Exercise 4.10

Q1

Evaluate:

$$\cot\left(\sin^{-1}\frac{3}{4}+\sec^{-1}\frac{4}{3}\right)$$

Solution

$$\cot\left(\sin^{-1}\frac{3}{4}+\sec^{-1}\frac{4}{3}\right)$$
$$=\cot\left(\sin^{-1}\frac{3}{4}+\cos^{-1}\frac{3}{4}\right)$$
$$=\cot\left(\frac{\pi}{2}\right)$$
$$=0$$

Q2

Evaluate:

$$\sin\left(\tan^{-1}x+\tan^{-1}\frac{1}{x}\right) \text{ for } x<0$$

Solution

$$\sin\left(\tan^{-1}x+\tan^{-1}\frac{1}{x}\right)$$
$$=\sin\left(-\pi+\tan^{-1}\left(\frac{x+\frac{1}{x}}{1-\frac{x}{x}}\right)\right)$$
$$=\sin\left(-\pi+\tan^{-1}(\infty)\right)$$
$$=\sin\left(-\pi+\frac{\pi}{2}\right)$$
$$=\sin\left(-\frac{\pi}{2}\right)$$
$$=-1$$

Q3

Evaluate:

$$\sin\left(\tan^{-1}x+\tan^{-1}\frac{1}{x}\right) \text{ for } x>0$$

Solution

$$\sin\left(\tan^{-1}x+\tan^{-1}\frac{1}{x}\right)$$
$$=\sin\left(\pi+\tan^{-1}\left(\frac{x+\frac{1}{x}}{1-\frac{x}{x}}\right)\right)$$
$$=\sin\left(\pi+\tan^{-1}(\infty)\right)$$
$$=\sin\left(\pi+\frac{\pi}{2}\right)$$
$$=-\sin\left(\frac{\pi}{2}\right)$$
$$=-1$$

Q4

Evaluate:

$$\cot\left(\tan^{-1}a+\cot^{-1}a\right)$$

Solution

$$\cot\left(\tan^{-1}a+\cot^{-1}a\right)$$
$$=\cot\left(\frac{\pi}{2}\right)$$
$$=0$$

Q5

Evaluate:

$$\cos\left(\sec^{-1}x+\operatorname{cosec}^{-1}x\right),|x|\geq 1$$

Solution

$$\cos\left(\sec^{-1}x+\operatorname{cosec}^{-1}x\right)$$
$$=\cos\left(\frac{\pi}{2}\right)$$
$$=0$$

Q6

If $\cos^{-1}x+\cos^{-1}y=\frac{\pi}{4}$, find the value of $\sin^{-1}x+\sin^{-1}y$.

Solution

$[\Pi/2 - \sin^{-1} x] + [\Pi/2 - \sin^{-1} y] = \Pi/4$

$\sin^{-1}x + \sin^{-1}y = \Pi - \Pi/4$

$\sin^{-1}x + \sin^{-1}y = 3\Pi/4$

Q7

if $\sin^{-1} x + \sin^{-1} y = \frac{\pi}{3}$ and $\cos^{-1} x - \cos^{-1} y = \frac{\pi}{6}$, find the values of x and y.

Solution

$\sin^{-1} x + \sin^{-1} y = \frac{\pi}{3}$.....(i)

$\cos^{-1} x - \cos^{-1} y = \frac{\pi}{6}$.....(ii)

On adding both the equation

$\Pi/2 + \sin^{-1}y - \cos^{-1}y = \Pi/2$

$[\Pi/2 - \cos^{-1}y] - \cos^{-1}y = 0$

$\cos^{-1}y = \Pi/4$

$y = 1/\sqrt{2}$

on putting $y=1/\sqrt{2}$ in 2nd equation

$\cos^{-1}x - \Pi/4 = \Pi/6$

$\cos^{-1}x = \Pi/4 + \Pi/6$

$x = \cos(\Pi/4 + \Pi/6)$

$x = \cos(\Pi/4)\cos(\Pi/6) - \sin(\Pi/4)\sin(\Pi/6)$

$x = (\sqrt{3}-1)/2\sqrt{2}$

Q8

If $\cot\left(\cos^{-1}\frac{3}{5} + \sin^{-1}x\right) = 0$, find the values of x.

Solution

$\cot(z) = 0$ means $z = \Pi/2, 3\Pi/2, 5\Pi/2$ .....

$\cos^{-1}(3/5) + \sin^{-1}x = n\Pi + \Pi/2$

$\sin^{-1}x = n\Pi + \Pi/2 - \cos^{-1}(3/5)$

$\sin^{-1}x = n\Pi + \sin^{-1}(3/5)$

$x = \sin(n\Pi + \sin^{-1}(3/5)) = (-1)^n \sin(\sin^{-1}(3/5))$

$x = (-1)^n 3/5$

Q9

If $\left(\sin^{-1}x\right)^2 + \left(\cos^{-1}x\right)^2 = \frac{17\pi^2}{36}$, find x.

Solution

$[\Pi/2 - \cos^{-1}x]^2 + (\cos^{-1}x)^2 = 17\Pi^2/36$

$\Pi^2/4 - \Pi\cos^{-1}x + 2(\cos^{-1}x)^2 = 17\Pi^2/36$

Let, $\cos^{-1}x = u$

$2u^2 - \Pi u + \Pi^2/4 - 17\Pi^2/36 = 0$

$2u^2 - \Pi u - 2\Pi^2/9 = 0$

$18u^2 - 9\Pi u - 2\Pi^2 = 0$

On factorizing

$18u^2 - 12\Pi u + 3\Pi u - 2\Pi^2 = 0$

$6u(3u - 2\Pi) + \Pi(3u - 2\Pi) = 0$

$(3u - 2\Pi)(6u + \Pi) = 0$

$u = -\Pi/6, 2\Pi/3$

i.e. $\cos^{-1}x = -\Pi/6, 2\Pi/3$

but range of $\cos^{-1}x$ is $[0, \pi]$

$x = \cos(\Pi/2 + \Pi/6)$

$x = -1/2$

Q10

Solve:

$\sin\left\{\sin^{-1}\frac{1}{5} + \cos^{-1}x\right\} = 1$

Solution

$\sin^{-1}(1/5) + [\ \Pi/2 - \sin^{-1}x\ ] = \sin^{-1}1$

$\sin^{-1}(1/5) + \Pi/2 - \sin^{-1}x = \Pi/2$

$\sin^{-1}(1/5) - \sin^{-1}x = 0$

$x = 1/5$

Q11

Solve:

$$\sin^{-1} x = \frac{\pi}{6} + \cos^{-1} x$$

Solution

$\Pi/2 - \cos^{-1}x = \Pi/6 + \cos^{-1}x$

$\Pi/3 = 2\cos^{-1}x$

$\cos^{-1}x = \Pi/6$

$x = \sqrt{3}/2$

Q12

Solve:

$4 \sin^{-1}x = \Pi - \cos^{-1}x$

Solution

$4\sin^{-1}x+\cos^{-1}x=\Pi$
$3\sin^{-1}x+\sin^{-1}x+\cos^{-1}x=\Pi$
$3\sin^{-1}x=\Pi/2\ [\sin^{-1}x+\cos^{-1}x=\Pi/2]$
$\sin^{-1}x=\Pi/6$
$x = \sin\Pi/6=0.5$

Q13

Solve:

$$\tan^{-1} x + 2\cot^{-1} x = \frac{2\pi}{3}$$

Solution

$\tan^{-1}x+\cot^{-1}x=\Pi/2$ so the above equation reduces to
$\cot^{-1}x =2\Pi/3-\Pi/2 =\Pi/6$
$x= \cot\Pi/6 =\sqrt{3}$

Q14

Solve:

$5 \tan^{-1}x + 3 \cot^{-1}x = 2\Pi$

Solution

$2\tan^{-1}x+3(\Pi/2)=2\Pi$
$2\tan^{-1}x=2\Pi-3\Pi/2=\Pi/3$
$\tan^{-1}x=\Pi/6$
$x=\tan\Pi/6=1/\sqrt{3}$