \times \frac{1}{2} = \cos^{-1}x$

$\frac{\pi}{6} = \cos^{-1}x$

$x = \cos\frac{\pi}{6}$

$x = \frac{\sqrt{3}}{2}$

Q7

$\sin\left[\cot^{-1}\left\{\tan\left(\cos^{-1}x\right)\right\}\right]$ is equal to

a. x
b. $\sqrt{1-x^2}$
c. $\frac{1}{x}$
d. None of these

Solution

Correct option: (a)

Put $\cos^{-1}x = u$

$\sin\left[\cot^{-1}\left\{\tan\left(\cos^{-1}x\right)\right\}\right]$

$= \sin\left[\cot^{-1}\left\{\tan(u)\right\}\right]$

$= \sin\left[\cot^{-1}\left\{\cot\left(\frac{\pi}{2} - u\right)\right\}\right]$

$= \sin\left[\frac{\pi}{2} - u\right]$

$= \cos u$

$= x \qquad \left(\because \cos^{-1}x = u \Rightarrow x = \cos u\right)$

Q8

The number of solutions of the equation

$\tan^{-1} 2x + \tan^{-1} 3x = \frac{\pi}{4}$ is

a. 2
b. 3
c. 1
d. None of these

Solution

Correct option: (a)

$\tan^{-1} 2x + \tan^{-1} 3x = \frac{\pi}{4}$

$\Rightarrow \tan^{-1}\left(\frac{2x + 3x}{1 - 6x^2}\right) = \frac{\pi}{4}$

$\Rightarrow \frac{5x}{1 - 6x^2} = \tan\frac{\pi}{4}$

$\Rightarrow \frac{5x}{1 - 6x^2} = 1$

$\Rightarrow 5x = 1 - 6x^2$

$\Rightarrow 6x^2 + 5x - 1 = 0$

$\Rightarrow x = -1$ or $\frac{1}{6}$

Solutions of the given equation are 2.

Q9

If $\alpha = \tan^{-1}\left(\tan\frac{5\pi}{4}\right)$ and $\beta = \tan^{-1}\left(-\tan\frac{2\pi}{3}\right)$, then

a. $4\alpha = 3\beta$
b. $3\alpha = 4\beta$
c. $\alpha - \beta = \frac{7\pi}{12}$
d. None of these

Solution

Correct option: (a)

$\alpha = \tan^{-1}\left(\tan\frac{5\pi}{4}\right)$

$\Rightarrow \alpha = \tan^{-1}\left(\tan\left(\pi + \frac{\pi}{4}\right)\right)$

$\Rightarrow \alpha = \tan^{-1}\left(\tan\left(\frac{\pi}{4}\right)\right)$

$\Rightarrow \alpha = \frac{\pi}{4}$

and

$\beta = \tan^{-1}\left(-\tan\left(\pi - \frac{2\pi}{3}\right)\right)$

$\beta = \tan^{-1}\left(-\tan\left(\frac{\pi}{3}\right)\right)$

$\beta = \frac{\pi}{3}$

$4\alpha = 4 \times \frac{\pi}{4} = \pi \quad \ldots(i)$

$3\beta = 3 \times \frac{\pi}{3} = \pi \quad \ldots(ii)$

From (i) and (ii)

$4\alpha = 3\beta$

Q10

The number of real solutions of the equation $\sqrt{1 + \cos 2x} = \sqrt{2}\sin^{-1}(\sin x), -\pi \le x \le \pi$ is

a. 0
b. 1
c. 2
d. Infinite

Solution

Correct option: (c)

$\sqrt{1+\cos 2x} = \sqrt{2}\sin^{-1}(\sin x), -\pi \le x \le \pi$

$\Rightarrow \sqrt{2\cos^2 x} = \sqrt{2}(-\pi - x)$

$\Rightarrow |\cos x| = x$

If $\cos x$ is positive then $\cos x = -\pi - x$

It does not satisfy any value in the interval $\left(-\pi, -\frac{\pi}{2}\right)$

for the inetrval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$\cos x = x$

It gives one value of x in the $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

For the interval $\left[-\frac{\pi}{2}, \pi\right]$.

$-\cos x = \pi - x$

$\cos x = x - \pi$

It gives one value of x in the inetrval $\left[\frac{\pi}{2}, \pi\right]$.

Two real solutions in the interval $[-\pi, \pi]$

Q11

If $x < 0$, $y < 0$ such that $xy = 1$, then $\tan^{-1}x + \tan^{-1}y$ equals

a. $\frac{\pi}{2}$
b. $-\frac{\pi}{2}$
c. $-\pi$
d. None of these

Solution

Correct option: (b)

Given that $xy = 1$

Consider,

$\tan^{-1}x + \tan^{-1}y$

$= \tan^{-1}\left(\frac{x+y}{1-xy}\right)$

$= \tan^{-1}(-\infty) \quad .....(\because x < 0, y < 0)$

$= -\frac{\pi}{2}$

Q12

If $u = \cot^{-1}\left\{\sqrt{\tan\theta}\right\} - \tan^{-1}\left\{\sqrt{\tan\theta}\right\}$ then, $\tan\left(\frac{\pi}{4} - \frac{u}{2}\right) =$

a. $\sqrt{\tan\theta}$
b. $\sqrt{\cot\theta}$
c. Tan θ
d. Cot θ

Solution

Correct option: (a)

$u = \cot^{-1}\left\{\sqrt{\tan\theta}\right\} - \tan^{-1}\left\{\sqrt{\tan\theta}\right\}$

Put, $\sqrt{\tan\theta} = z$

$\Rightarrow u = \cot^{-1} z - \tan^{-1} z$

$\Rightarrow u = \frac{\pi}{2} - \tan^{-1} z - \tan^{-1} z$

$\Rightarrow u = \frac{\pi}{2} - 2\tan^{-1} z$

$\Rightarrow 2\tan^{-1} z = \frac{\pi}{2} - u$

$\Rightarrow \tan^{-1} z = \frac{\pi}{4} - \frac{u}{2}$

$\Rightarrow z = \tan\left(\frac{\pi}{4} - \frac{u}{2}\right)$

$\Rightarrow \sqrt{\tan\theta} = \tan\left(\frac{\pi}{4} - \frac{u}{2}\right)$

Q13

If $\cos^{-1}\frac{x}{3} + \cos^{-1}\frac{y}{2} = \frac{\theta}{2}$, then $4x^2 - 12xy\cos\frac{\theta}{2} + 9y^2 =$

a. 36
b. 36 - 36 cos θ
c. 18 - 18 cos θ
d. 18 + 18 cos θ

Solution

Correct option: (c)

$\cos^{-1} x + \cos^{-1} y = \cos^{-1}\left(xy - \sqrt{1-x^2}\sqrt{1-y^2}\right)$

$\Rightarrow \cos^{-1}\frac{x}{3} + \cos^{-1}\frac{y}{2} = \frac{\theta}{2}$

$\Rightarrow \cos^{-1}\left(\frac{x}{3} \times \frac{y}{2} - \sqrt{1-\left(\frac{x}{3}\right)^2}\sqrt{1-\left(\frac{y}{2}\right)^2}\right) = \frac{\theta}{2}$

$\Rightarrow \frac{xy}{6} - \sqrt{1-\left(\frac{x^2}{9}\right)}\sqrt{1-\left(\frac{y^2}{4}\right)} = \cos\frac{\theta}{2}$

$\Rightarrow \frac{xy - 6\cos\frac{\theta}{2}}{6} = \frac{\sqrt{9-x^2}\sqrt{4-y^2}}{6}$

$\Rightarrow xy - 6\cos\frac{\theta}{2} = \sqrt{9-x^2}\sqrt{4-y^2}$

Takings square on both sides,

$\Rightarrow x^2y^2 - 12xy\cos\frac{\theta}{2} + 36\cos^2\frac{\theta}{2} = (9-x^2)(4-y^2)$

$\Rightarrow x^2y^2 - 12xy\cos\frac{\theta}{2} + 36\cos^2\frac{\theta}{2} = 36 - 9y^2 - 4x^2 + x^2y^2$

$\Rightarrow 4x^2 + 9y^2 - 12xy\cos^2\frac{\theta}{2} = 36\left(1-\cos^2\frac{\theta}{2}\right)$

$\Rightarrow 4x^2 + 9y^2 - 12xy\cos^2\frac{\theta}{2} = 36\left(1-\frac{1+\cos\theta}{2}\right)$

$\Rightarrow 4x^2 + 9y^2 - 12xy\cos^2\frac{\theta}{2} = 18 - 18\cos\theta$

Q14

If $\alpha = \tan^{-1}\left(\frac{\sqrt{3}x}{2y-x}\right), \beta = \tan^{-1}\left(\frac{2x-y}{\sqrt{3}y}\right)$, then $\alpha - \beta =$

a. $\frac{\pi}{6}$

b. $\frac{\pi}{3}$

c. $\frac{\pi}{2}$

d. $-\frac{\pi}{3}$

Solution

Correct option: (a)

$$\alpha = \tan^{-1}\left(\frac{\sqrt{3}x}{2y-x}\right), \beta = \tan^{-1}\left(\frac{2x-y}{\sqrt{3}y}\right)$$

$$\alpha - \beta = \tan^{-1}\left(\frac{\sqrt{3}x}{2y-x}\right) - \tan^{-1}\left(\frac{2x-y}{\sqrt{3}y}\right)$$

$$\alpha - \beta = \tan^{-1}\left(\frac{\frac{\sqrt{3}x}{2y-x} - \frac{2x-y}{\sqrt{3}y}}{1 + \frac{\sqrt{3}x}{2y-x} \times \frac{2x-y}{\sqrt{3}y}}\right)$$

$$\alpha - \beta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

$$\alpha - \beta = \frac{\pi}{6}$$

Q15

Let $f(x) = e^{\cos^{-1}\{\sin(x+\pi/3)\}}$. Then, $f(8\pi/9) =$

a. $e^{5\pi/18}$
b. $e^{13\pi/18}$
c. $e^{-2\pi/18}$
d. None of these

Solution

Correct option: (b)

$$f(x) = e^{\cos^{-1}\{\sin(x+\pi/3)\}}$$

$$f\left(\frac{8\pi}{9}\right) = e^{\cos^{-1}\left\{\sin\left(\frac{8\pi}{9}+\frac{\pi}{3}\right)\right\}}$$

$$\Rightarrow f\left(\frac{8\pi}{9}\right) = e^{\cos^{-1}\sin\left(\frac{11\pi}{9}\right)}$$

$$\Rightarrow f\left(\frac{8\pi}{9}\right) = e^{\cos^{-1}\sin\left(\frac{\pi}{2}+\frac{13\pi}{18}\right)}$$

$$\Rightarrow f\left(\frac{8\pi}{9}\right) = e^{\frac{13\pi}{18}}$$

Q16

$\tan^{-1}\frac{1}{11} + \tan^{-1}\frac{2}{11}$ is equal to

a. 0
b. 1/2
c. -1
d. none of these

Solution

Correct option: (d)

$\tan^{-1}\frac{1}{11}+\tan^{-1}\frac{2}{11}$

$=\tan^{-1}\left(\frac{\frac{1}{11}+\frac{2}{11}}{1-\frac{2}{11}\times\frac{1}{11}}\right)$

$=\tan^{-1}\left(\frac{\frac{3}{11}}{1-\frac{2}{121}}\right)$

$=\tan^{-1}\left(\frac{33}{119}\right)$

Q17

If $\cos^{-1}\frac{x}{2}+\cos^{-1}\frac{y}{3}=\theta$, then $9x^2-12xy\cos\theta+4y^2$ is equal to

a. 36
b. $-36\sin^2\theta$
c. $36\sin^2\theta$
d. $36\cos^2\theta$

Solution

Correct option: (c)

$\cos^{-1}\frac{x}{2}+\cos^{-1}\frac{y}{3}=\theta$

We know that

$\cos^{-1}x+\cos^{-1}y=\cos^{-1}\left(xy-\sqrt{1-x^2}\sqrt{1-y^2}\right)$

$\cos^{-1}\left(\frac{x}{2}\times\frac{y}{3}-\sqrt{1-\frac{x^2}{4}}\sqrt{1-\frac{y^2}{9}}\right)=\theta$

$xy-\sqrt{1-x^2}\sqrt{1-y^2}=6\cos\theta$

$xy-6\cos\theta=\sqrt{1-x^2}\sqrt{1-y^2}$

$(xy-6\cos\theta)^2=(1-x^2)(1-y^2)$

Simplifying this you will get

$9x^2-12xy\cos\theta+4y^2=36\sin^2\theta$

Q18

If $\tan^{-1}3+\tan^{-1}x=\tan^{-1}8$, then x =

a. 5
b. 1/5
c. 5/14
d. 14/5

Solution

Correct option: (b)

$\tan^{-1}3 + \tan^{-1}x = \tan^{-1}8$

$\tan^{-1}\left(\frac{3+x}{1-3x}\right) = \tan^{-1}8$

$\frac{3+x}{1-3x} = 8$

$3+x = 8-24x$

$25x = 5$

$x = \frac{1}{5}$

Q19

The value of $\sin^{-1}\left(\cos\frac{33\pi}{5}\right)$ is

a. $\frac{3\pi}{5}$

b. $-\frac{\pi}{10}$

c. $\frac{\pi}{10}$

d. $\frac{7\pi}{5}$

Solution

Correct option: (b)

$\sin^{-1}\left(\cos\frac{33\pi}{5}\right)$

$= \sin^{-1}\left(\cos\left(6\pi + \frac{3\pi}{5}\right)\right)$

$= \sin^{-1}\left(\cos\left(\frac{3\pi}{5}\right)\right)$

$= \sin^{-1}\left(\sin\left(\frac{\pi}{2} - \frac{3\pi}{5}\right)\right)$

$= \frac{\pi}{2} - \frac{3\pi}{5}$

$= \frac{-\pi}{10}$

Q20

The value of $\cos^{-1}\left(\cos\frac{5\pi}{3}\right)+\sin^{-1}\left(\sin\frac{5\pi}{3}\right)$ is

a. $\frac{\pi}{2}$
b. $\frac{5\pi}{3}$
c. $\frac{10\pi}{3}$
d. 0

Solution

Correct option: (d)

$\cos^{-1}\left(\cos\frac{5\pi}{3}\right)+\sin^{-1}\left(\sin\frac{5\pi}{3}\right)$

$=\cos^{-1}\left(\cos\left(2\pi-\frac{\pi}{3}\right)\right)+\sin^{-1}\left(\sin\left(2\pi-\frac{\pi}{3}\right)\right)$

$=\cos^{-1}\left(\cos\left(\frac{\pi}{3}\right)\right)+\sin^{-1}\left(-\sin\left(\frac{\pi}{3}\right)\right)$

$=\cos^{-1}\left(\cos\left(\frac{\pi}{3}\right)\right)-\sin^{-1}\left(\sin\left(\frac{\pi}{3}\right)\right)$

$=\frac{\pi}{3}-\frac{\pi}{3}$

$=0$

Q21

$\sin\left\{2\cos^{-1}\left(\frac{-3}{5}\right)\right\}$ is equal to

a. $\frac{6}{25}$
b. $\frac{24}{25}$
c. $\frac{4}{5}$
d. $-\frac{24}{25}$

Solution

Correct option: (d)

To find $\sin\left\{2\cos^{-1}\left(\frac{-3}{5}\right)\right\}$

Let, $\cos^{-1}\left(\frac{-3}{5}\right) = y$

$\Rightarrow \frac{-3}{5} = \cos y$

$\Rightarrow \sin y = \sqrt{1-\cos^2 y}$

$\Rightarrow \sin y = \sqrt{1-\left(\frac{-3}{5}\right)^2}$

$\Rightarrow \sin y = \frac{4}{5}$

$\sin\left\{2\cos^{-1}\left(\frac{-3}{5}\right)\right\} = \sin 2y$

$\Rightarrow \sin\left\{2\cos^{-1}\left(\frac{-3}{5}\right)\right\} = 2\sin y\cos y$

$\Rightarrow \sin\left\{2\cos^{-1}\left(\frac{-3}{5}\right)\right\} = 2\times\frac{4}{5}\times\frac{-3}{5}$

$\Rightarrow \sin\left\{2\cos^{-1}\left(\frac{-3}{5}\right)\right\} = \frac{-24}{25}$

Q22

If $\theta = \sin^{-1}\{\sin(-600°)\}$, then one of the possible values of θ is

a. $\frac{\pi}{3}$

b. $\frac{\pi}{2}$

c. $\frac{2\pi}{3}$

d. $-\frac{2\pi}{3}$

Solution

Correct option: (a)

$\theta = \sin^{-1}\{\sin(-600°)\}$

$\theta = \sin^{-1}\left[-\sin(600°)\right]$

$\theta = \sin^{-1}\left[-\sin(180°\times 3+60)\right]$

$\theta = \sin^{-1}\left[-\{-\sin(60°)\}\right]$

$\theta = \sin^{-1}\left(\sin(60°)\right)$

$\theta = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$\theta = \frac{\pi}{3}$

Q23

If $3\sin^{-1}\left(\frac{2x}{1+x^2}\right) - 4\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) + 2\tan^{-1}\left(\frac{2x}{1-x^2}\right) = \frac{\pi}{3}$, then x is equal to

a. $\frac{1}{\sqrt{3}}$

b. $-\frac{1}{\sqrt{3}}$

c. $\sqrt{3}$

d. $-\frac{\sqrt{3}}{4}$

Solution

Correct option: (a)

$3\sin^{-1}\left(\frac{2x}{1+x^2}\right) - 4\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) + 2\tan^{-1}\left(\frac{2x}{1-x^2}\right) = \frac{\pi}{3}$

Let, $x = \tan\theta \Rightarrow \theta = \tan^{-1}x$

$\Rightarrow 3\sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right) - 4\cos^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right) + 2\tan^{-1}\left(\frac{2\tan\theta}{1-\tan^2\theta}\right) = \frac{\pi}{3}$

$\Rightarrow 3\sin^{-1}(\sin 2\theta) - 4\cos^{-1}(\cos 2\theta) + 2\tan^{-1}(\tan 2\theta) = \frac{\pi}{3}$

$\Rightarrow 3\times 2\theta - 4\times 2\theta + 2\times 2\theta = \frac{\pi}{3}$

$\Rightarrow 2\theta = \frac{\pi}{3}$

$\Rightarrow \theta = \frac{\pi}{6}$

$\Rightarrow \tan^{-1}x = \frac{\pi}{6}$

$\Rightarrow x = \tan\frac{\pi}{6}$

$\Rightarrow x = \frac{1}{\sqrt{3}}$

Q24

If $4\cos^{-1}x + \sin^{-1}x = \pi$, then the value of x is

a. $\frac{3}{2}$

b. $\frac{1}{\sqrt{2}}$

c. $\frac{\sqrt{3}}{2}$

d. $\frac{2}{\sqrt{3}}$

Solution

Correct option: (c)

$4\cos^{-1}x + \sin^{-1}x = \pi$

$\Rightarrow 3\cos^{-1}x + \cos^{-1}x + \sin^{-1}x = \pi$

$\Rightarrow 3\cos^{-1}x + \frac{\pi}{2} = \pi$

$\Rightarrow \cos^{-1}x = \frac{\pi}{6}$

$\Rightarrow x = \cos\frac{\pi}{6}$

$\Rightarrow x = \frac{\sqrt{3}}{2}$

Q25

If $\tan^{-1}\frac{x+1}{x-1} + \tan^{-1}\frac{x-1}{x} = \tan^{-1}(-7)$, then the value of x is

a. 0
b. -2
c. 1
d. 2

Solution

Correct option: (d)

$\tan^{-1}\frac{x+1}{x-1} + \tan^{-1}\frac{x-1}{x} = \tan^{-1}(-7)$

$\Rightarrow \tan^{-1}\left(\frac{\frac{x+1}{x-1} + \frac{x-1}{x}}{1 - \frac{x+1}{x-1} \times \frac{x-1}{x}}\right) = \tan^{-1}(-7)$

$\Rightarrow \tan^{-1}\left(\frac{x^2 + x + x^2 - 2x + 1}{x^2 - x - (x^2 - 1)}\right) = \tan^{-1}(-7)$

$\Rightarrow \frac{2x^2 - x + 1}{-x + 1} = -7$

$\Rightarrow 2x^2 - x + 1 = 7x - 7$

$\Rightarrow 2x^2 - 8x + 8 = 0$

$\Rightarrow x^2 - 4x + 4 = 0$

$\Rightarrow (x-2)^2 = 0$

$\Rightarrow x = 2$

Q26

If $\cos^{-1} x > \sin^{-1}x$, then

a. $\frac{1}{\sqrt{2}} < x \le 1$

b. $0 \le x < \frac{1}{\sqrt{2}}$

c. $-1 \le x < \frac{1}{\sqrt{2}}$

d. $x > 0$

Solution

Correct option: (a)

$\cos^{-1}x > \sin^{-1}x$

$\cos^{-1}x > \frac{\pi}{2} - \cos^{-1}x$

$2\cos^{-1} x > \frac{\pi}{2}$

$\cos^{-1} x > \frac{\pi}{4}$

$x > \cos\frac{\pi}{4}$

$x > \frac{1}{\sqrt{2}}$

Hence; $\frac{1}{\sqrt{2}} < x \le 1$

Q27

In a ΔABC, if C is a right angle, then $\tan^{-1}\left(\frac{a}{b+c}\right) + \tan^{-1}\left(\frac{b}{c+a}\right) =$

a. $\frac{\pi}{3}$

b. $\frac{\pi}{4}$

c. $\frac{5\pi}{2}$

d. $\frac{\pi}{6}$

Solution

Correct option: (b)

$$\tan^{-1}\left(\frac{a}{b+c}\right)+\tan^{-1}\left(\frac{b}{c+a}\right)$$

$$=\tan^{-1}\left(\frac{\frac{a}{b+c}+\frac{b}{c+a}}{1-\frac{a}{b+c}\times\frac{b}{c+a}}\right)$$

$$=\tan^{-1}\left(\frac{ac+a^2+b^2+bc}{bc+ba+c^2+ca-ab}\right)$$

$$=\tan^{-1}\left(\frac{ac+c^2+bc}{bc+ba+c^2+ca-ab}\right) \quad \cdots\cdots \left(\begin{array}{l}\because \Delta ABC \text{ is right angle at } C\\ c^2=a^2+b^2\end{array}\right)$$

$$=\tan^{-1}\left(\frac{ac+c^2+bc}{bc+c^2+ca}\right)$$

$$=\tan^{-1}(1)$$

$$=\frac{\pi}{4}$$

Q28

The value of $\sin\left(\frac{1}{4}\sin^{-1}\frac{\sqrt{63}}{8}\right)$ is

a. $\frac{1}{\sqrt{2}}$

b. $\frac{1}{\sqrt{3}}$

c. $\frac{1}{2\sqrt{2}}$

d. $\frac{1}{3\sqrt{3}}$

Solution

Correct option: (c)

$\sin\left(\frac{1}{4}\sin^{-1}\frac{\sqrt{63}}{8}\right)$

Let, $\sin^{-1}\frac{\sqrt{63}}{8} = x$

$\sin x = \frac{\sqrt{63}}{8}$

$\cos x = \sqrt{1-\sin^2 x}$

$\cos x = \sqrt{1-\frac{63}{64}}$

$\cos x = \frac{1}{8}$

Consider,

$\sin\left(\frac{1}{4}\sin^{-1}\frac{\sqrt{63}}{8}\right)$

$= \sin\left(\frac{1}{4}x\right)$

$= \sqrt{\frac{1-\cos\frac{x}{2}}{2}} \qquad \left(\because \sin x = \frac{1-\cos 2x}{2}\right)$

$= \sqrt{\frac{1-\sqrt{\frac{1+\cos x}{2}}}{2}} \qquad \left(\because \cos x = \frac{1+\cos 2x}{2}\right)$

$= \sqrt{\frac{1-\sqrt{\frac{1+\frac{1}{8}}{2}}}{2}}$

$= \sqrt{\frac{1-\frac{3}{4}}{2}}$

$= \sqrt{\frac{1}{8}}$

$= \frac{1}{2\sqrt{2}}$

Q29

$\cot\left(\frac{\pi}{4} - 2\cot^{-1}3\right) =$

a. 7
b. 6
c. 5
d. None of these

Solution

Correct option:(a)

$\cot\left(\frac{\pi}{4}-2\cot^{-1}3\right)$

Put, $2\cot^{-1}3 = x$

$\Rightarrow \cot^{-1}3 = \frac{x}{2}$

$\Rightarrow \cot\frac{x}{2} = 3$

$\Rightarrow \tan\frac{x}{2} = \frac{1}{3}$

$\cot\left(\frac{\pi}{4}-2\cot^{-1}3\right)$

$= \cot\left(\frac{\pi}{4}-x\right)$

We will find $\tan\left(\frac{\pi}{4}-x\right)$ then $\cot\left(\frac{\pi}{4}-x\right)$

$\tan\left(\frac{\pi}{4}-x\right)$

$= \frac{1-\tan x}{1+\tan x}$

$= \frac{1-\frac{3}{4}}{1+\frac{3}{4}} \qquad \left(\because \tan x = \frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}\right)$

$= \frac{1}{7}$

$\Rightarrow \tan\left(\frac{\pi}{4}-x\right) = \frac{1}{7}$

$\cot\left(\frac{\pi}{4}-x\right) = 7$

Q30

If $\tan^{-1}(\cot\theta) = 2\theta$, then $\theta =$

a. $\pm\frac{\pi}{3}$

b. $\pm\frac{\pi}{4}$

c. $\pm\frac{\pi}{6}$

d. none of these

Solution

Correct option: (c)

$\tan^{-1}(\cot\theta) = 2\theta$

$\cot\theta = \tan 2\theta$

$\frac{\cos\theta}{\sin\theta} = \frac{\sin 2\theta}{\cos 2\theta}$

$\frac{\cos\theta}{\sin\theta} = \frac{2\sin\theta\cos\theta}{\cos^2\theta - \sin^2\theta}$

$\cos^2\theta - \sin^2\theta = 2\sin^2\theta$

$\cos^2\theta = 3\sin^2\theta$

$\tan^2\theta = \frac{1}{3}$

$\tan\theta = \pm\frac{1}{\sqrt{3}}$

$\theta = \pm\frac{\pi}{6}$

Q31

If $\sin^{-1}\left(\frac{2a}{1+a^2}\right) + \cos^{-1}\left(\frac{1-a^2}{1+a^2}\right) = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$, where $a, x \in (0, 1)$, then, the value of x is

a. 0

b. $\frac{a}{2}$

c. a

d. $\frac{2a}{1-a^2}$

Solution

Correct option: (d)

$\sin^{-1}\left(\frac{2a}{1+a^2}\right) + \cos^{-1}\left(\frac{1-a^2}{1+a^2}\right) = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$

Let, $a = \tan\theta \Rightarrow \theta = \tan^{-1} a$

$\sin^{-1}(\sin 2\theta) + \cos^{-1}(\cos 2\theta) = 2\tan^{-1}(x)$

$2\theta + 2\theta = 2\tan^{-1}(x)$

$4\theta = 2\tan^{-1}(x)$

$2\tan^{-1} a = \tan^{-1}(x)$

$\tan^{-1}\left(\frac{2a}{1-a^2}\right) = \tan^{-1}(x)$

$x = \frac{2a}{1-a^2}$

Q32

The value of sin $(2(\tan^{-1}0.75))$ is equal to

a. 0.75
b. 1.5
c. 0.96
d. $\sin^{-1}1.5$

Solution

Correct option: (c)

$\sin\{2(\tan^{-1}0.75)\}$

Let, $\tan^{-1}0.75 = x$

$\Rightarrow \tan^{-1}\frac{3}{4} = x$

$\Rightarrow \tan x = \frac{3}{4}$

Using trigonometric identities,

$\sin x = \frac{3}{5},\ \cos x = \frac{4}{5}$

$\sin\{2(\tan^{-1}0.75)\}$

$= \sin(2x)$

$= 2\sin x\cos x$

$= 2\times\frac{3}{5}\times\frac{4}{5}$

$= \frac{24}{25} = 0.96$

Q33

If $x > 1$, then $2\tan^{-1}x + \sin^{-1}\left(\frac{2x}{1+x^2}\right)$ is equal to

a. $4\tan^{-1}x$
b. 0
c. $\frac{\pi}{2}$
d. π

Solution

Correct option: (a)

$2\tan^{-1}x + \sin^{-1}\left(\frac{2x}{1+x^2}\right)$

Let, $x = \tan\theta \Rightarrow \theta = \tan^{-1}x$

$\Rightarrow 2\tan^{-1}x + \sin^{-1}(\sin 2\theta)$

$\Rightarrow 2\theta + 2\theta$

$\Rightarrow 4\theta$

$\Rightarrow 4\tan^{-1}x$

Q34

The domain of $\cos^{-1}(x^2 - 4)$ is

a. [3, 5]
b. [-1, 1]
c. $\left[-\sqrt{5}, -\sqrt{3}\right] \cup \left[\sqrt{3}, \sqrt{5}\right]$
d. $\left[-\sqrt{3}, -\sqrt{3}\right] \cap \left[-\sqrt{5}, \sqrt{3}\right]$

Solution

Correct option: (c)

Let, $\cos^{-1}\left(x^2 - 4\right) = y$

$\Rightarrow \cos y = x^2 - 4$

$\Rightarrow -1 \le x^2 - 4 \le 1$

$\Rightarrow 3 \le x^2 \le 5$

$\Rightarrow \pm\sqrt{3} \le x \le \pm\sqrt{5}$

$x \in \left[-\sqrt{5}, -\sqrt{3}\right] \cup \left[\sqrt{3}, \sqrt{5}\right]$

Q35

The value of $\tan\left(\cos^{-1}\frac{3}{5} + \tan^{-1}\frac{1}{4}\right)$ is

a. $\frac{19}{8}$
b. $\frac{8}{19}$
c. $\frac{19}{12}$
d. $\frac{3}{4}$

Solution

Correct option: (a)

$\tan\left(\cos^{-1}\frac{3}{5}+\tan^{-1}\frac{1}{4}\right)$

Let, $\cos^{-1}\frac{3}{5}=x$ and $\tan^{-1}\frac{1}{4}=y$

$\Rightarrow \frac{3}{5}=\cos x$ and $\frac{1}{4}=\tan y$

Using trigonometric identities,

$\tan x=\frac{4}{3}$

Consider,

$\tan\left(\cos^{-1}\frac{3}{5}+\tan^{-1}\frac{1}{4}\right)$

$=\tan(x+y)$

$=\frac{\tan x+\tan y}{1-\tan x\tan y}$

$=\frac{\frac{4}{3}+\frac{1}{4}}{1-\frac{4}{3}\times\frac{1}{4}}$

$=\frac{19}{8}$