Indefinite Integrals Ex 19.9 Q1

Let $I = \int \frac{\log x}{x} dx$

Let $\log x = t$ then,

$d(\log x) = dt$

$\Rightarrow \frac{1}{x} dx = dt$

$\Rightarrow dx = x\,dt$

Putting $\log x = t$ and $dx = x\,dt$, we get

$$I = \int \frac{t}{x} \times x\,dt$$

$$= \int t\,dt$$

$$= \frac{t^2}{2} + c$$

$$= \frac{(\log x)^2}{2} + c$$

$$\therefore \quad I = \frac{(\log x)^2}{2} + c$$

Indefinite Integrals Ex 19.9 Q2

Let $I = \int \frac{\log\left(1+\frac{1}{x}\right)}{x(1+x)} dx$ - - - - - - (i)

Let $\log\left(1+\frac{1}{x}\right) = t$ then,

$$d\left[\log\left(1+\frac{1}{x}\right)\right] = dt$$

$$\Rightarrow \quad \frac{1}{1+\frac{1}{x}} \times \frac{-1}{x^2} dx = dt$$

$$\Rightarrow \quad \frac{1}{\frac{x+1}{x}} \times \frac{-1}{x^2} dx = dt$$

$$\Rightarrow \quad \frac{-x}{x^2(x+1)} dx = -dt$$

$$\Rightarrow \quad \frac{dx}{x(x+1)} = -dt$$

Putting $\log\left(1+\frac{1}{x}\right) = t$ and $\frac{dx}{x(x+1)} = -dt$ in equation (i), we get

$$I = \int t \times -dt$$

$$= -\frac{t^2}{2} + c$$

$$= -\frac{1}{2}\left[\log\left(1+\frac{1}{x}\right)\right]^2 + c$$

$$\therefore \quad I = -\frac{1}{2}\left[\log\left(1+\frac{1}{x}\right)\right]^2 + c$$

Let $I = \int \frac{\log\left(1+\frac{1}{x}\right)}{x(1+x)} dx$ - - - - - - (i)

Let $\log\left(1+\frac{1}{x}\right) = t$ then,

$$d\left[\log\left(1+\frac{1}{x}\right)\right] = dt$$

$$\Rightarrow \quad \frac{1}{1+\frac{1}{x}} \times \frac{-1}{x^2} dx = dt$$

$$\Rightarrow \quad \frac{1}{\frac{x+1}{x}} \times \frac{-1}{x^2} dx = dt$$

$$\Rightarrow \quad \frac{-x}{x^2(x+1)} dx = dt$$

$$\Rightarrow \quad \frac{dx}{x(x+1)} = -dt$$

Putting $\log\left(1+\frac{1}{x}\right) = t$ and $\frac{dx}{x(x+1)} = -dt$ in equation (i), we get

$$I = \int t \times -dt$$

$$= -\frac{t^2}{2} + c$$

$$= -\frac{1}{2}\left[\log\left(1+\frac{1}{x}\right)\right]^2 + c$$

$$\therefore \quad I = -\frac{1}{2}\left[\log\left(1+\frac{1}{x}\right)\right]^2 + c$$

Indefinite Integrals Ex 19.9 Q3

Let $I = \int \frac{\left(1+\sqrt{x}\right)^2}{\sqrt{x}} dx$

Let $\left(1+\sqrt{x}\right) = t$ then,

$d\left(1+\sqrt{x}\right) = dt$

$\Rightarrow \quad \frac{1}{2\sqrt{x}} dx = dt$

$\Rightarrow \quad dx = dt \times 2\sqrt{x}$

Putting $\left(1+\sqrt{x}\right) = t$ and $dx = dt \times 2\sqrt{x}$, we get

$$I = \int \frac{t^2}{\sqrt{x}} \times dt \times 2\sqrt{x}$$
$$= 2\int t^2\, dt$$
$$= 2 \times \frac{t^3}{3} + c$$
$$= \frac{2}{3}\left[1+\sqrt{x}\right]^3 + c$$

$\therefore \quad I = \frac{2}{3}\left(1+\sqrt{x}\right)^3 + c$

Indefinite Integrals Ex 19.9 Q4

Let $I = \int \sqrt{1+e^x}\, e^x dx$ - - - - - - (i)

Let $1+e^x = t$ then,

$d\left(1+e^x\right) = dt$

$\Rightarrow \quad e^x dx = dt$

$\Rightarrow \quad dx = \frac{dt}{e^x}$

Putting $1+e^x = t$ and $dx = \frac{dt}{e^x}$ in equation (i), we get

$I = \int \sqrt{t}\, e^x \frac{dt}{e^x}$

$= \int t^{\frac{1}{2}} dt$

$= \frac{2}{3} \times \frac{t^{\frac{3}{2}}}{} + c$

$= \frac{2}{3}\left(1+e^x\right)^{\frac{3}{2}} + c$

Indefinite Integrals Ex 19.9 Q5

Let $\quad I = \int \sqrt[3]{\cos^2 x}\, \sin x\, dx$ ----- (i)

Let $\cos x = t$ then,

$d(\cos x) = dt$

$\Rightarrow \quad -\sin x\, dx = dt$

$\Rightarrow \quad dx = -\dfrac{dt}{\sin x}$

Putting $\cos x = t$ and $dx = -\dfrac{dt}{\sin x}$ in equation (i), we get

$$I = \int \sqrt[3]{t^2} \sin x \times \frac{-dt}{\sin x}$$

$$= -\int t^{\frac{2}{3}} \sin x \frac{dt}{\sin x}$$

$$= -\int t^{\frac{2}{3}} dt$$

$$= -\frac{3}{5} \times t^{\frac{5}{3}} + c$$

$$= -\frac{3}{5}(\cos x)^{\frac{5}{3}} + c$$

$$\therefore \quad I = -\frac{3}{5}(\cos x)^{\frac{5}{3}} + c$$

Indefinite Integrals Ex 19.9 Q6

Let $\quad I = \int \frac{e^x}{\left(1+e^x\right)^2} dx$ - - - - - (i)

Let $1+e^x = t$ then,

$d\left(1+e^x\right) = dt$

$\Rightarrow \quad e^x\,dx = dt$

$\Rightarrow \quad dx = \frac{dt}{e^x}$

Putting $1+e^x = t$ and $dx = \frac{dt}{e^x}$ in equation (i), we get

$I = \int \frac{e^x}{t^2} \times \frac{dt}{e^x}$

$= \int \frac{dt}{t^2}$

$= \int t^{-2}\,dt$

$= -t^{-1} + c$

$= -\frac{1}{t} + c$

$= -\frac{1}{1+e^x} + c$

$\therefore \quad I = -\frac{1}{1+e^x} + c$

Indefinite Integrals Ex 19.9 Q7

Let $I = \int \cot^3 x \operatorname{cosec}^2 x \, dx$ - - - - - (i)

Let $\cot x = t$ then,

$d(\cot x) = dt$

$\Rightarrow$ $-\operatorname{cosec}^2 x dx = dt$

$\Rightarrow$ $dx = -\dfrac{dt}{\operatorname{cosec}^2 x}$

Putting $\cot x = t$ and $dx = -\dfrac{dt}{\operatorname{cosec}^2 x}$ in equation (i), we get

$$I = \int t^3 \operatorname{cosec}^2 x \times \frac{-dt}{\operatorname{cosec}^2 x}$$

$$= -\int t^3 dt$$

$$= -\frac{t^4}{4} + c$$

$$= -\frac{\cot^4 x}{4} + c$$

$\therefore$ $I = -\dfrac{\cot^4 x}{4} + c$

Indefinite Integrals Ex 19.9 Q8

Let $\quad I = \int \frac{\left\{e^{\sin^{-1}x}\right\}^2}{\sqrt{1-x^2}}dx \text{ - - - - - - (i)}$

Let $\sin^{-1}x = t$ then,

$d\left(\sin^{-1}x\right) = dt$

$\Rightarrow \quad \frac{1}{\sqrt{1-x^2}}dx = dt$

$\Rightarrow \quad dx = \sqrt{1-x^2}\,dt$

Putting $\sin^{-1}x = t$ and $dx = \sqrt{1-x^2}\,dt$ in equation (i), we get

$$I = \int \frac{\left(e^t\right)^2}{\sqrt{1-x^2}} \times \sqrt{1-x^2}\,dt$$

$$= \int e^{2t}dt$$

$$= \frac{e^{2t}}{2} + c$$

$$= \frac{e^{2\sin^{-1}x}}{2} + c$$

$$\therefore \quad I = \frac{\left\{e^{\sin^{-1}x}\right\}^2}{2} + c$$

Indefinite Integrals Ex 19.9 Q9

Let $I = \int \frac{1+\sin x}{\sqrt{x-\cos x}}\,dx$ ------ (i)

Let $x - \cos x = t$ then,

$d(x - \cos x) = dt$

$\Rightarrow$ $[1-(-\sin x)]dx = dt$

$\Rightarrow$ $(1+\sin x)dx = dt$

Putting $x - \cos x = t$ and $(1+\sin x)dx = dt$ in equation (i), we get

$$I = \int \frac{dt}{\sqrt{t}}$$

$$= \int t^{\frac{-1}{2}}dt$$

$$= 2t^{\frac{1}{2}} + c$$

$$= 2(x - \cos x)^{\frac{1}{2}} + c$$

$\therefore$ $I = 2\sqrt{x - \cos x} + c$

Indefinite Integrals Ex 19.9 Q10

Let $I = \int \frac{1}{\sqrt{1-x^2}\left(\sin^{-1} x\right)^2} dx$ - - - - - (i)

Let $\sin^{-1} x = t$ then,

$d\left(\sin^{-1} x\right) = dt$

$\Rightarrow \quad \frac{1}{\sqrt{1-x^2}} dx = dt$

Putting $\sin^{-1} x = t$ and $\frac{1}{\sqrt{1-x^2}} dx = dt$ in equation (i), we get

$$I = \int \frac{dt}{t^2}$$

$$= \int t^{-2} dt$$

$$= -1 t^{-1} + c$$

$$= \frac{-1}{t} + c$$

$$= \frac{-1}{\sin^{-1} x} + c$$

$\therefore \quad I = \frac{-1}{\sin^{-1} x} + c$

Indefinite Integrals Ex 19.9 Q11

Let $I = \int \frac{\cot x}{\sqrt{\sin x}} dx$ ------ (i)

Let $\sin x = t$ then,

$d(\sin x) = dt$

$\Rightarrow$ $\cos x\, dx = dt$

Now, $I = \int \frac{\cot x}{\sqrt{\sin x}} dx$

$= \int \frac{\cos x}{\sin x \sqrt{\sin x}} dx$

$= \int \frac{\cos x}{(\sin x)^{\frac{3}{2}}} dx$

$\Rightarrow$ $= \int \frac{\cos x}{(\sin x)^{\frac{3}{2}}} dx$ ------- (ii)

Putting $\sin x = t$ and $\cos x\, dx = dt$ in equation (ii), we get

$I = \int \frac{dt}{t^{\frac{3}{2}}}$

$= \int t^{-\frac{3}{2}} dt$

$= -2t^{-\frac{1}{2}} + c$

$= \frac{-2}{\sqrt{t}} + c$

$= \frac{-2}{\sqrt{\sin x}} + c$

$\therefore$ $I = \frac{-2}{\sqrt{\sin x}} + c$

Indefinite Integrals Ex 19.9 Q12

Let $I = \int \frac{\tan x}{\sqrt{\cos x}} dx$

$\therefore$ $I = \int \frac{\sin x}{\cos x \sqrt{\cos x}} dx$

$= \int \frac{\sin x}{(\cos x)^{\frac{3}{2}}} dx$

$\Rightarrow$ $I = \int \frac{\sin x}{(\cos x)^{\frac{3}{2}}} dx$ ------ (i)

Let $\cos x = t$ then,

$d(\cos x) = dt$

$\Rightarrow$ $-\sin x \, dx = dt$

$\Rightarrow$ $\sin x \, dx = -dt$

Putting $\cos x = t$ and $\sin x \, dx = -dt$ in equation (i), we get

$I = \int \frac{-dt}{t^{\frac{3}{2}}}$

$= -\int t^{-\frac{3}{2}} dt$

$= -\left[-2t^{-\frac{1}{2}}\right] + c$

$= \frac{2}{t^{\frac{1}{2}}} + c$

$= \frac{2}{\sqrt{\cos x}} + c$

$\therefore$ $I = \frac{2}{\sqrt{\cos x}} + c$

Indefinite Integrals Ex 19.9 Q13

Let $I = \int \frac{\cos^3 x}{\sqrt{\sin x}} dx$

$\therefore$ $I = \int \frac{\cos^2 x \cos x}{\sqrt{\sin x}} dx$

$= \int \frac{\left(1 - \sin^2 x\right) \cos x}{\sqrt{\sin x}} dx$

$\therefore$ $I = \int \frac{\left(1 - \sin^2 x\right)}{\sqrt{\sin x}} \cos x \, dx$ - - - - - - (i)

Let $\sin x = t$ then,

$d\left(\sin x\right) = dt$

$\Rightarrow$ $\cos x \, dx = dt$

Putting $\sin x = t$ and $\cos x \, dx = dt$ in equation (i), we get

$I = \int \frac{1 - t^2}{\sqrt{t}} dt$

$= \int \left(t^{\frac{-1}{2}} - t^2 \times t^{\frac{-1}{2}}\right) dt$

$= \int \left(t^{\frac{-1}{2}} - t^{\frac{3}{2}}\right) dt$

$= 2t^{\frac{1}{2}} - \frac{2}{5} t^{\frac{5}{2}} + c$

$\Rightarrow$ $I = 2\left(\sin x\right)^{\frac{1}{2}} - \frac{2}{5}\left(\sin x\right)^{\frac{5}{2}} + c$

$\therefore$ $I = 2\sqrt{\sin x} - \frac{2}{5}\left(\sin x\right)^{\frac{5}{2}} + c$

Indefinite Integrals Ex 19.9 Q14

Let $\quad I = \int \frac{\sin^3 x}{\sqrt{\cos x}} dx$

$\therefore \quad I = \int \frac{\sin^2 x \sin x}{\sqrt{\cos x}} dx$

$\Rightarrow \quad I = \int \frac{\left(1 - \cos^2 x\right)}{\sqrt{\cos x}} \sin x \, dx \text{ ------ } (i)$

Let $\quad \cos x = t \quad$ then,

$d\left(\cos x\right) = dt$

$\Rightarrow \quad -\sin x \, dx = dt$

$\Rightarrow \quad \sin x \, dx = -dt$

Putting $\cos x = t$ and $\sin x \, dx = -dt$ in equation (i), we get

$$I = \int \frac{\left(1 - t^2\right)}{\sqrt{t}} \times -dt$$

$$= \int \frac{t^2 - 1}{\sqrt{t}} dt$$

$$= \int \left(\frac{t^2}{t^{\frac{1}{2}}} - \frac{1}{t^{\frac{1}{2}}} \right) dx$$

$$= \int \left(t^{2 - \frac{1}{2}} - t^{\frac{-1}{2}} \right) dt$$

$$= \int \left(t^{\frac{3}{2}} - t^{-\frac{1}{2}} \right) dt$$

$$= \frac{2}{5} t^{\frac{5}{2}} - 2t^{\frac{1}{2}} + c$$

$\therefore \quad I = \frac{2}{5} \cos^{\frac{5}{2}} x - 2 \cos^{\frac{1}{2}} x + c$

$\therefore \quad I = \frac{2}{5} \cos^{\frac{5}{2}} x - 2\sqrt{\cos x} + c$

Indefinite Integrals Ex 19.9 Q15

Let $I = \int \frac{1}{\sqrt{\tan^{-1} x}\left(1+x^2\right)} dx$ - - - - - (i)

Let $\tan^{-1} x = t$ then,

$d\left(\tan^{-1} x\right) = dt$

$\Rightarrow \quad \frac{1}{1+x^2} dx = dt$

Putting $\tan^{-1} x = t$ and $\frac{1}{1+x^2} dx = dt$ in equation (i), we get

$$I = \int \frac{1}{\sqrt{t}} dt$$
$$= \int t^{-\frac{1}{2}} dt$$
$$= 2t^{\frac{1}{2}} + c$$
$$= 2\sqrt{\tan^{-1} x} + c$$

$\therefore \quad I = 2\sqrt{\tan^{-1} x} + c$

Indefinite Integrals Ex 19.9 Q16

$$\text{Let } I = \int \frac{\sqrt{\tan x}}{\sin x \cos x} dx$$
$$= \int \frac{\sqrt{\tan x} \times \cos x}{\sin x \cos x \times \cos x} dx$$
$$= \int \frac{\sqrt{\tan x}}{\tan x \cos^2 x} dx$$
$$= \int \frac{\sec^2 x \, dx}{\sqrt{\tan x}}$$

Let $\tan x = t \Rightarrow \sec^2 x \, dx = dt$

$$\therefore I = \int \frac{dt}{\sqrt{t}}$$
$$= 2\sqrt{t} + C$$
$$= 2\sqrt{\tan x} + C$$

Indefinite Integrals Ex 19.9 Q17

Let $I = \int \frac{1}{x}(\log x)^2 dx$ - - - - - (i)

Let $\log x = t$ then,

$d(\log x) = dt$

$\Rightarrow$ $\frac{1}{x} dx = dt$

Putting $\log x = t$ and $\frac{1}{x} dx = dt$ in equation (i), we get

$I = \int t^2 dt$

$= \frac{t^3}{3} + c$

$= \frac{(\log x)^3}{3} + c$

$\therefore$ $I = \frac{1}{3}(\log x)^3 + c$

Indefinite Integrals Ex 19.9 Q18

Let $I = \int \sin^5 x \cos x \, dx$ - - - - - (i)

Let $\sin x = t$ then,

$d(\sin x) = dt$

$\Rightarrow$ $\cos x \, dx = dt$

Putting $\sin x = t$ and $\cos x \, dx = dt$ in equation (i), we get

$I = \int t^5 dt$

$= \frac{t^6}{6} + c$

$= \frac{\sin^6 x}{6} + c$

$\therefore$ $I = \frac{1}{6}\sin^6 x + c$

Indefinite Integrals Ex 19.9 Q19

Let $I = \int \tan^{\frac{3}{2}} x \sec^2 x \, dx$ - - - - - (i)

Let $\tan x = t$ then,

$d(\tan x) = dt$

$\Rightarrow \sec^2 x \, dx = dt$

Putting $\tan x = t$ and $\sec^2 x \, dx = dt$ in equation (i), we get

$$I = \int t^{\frac{3}{2}} dt$$
$$= \frac{2}{5} t^{\frac{5}{2}} + c$$
$$= \frac{2}{5} (\tan x)^{\frac{5}{2}} + c$$

$$\therefore \quad I = \frac{2}{5} \tan^{\frac{5}{2}} x + c$$

Indefinite Integrals Ex 19.9 Q20

Let $I = \int \frac{x^3}{(x^2+1)^3} x \, dx$ - - - - - (i)

Let $1 + x^2 = t$ then,

$d(1+x^2) = dt$

$\Rightarrow 2x \, dx = dt$

$\Rightarrow x \, dx = \frac{dt}{2}$

Putting $1 + x^2 = t$ and $x \, dx = \frac{dt}{2}$ in equation (i), we get

$$I = \int \frac{x^2}{t^3} \times \frac{dt}{2}$$
$$= \frac{1}{2} \int \frac{(t-1)}{t^3} dt \qquad [\because \quad 1 + x^2 = t]$$
$$= \frac{1}{2} \int \left[\left(\frac{t}{t^3} - \frac{1}{t^3} \right) dt \right]$$
$$= \frac{1}{2} \int \left(t^{-2} - t^{-3} \right) dt$$
$$= \frac{1}{2} \left[-1t^{-1} - \frac{t^{-2}}{-2} \right] + c$$

$$= \frac{1}{2}\left[-\frac{1}{t}+\frac{1}{2t^2}\right]+c$$

$$= -\frac{1}{2t}+\frac{1}{4t^2}+c$$

$$= -\frac{1}{2\left(1+x^2\right)}+\frac{1}{4\left(1+x^2\right)^2}+c$$

$$= \frac{-2\left(1+x^2\right)+1}{4\left(1+x^2\right)^2}+c$$

$$= \frac{-2-2x^2+1}{4\left(1+x^2\right)^2}+c$$

$$= \frac{-2x^2-1}{4\left(1+x^2\right)^2}+c$$

$$= -\frac{\left(1+2x^2\right)}{4\left(x^2+1\right)^2}+c$$

$$\therefore \qquad I = \frac{-\left(1+2x^2\right)}{4\left(x^2+1\right)^2}+c$$

Indefinite Integrals Ex 19.9 Q21

Let $x^2+x+1=t$

$(2x+1)dx = dt$

$$\int (4x+2)\sqrt{x^2+x+1}\,dx$$

$$= \int 2\sqrt{t}\,dt$$

$$= 2\int \sqrt{t}\,dt$$

$$= 2\left(\frac{t^{\frac{3}{2}}}{\frac{3}{2}}\right)+C$$

$$= \frac{4}{3}\left(x^2+x+1\right)^{\frac{3}{2}}+C$$

Indefinite Integrals Ex 19.9 Q22

Let $I = \int \frac{4x+3}{\sqrt{2x^3+3x+1}}\,dx$ - - - - - - (i)

Let $2x^3+3x+1 = t$ then,

$d\left(2x^3+3x+1\right) = dt$

$\Rightarrow$ $(4x+3)\,dx = dt$

Putting $2x^3+3x+1 = t$ and $(4x+3)\,dx = dt$ in equation (i), we get

$I = \int \frac{dt}{\sqrt{t}}$

$= \int t^{\frac{-1}{2}}\,dt$

$= 2t^{\frac{1}{2}} + c$

$= 2\sqrt{t} + c$

$\therefore$ $I = 2\sqrt{2x^3+3x+1} + c$

Indefinite Integrals Ex 19.9 Q23

Let $I = \int \frac{1}{1+\sqrt{x}} dx$ ------ (i)

Let $x = t^2$ then,

$dx = d\left(t^2\right)$

$\Rightarrow$ $dx = 2t\,dt$

Putting $x = t^2$ and $dx = 2t\,dt$ in equation (i), we get

$$I = \int \frac{2t}{1+\sqrt{t^2}} dt$$
$$= \int \frac{2t}{1+t} dt$$
$$= 2\int \frac{t}{1+t} dt$$
$$= 2\int \frac{1+t-1}{1+t} dt$$
$$= 2\int \left[\frac{1+t}{1+t} - \frac{1}{1+t}\right] dt$$
$$= 2\int dt - 2\int \frac{1}{1+t} dt$$
$$= 2t - 2\log\left(1+t\right) + c$$
$$= 2\sqrt{x} - 2\log\left(1+\sqrt{x}\right) + c$$

$\therefore$ $I = 2\sqrt{x} - 2\log\left(1+\sqrt{x}\right) + c$

Indefinite Integrals Ex 19.9 Q24

Let $\quad I = \int e^{\cos^2 x} \sin 2x \, dx$ - - - - (i)

Let $\quad \cos^2 x = t$ then,

$d\left(\cos^2 x\right) = dt$

$\Rightarrow \quad -2\cos x \sin x \, dx = dt$

$\Rightarrow \quad -\sin 2x \, dx = dt$

$\Rightarrow \quad \sin 2x \, dx = -dt$

Putting $\cos^2 x = t$ and $\sin 2x \, dx = -dt$ in equation (i), we get

$$I = \int e^t (-dt)$$
$$= -e^t + c$$
$$= -e^{\cos^2 x} + c$$

$\therefore \quad I = -e^{\cos^2 x} + c$

Indefinite Integrals Ex 19.9 Q25

Let $\quad I = \int \frac{1+\cos x}{(x+\sin x)^3} dx$ - - - - - (i)

Let $\quad x + \sin x = t$ then,

$d(x+\sin x) = dt$

$\Rightarrow \quad (1+\cos x)dx = dt$

Putting $x + \sin x = t$ and $(1+\cos x)dx = dt$ in equation (i), we get

$$I = \int \frac{dt}{t^3}$$

$$= \int t^{-3} dt$$

$$= \frac{t^{-2}}{-2} + c$$

$$= -\frac{1}{2t^2} + c$$

$$= \frac{-1}{2(x+\sin x)^2} + c$$

$$\therefore \quad I = \frac{-1}{2(x+\sin x)^2} + c$$

Indefinite Integrals Ex 19.9 Q26

$$\frac{\cos x - \sin x}{1+\sin 2x} = \frac{\cos x - \sin x}{\left(\sin^2 x + \cos^2 x\right) + 2\sin x \cos x}$$

$$\left[\sin^2 x + \cos^2 x = 1;\ \ \sin 2x = 2\sin x \cos x\right]$$

$$= \frac{\cos x - \sin x}{\left(\sin x + \cos x\right)^2}$$

Let $\sin x + \cos x = t$

$\therefore \left(\cos x - \sin x\right) dx = dt$

$$\Rightarrow \int \frac{\cos x - \sin x}{1+\sin 2x} dx = \int \frac{\cos x - \sin x}{\left(\sin x + \cos x\right)^2} dx$$

$$= \int \frac{dt}{t^2}$$

$$= \int t^{-2} dt$$

$$= -t^{-1} + C$$

$$= -\frac{1}{t} + C$$

$$= \frac{-1}{\sin x + \cos x} + C$$

Indefinite Integrals Ex 19.9 Q27

Let $I = \int \frac{\sin 2x}{(a + b\cos 2x)^2} dx$ - - - - - (i)

Let $a + b\cos 2x = t$ then,

$d(a + b\cos 2x) = dt$

$\Rightarrow$ $b(-2\sin 2x)dx = dt$

$\Rightarrow$ $\sin 2x\, dx = -\frac{dt}{2b}$

Putting $a + b\cos 2x = t$ and $\sin 2x\, dx = -\frac{dt}{2b}$ in equation (i), we get

$$I = \int \frac{1}{t^2} \times \frac{-dt}{2b}$$

$$= \frac{-1}{2b}\int t^{-2}dt$$

$$= -\frac{1}{2b}\left(-1t^{-1}\right) + c$$

$$= \frac{1}{2bt} + c$$

$$= \frac{1}{2b(a + b\cos 2x)} + c$$

$\therefore$ $I = \frac{1}{2b(a + b\cos 2x)} + c$

Indefinite Integrals Ex 19.9 Q28

Let $I = \int \frac{\log x^2}{x} dx$ ----- (i)

Let $\log x = t$ then,

$d(\log x) = dt$

$\Rightarrow \frac{1}{x} dx = dt$

$\Rightarrow \frac{dx}{x} = dt$

Now, $I = \int \frac{\log x^2}{x} dx$

$= \int \frac{2\log x}{x} dx$

$= 2\int \frac{\log x}{x} dx$ ---- (ii)

Putting $\log x = t$ and $\frac{dx}{x} = dt$ in equation (ii), we get

$I = 2\int t\, dt$

$= \frac{2t^2}{2} + c$

$= t^2 + c$

$\therefore I = (\log x)^2 + c$

Indefinite Integrals Ex 19.9 Q29

Let $I = \int \frac{\sin x}{(1+\cos x)^2} dx$ - - - - - (i)

Let $1+\cos x = t$ then,

$d(1+\cos x) = dt$

$\Rightarrow$ $-\sin x\, dx = dt$

$\Rightarrow$ $\sin x\, dx = -dt$

Putting $1+\cos x = t$ and $\sin dx = -dt$ in equation (ii), we get

$$I = \int \frac{-dt}{t^2}$$
$$= -\int t^{-2} dt$$
$$= -\left(-1t^{-1}\right) + c$$
$$= \frac{1}{t} + c$$
$$= \frac{1}{1+\cos x} + c$$

$$\therefore \quad I = \frac{1}{1+\cos x} + c$$

Indefinite Integrals Ex 19.9 Q30

Let $\log \sin x = t$

$$\Rightarrow \frac{1}{\sin x} \cdot \cos x\, dx = dt$$
$$\therefore \cot x\, dx = dt$$

$$\Rightarrow \int \cot x \log \sin x\, dx = \int t\, dt$$
$$= \frac{t^2}{2} + C$$
$$= \frac{1}{2}(\log \sin x)^2 + C$$

Indefinite Integrals Ex 19.9 Q31

Let $\quad I = \int \sec x \cdot \log(\sec x + \tan x)\,dx$ - - - - - (i)

Let $\quad \log(\sec x + \tan x) = t$ then,

$d[\log(\sec x + \tan x)] = dt$

$\Rightarrow \quad \sec x\,dx = dt \quad \left[\because \quad \frac{d}{dx}(\log(\sec x + \tan x)) = \sec x\right]$

Putting $\log(\sec x + \tan x) = t$ and $\sec x\,dx = dt$ in equation (i), we get

$$I = \int t\,dt$$
$$= \frac{t^2}{2} + c$$
$$= \frac{1}{2}[\log(\sec x + \tan x)]^2 + c$$

$\therefore \quad I = \frac{1}{2}[\log(\sec x + \tan x)]^2 + c$

Indefinite Integrals Ex 19.9 Q32

Let $\quad I = \int \operatorname{cosec} x \log(\operatorname{cosec} x - \cot x)\,dx$ - - - - - (i)

Let $\quad \log(\operatorname{cosec} x - \cot x) = t$ then,

$dx[\log(\operatorname{cosec} x - \cot x)] = dt$

$\Rightarrow \quad \operatorname{cosec} x\,dx = dt \quad \left[\because \quad \frac{d}{dx}(\log(\operatorname{cosec} x - \cot x)) = \operatorname{cosec} x\right]$

Putting $\log(\operatorname{cosec} x - \cot x) = t$ and $\operatorname{cosec} x\,dx = dt$ in equation (i), we get

$$I = \int t\,dt$$
$$= \frac{t^2}{2} + c$$

$\therefore \quad I = \frac{1}{2}[\log(\operatorname{cosec} x - \cot x)]^2 + c$

Indefinite Integrals Ex 19.9 Q33

Let $I = \int x^3 \cos x^4 dx$ ----- (i)

Let $x^4 = t$ then,

$dx\left(x^4\right) = dt$

$\Rightarrow \quad 4x^3 dx = dt$

$\Rightarrow \quad x^3 = \frac{dt}{4}$

Putting $x^4 = t$ and $x^3 dx = \frac{dt}{4}$ in equation (i), we get

$I = \int \cos t \frac{dt}{4}$

$= \frac{1}{4}\sin t + c$

$\therefore \quad I = \frac{1}{4}\sin x^4 + c$

Indefinite Integrals Ex 19.9 Q34

Let $I = \int x^3 \sin x^4 dx$ ----- (i)

Let $x^4 = t$ then,

$d\left(x^4\right) = dt$

$\Rightarrow \quad 4x^3 dx = dt$

$\Rightarrow \quad x^3 = \frac{dt}{4}$

Putting $x^4 = t$ and $x^3 dx = \frac{dt}{4}$ in equation (i), we get

$I = \int \sin t \frac{dt}{4}$

$= \frac{1}{4}\int \sin t\, dt$

$= -\frac{1}{4}\cos t + c$

$= -\frac{1}{4}\cos x^4 + c$

$\therefore \quad I = -\frac{1}{4}\cos x^4 + c$

Indefinite Integrals Ex 19.9 Q35

Let $I = \int \frac{x \sin^{-1} x^2}{\sqrt{1 - x^4}} dx \text{ ----- (i)}$

Let $\sin^{-1} x^2 = t$ then,

$d\left(\sin^{-1} x^2\right) = dt$

$\Rightarrow \quad 2x \times \frac{1}{\sqrt{1 - x^4}} dx = dt$

$\Rightarrow \quad \frac{x}{\sqrt{1 - x^4}} dx = \frac{dt}{2}$

Putting $\sin^{-1} x^2 = t$ and $\frac{x}{\sqrt{1 - x^4}} dx = \frac{dt}{2}$ in equation (i),

we get

$$I = \int t \frac{dt}{2}$$

$$= \frac{1}{2} \times \frac{t^2}{2} + c$$

$$= \frac{1}{4}\left(\sin^{-1} x^2\right)^2 + c$$

$$\therefore \quad I = \frac{1}{4}\left(\sin^{-1} x^2\right)^2 + c$$

Indefinite Integrals Ex 19.9 Q36

Let $I = \int x^3 \sin\left(x^4 + 1\right) dx$ ----- (i)

Let $x^4 + 1 = t$ then,

$d\left(x^4 + 1\right) = dt$

$\Rightarrow \quad x^4 dx = dt$

$\Rightarrow \quad x^3 dx = \frac{dt}{4}$

Putting $x^4 + 1 = t$ and $x^3 dx = \frac{dt}{4}$ in equation (i), we get

$$I = \int \sin t \frac{dt}{4}$$

$$= -\frac{1}{4}\cos t + c$$

$$= -\frac{1}{4}\cos\left(x^4 + 1\right) + c$$

$\therefore \quad I = -\frac{1}{4}\cos\left(x^4 + 1\right) + c$

Indefinite Integrals Ex 19.9 Q37

Let $I = \int \frac{(x+1)e^x}{\cos^2(xe^x)} dx$ - - - - - - (i)

Let $xe^x = t$ then,

$d(xe^x) = dt$

$\Rightarrow$ $(e^x + xe^x)dx = dt$

$\Rightarrow$ $(x+1)e^x dx = dt$

Putting $xe^x = t$ and $(x+1)e^x dx = dt$ in equation (i), we get

$$I = \int \frac{dt}{\cos^2 t}$$
$$= \int \sec^2 t\, dt$$
$$= \tan t + c$$
$$= \tan(xe^x) + c$$

$\therefore$ $I = \tan(xe^x) + c$

Indefinite Integrals Ex 19.9 Q38

Let $I = \int x^2 e^{x^3} \cos\left(e^{x^3}\right) dx$ - - - - - (i)

Let $e^{x^3} = t$ then,

$d\left(e^{x^3}\right) = dt$

$\Rightarrow \quad 3x^2 e^{x^3} dx = dt$

$\Rightarrow \quad x^2 e^{x^3} dx = \frac{dt}{3}$

Putting $e^{x^3} = t$ and $x^2 e^{x^3} dx = \frac{dt}{3}$ in equation (i), we get

$$I = \int \cos t \frac{dt}{3}$$

$$= \frac{\sin t}{3} + c$$

$$= \frac{\sin\left(e^{x^3}\right)}{3} + c$$

$\therefore \quad I = \frac{1}{3} \sin\left(e^{x^3}\right) + c$

Indefinite Integrals Ex 19.9 Q39

Let $I = \int 2x \sec^3\left(x^2 + 3\right) \tan\left(x^2 + 3\right) dx$ - - - - - (i)

Let $\sec\left(x^2 + 3\right) = t$ then,

$d\left[\sec\left(x^2 + 3\right)\right] = dt$

$\Rightarrow \quad 2x \sec\left(x^2 + 3\right) \tan\left(x^2 + 3\right) dx = dt$

Putting $\sec\left(x^2 + 3\right) = t$ and $2x \sec\left(x^2 + 3\right) \tan\left(x^2 + 3\right) dx = dt$ in equation (i), we get

$$I = \int t^2 dt$$

$$= \frac{t^3}{3} + c$$

$$= \frac{1}{3}\left[\sec\left(x^2 + 3\right)\right]^3 + c$$

$\therefore \quad I = \frac{1}{3}\left[\sec\left(x^2 + 3\right)\right]^3 + c$

Indefinite Integrals Ex 19.9 Q40

$$\frac{(x+1)(x+\log x)^2}{x} = \left(\frac{x+1}{x}\right)(x+\log x)^2 = \left(1+\frac{1}{x}\right)(x+\log x)^2$$

Let $(x+\log x)=t$

$$\Rightarrow \left(1+\frac{1}{x}\right)dx = dt$$

$$\Rightarrow \int\left(1+\frac{1}{x}\right)(x+\log x)^2\,dx = \int t^2\,dt$$

$$= \frac{t^3}{3}+C$$

$$= \frac{1}{3}(x+\log x)^3 + C$$

Indefinite Integrals Ex 19.9 Q41

Let $\quad I = \int \tan x \sec^2 x \sqrt{1 - \tan^2 x}\, dx$ - - - - - (i)

Let $\quad 1 - \tan^2 x = t$ then,

$d\left(1 - \tan^2 x\right) = dt$

$\Rightarrow \quad -2 \tan x \sec^2 x\, dx = dt$

$\Rightarrow \quad \tan x \sec^2 x dx = \frac{-dt}{2}$

Putting $1 - \tan^2 x = t$ and $\tan x \sec^2 x\, dx = -\frac{dt}{2}$ in equation (i),

we get

$$I = \int \sqrt{t} \times \frac{-dt}{2}$$

$$= -\frac{1}{2} \int t^{\frac{1}{2}} dt$$

$$= -\frac{1}{2} \times \frac{t^{\frac{3}{2}}}{\frac{3}{2}} + c$$

$$= -\frac{1}{3} t^{\frac{3}{2}} + c$$

$\therefore \quad I = -\frac{1}{3}\left[1 - \tan^2 x\right]^{\frac{3}{2}} + c$

Indefinite Integrals Ex 19.9 Q42

Let $I = \int \log x \frac{\sin\left(1+(\log x)^2\right)}{x} dx$ ----- (i)

Let $1+(\log x)^2 = t$ then,

$d\left(1+(\log x)^2\right) = dt$

$\Rightarrow \quad 2\log x \frac{1}{x} dx = dt$

$\Rightarrow \quad \frac{\log x}{x} dx = \frac{dt}{2}$

Putting $1+(\log x)^2 = t$ and $\frac{\log x}{x} dx = \frac{dt}{2}$ in equation (i),

we get

$$I = \int \sin t \times \frac{dt}{2}$$

$$= \frac{1}{2}\int \sin t \, dt$$

$$\therefore \quad I = -\frac{1}{2}\cos t + c$$

$$= -\frac{1}{2}\cos\left[1+(\log x)^2\right] + c$$

$$\therefore \quad I = -\frac{1}{2}\cos\left[1+(\log x)^2\right] + c$$

Indefinite Integrals Ex 19.9 Q43

Let $\quad I = \int \frac{1}{x^2} \cos^2\left(\frac{1}{x}\right) dx$ - - - - - (i)

Let $\quad \frac{1}{x} = t$ then,

$$d\left(\frac{1}{x}\right) = dt$$

$\Rightarrow \quad \frac{-1}{x^2} dx = dt$

$\Rightarrow \quad \frac{1}{x^2} dx = -dt$

Putting $\frac{1}{x} = t$ and $\frac{1}{x^2} dx = -dt$ in equation (i),

we get

$$I = \int \cos^2 t \, (-dt)$$
$$= -\int \cos^2 t \, dt$$
$$= -\int \frac{\cos^2 2t + 1}{2} dt$$
$$= -\frac{1}{2} \int \cos 2t \, dt - \frac{1}{2} \int dt$$
$$= -\frac{1}{2} \times \frac{\sin 2t}{2} - \frac{1}{2} t + c$$

$\therefore \quad I = -\frac{1}{4} \sin 2t - \frac{1}{2} t + c$

$$= -\frac{1}{4} \sin 2 \times \frac{1}{x} - \frac{1}{2} \times \frac{1}{x} + c$$

$\therefore \quad I = -\frac{1}{4} \sin\left(\frac{2}{x}\right) - \frac{1}{2}\left(\frac{1}{x}\right) + c$

Indefinite Integrals Ex 19.9 Q44

Let $I = \int \sec^4 x \tan x \, dx$ - - - -(i)

Let $\tan x = t$ then,

$d(\tan x) = dt$

$\Rightarrow \sec^2 x \, dx = dt$

$\Rightarrow dx = \dfrac{dt}{\sec^2 x}$

Putting $\tan x = t$ and $dx = \dfrac{dt}{\sec^2 x}$ in equation (i), we get

$$I = \int \sec^4 x \tan x \frac{dt}{\sec^2 x}$$

$$= \int \sec^2 x \, t dt$$

$$= \int \left(1 + \tan^2 x\right) t \, dt$$

$$= \int \left(1 + t^2\right) t \, dt$$

$$= \int \left(t + t^3\right) dt$$

$$= \frac{t^2}{2} + \frac{t^4}{4} + c$$

$$= \frac{\tan^2 x}{2} + \frac{\tan^4 x}{4} + c$$

$$\therefore \quad I = \frac{1}{2}\tan^2 x + \frac{1}{4}\tan^4 x + c$$

Indefinite Integrals Ex 19.9 Q45

Let $\quad I = \int \frac{e^{\sqrt{x}} \cos\left(e^{\sqrt{x}}\right)}{\sqrt{x}} dx$ - - - - - (i)

Let $\quad e^{\sqrt{x}} = t \quad$ then,

$d\left(e^{\sqrt{x}}\right) = dt$

$\Rightarrow \quad e^{\sqrt{x}} \frac{1}{2\sqrt{x}} dx = dt$

$\Rightarrow \quad \frac{e^{\sqrt{x}}}{\sqrt{x}} dx = 2dt$

Putting $e^{\sqrt{x}} = t$ and $\frac{e^{\sqrt{x}}}{\sqrt{x}} dx = 2dt$ in equation (i),

we get

$$I = \int \cos t \times 2dt$$
$$= 2\int \cos t \, dt$$
$$= 2\sin t + c$$
$$= 2\sin\left(e^{\sqrt{x}}\right) + c$$

$\therefore \quad I = 2\sin\left(e^{\sqrt{x}}\right) + c$

Indefinite Integrals Ex 19.9 Q46

Let $I = \int \frac{\cos^5 x}{\sin x} dx$ ---- (i)

Let $\sin x = t$ then,

$d(\sin x) = dt$

$\Rightarrow \cos x \, dx = dt$

$\Rightarrow dx = \frac{dt}{\cos x}$

Putting $\sin x = t$ and $dx = \frac{dt}{\cos x}$ in equation (i),

we get

$$I = \int \frac{\cos^5 x}{t} \times \frac{dt}{\cos x}$$

$$= \int \frac{\cos^4 x}{t} dt$$

$$= \int \frac{\left(1 - \sin^2 x\right)^2}{t} dt$$

$$= \int \frac{\left(1 - t^2\right)^2}{t} dt$$

$$= \int \frac{1 + t^4 - 2t^2}{t} dt$$

$$= \int \frac{1}{t} dt + \int \frac{t^4}{t} dt - 2\int \frac{t^2}{t} dt$$

$$= \log|t| + \frac{t^4}{4} - \frac{2t^2}{2} + c$$

$$= \log|\sin x| + \frac{\sin^4 x}{4} - \sin^2 x + c$$

$\therefore \quad I = \frac{1}{4}\sin^4 x - \sin^2 x + \log|\sin x| + c$

Indefinite Integrals Ex 19.9 Q47

Let $\sqrt{x} = t$

$\Rightarrow \frac{1}{2\sqrt{x}}dx = dt$

$\Rightarrow \frac{1}{\sqrt{x}}dx = 2dt$

Therefore,

$\int \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx$

$=2\int \sin t\,dt$

$= -2\cos t + C$

$= -2\cos\sqrt{x} + C$

Indefinite Integrals Ex 19.9 Q48

Let $I = \int \frac{(x+1)e^x}{\sin^2(xe^x)}dx$ ------ (i)

Let $xe^x = t$ then,

$d(xe^x) = dt$

$\Rightarrow (xe^x + e^x)dx = dt$

$\Rightarrow (x+1)e^x dx = dt$

Putting $xe^x = t$ and $(x+1)e^x dx = dt$ in equation (i), we get

$I = \int \frac{dt}{\sin^2 t}$

$= \int \text{cosec}^2 t\,dt$

$= -\cot t + c$

$= -\cot(xe^x) + c$

$\therefore \quad I = -\cot(xe^x) + c$

Indefinite Integrals Ex 19.9 Q49

Let $I = \int 5^{x+\tan^{-1}x}\left(\frac{x^2+2}{x^2+1}\right)dx$ ------ (i)

Let $x + \tan^{-1}x = t$ then,

$d\left(x + \tan^{-1}x\right) = dt$

$\Rightarrow \left(1+\frac{1}{1+x^2}\right)dx = dt$

$\Rightarrow \left(\frac{1+x^2+1}{1+x^2}\right)dx = dt$

$\Rightarrow \frac{\left(x^2+2\right)}{\left(x^2+1\right)}dx = dt$

Putting $x + \tan^{-1}x = t$ and $\left(\frac{x^2+2}{x^2+1}\right)dx = dt$ in equation (i),

we get

$$I = \int 5^t\,dt$$

$$= \frac{5^t}{\log 5} + c$$

$$= \frac{5^{x+\tan^{-1}x}}{\log 5} + c$$

$$\therefore \quad I = \frac{5^{x+\tan^{-1}x}}{\log 5} + c$$

Indefinite Integrals Ex 19.9 Q50

Let $I = \int \frac{e^{m \sin^{-1} x}}{\sqrt{1-x^2}} dx$ - - - - - (i)

Let $m \sin^{-1} x = t$ then,

$d\left(m \sin^{-1} x\right) = dt$

$\Rightarrow \quad m \frac{1}{\sqrt{1-x^2}} dx = dt$

$\Rightarrow \quad \frac{dx}{\sqrt{1-x^2}} = \frac{dt}{m}$

Putting $m \sin^{-1} x = t$ and $\frac{dx}{\sqrt{1-x^2}} = \frac{dt}{m}$ in equation (i),

we get

$$I = \int e^t \frac{dt}{m}$$

$$= \frac{1}{m} e^t + c$$

$$= \frac{1}{m} e^{m \sin^{-1} x} + c$$

$$\therefore \quad I = \frac{1}{m} e^{m \sin^{-1} x} + c$$

Indefinite Integrals Ex 19.9 Q51

Let $\sqrt{x} = t$

$$\Rightarrow \frac{1}{2\sqrt{x}} dx = dt$$

$$\Rightarrow \int \frac{\cos\sqrt{x}}{\sqrt{x}} dx = 2\int \cos t \, dt$$

$$= 2\sin t + C$$

$$= 2\sin\sqrt{x} + C$$

Indefinite Integrals Ex 19.9 Q52

Let $I = \int \frac{\sin\left(\tan^{-1} x\right)}{1+x^2} dx$ ----- (i)

Let $\tan^{-1} x = t$ then,

$d\left(\tan^{-1} x\right) = dt$

$\Rightarrow \frac{1}{1+x^2} dx = dt$

Putting $\tan^{-1} x = t$ and $\frac{dx}{1+x^2} = dt$ in equation (i),

we get

$I = \int \sin t\, dt$

$= -\cos t + c$

$= -\cos\left(\tan^{-1} x\right) + c$

$\therefore I = -\cos\left(\tan^{-1} x\right) + c$

Indefinite Integrals Ex 19.9 Q53

Let $I = \int \frac{\sin(\log x)}{x} dx$ ------ (i)

Let $\log x = t$ then,

$d(\log x) = dt$

$\Rightarrow \frac{1}{x} dx = dt$

Putting $\log x = t$ and $\frac{1}{x} dx = dt$ in equation (i),

we get

$I = \int \sin t\, dt$

$= -\cos t + c$

$= -\cos(\log x) + c$

$\therefore I = -\cos(\log x) + c$

Indefinite Integrals Ex 19.9 Q54

$$\text{Let } \tan^{-1}x = t$$

Differentiating the above function with respect to, w, we have,

$$\frac{1}{1+x^2}dx = dt$$

$$\Rightarrow \int \frac{e^{m\tan^{-1}x}}{1+x^2} = \int e^{mt} \times dt$$

$$\Rightarrow \int \frac{e^{m\tan^{-1}x}}{1+x^2} = \frac{e^{mt}}{m}$$

Resubstituting the value of t in the above solution, we have,

$$\Rightarrow \int \frac{e^{m\tan^{-1}x}}{1+x^2} = \frac{e^{m\tan^{-1}x}}{m} + C$$

Indefinite Integrals Ex 19.9 Q55

Let $\quad I = \int \frac{x}{\sqrt{x^2+a^2}+\sqrt{x^2-a^2}}dx$

$$\therefore \quad I = \int \frac{x}{\sqrt{x^2+a^2}+\sqrt{x^2-a^2}} \times \frac{\sqrt{x^2+a^2}-\sqrt{x^2-a^2}}{\sqrt{x^2+a^2}-\sqrt{x^2-a^2}}dx$$

$$= \int \frac{x\left(\sqrt{x^2+a^2}-\sqrt{x^2-a^2}\right)}{x^2+a^2-x^2+a^2}dx$$

$$= \int \frac{x}{2a^2}\left(\sqrt{x^2+a^2}-\sqrt{x^2-a^2}\right)dx$$

$$\therefore \quad I = \frac{1}{2a^2}\int x\left(\sqrt{x^2+a^2}-\sqrt{x^2-a^2}\right)dx \text{ - - - - - - - (i)}$$

Let $\quad x^2 = t \quad$ then,

$d\left(x^2\right) = dt$

$\Rightarrow \quad 2x\,dx = dt$

$\Rightarrow \quad x\,dx = \frac{dt}{2}$

Putting $x^2 = t$ and $x\,dx = \frac{dt}{2}$ in equation (i),

we get

$$I = \frac{1}{2a^2}\int\left(\sqrt{t+a^2}-\sqrt{t-a^2}\right)\frac{dx}{2}$$

$$= \frac{1}{4a^2}\left[\frac{2}{3}\left(t+a^2\right)^{\frac{3}{2}}-\frac{2}{3}\left(t-a^2\right)^{\frac{3}{2}}\right]+c$$

$$\therefore \quad I = \frac{1}{4a^2}\left[\frac{2}{3}\left(x^2+a^2\right)^{\frac{3}{2}}-\frac{2}{3}\left(x^2-a^2\right)^{\frac{3}{2}}\right]+c$$

$$= \frac{1}{6a^2}\left[\left(x^2+a^2\right)^{\frac{3}{2}}-\left(x^2-a^2\right)^{\frac{3}{2}}\right]+c$$

Indefinite Integrals Ex 19.9 Q56

Let $I = \int x \frac{\tan^{-1} x^2}{1+x^4} dx$ ------ (i)

Let $\tan^{-1} x^2 = t$ then,

$d\left(\tan^{-1} x^2\right) = dt$

$\Rightarrow \frac{1 \times 2x}{1+\left(x^2\right)^2} dx = dt$

$\Rightarrow \frac{1 \times x}{1+x^4} dx = \frac{dt}{2}$

Putting $\tan^{-1}x^2 = t$ and $\frac{x}{1+x^4} dx = \frac{dt}{2}$ in equation (i), we get

$I = \int t \frac{dx}{2}$

$= \frac{1}{2} \int t\, dt$

$= \frac{1}{2} \times \frac{t^2}{2} + c$

$\therefore I = \frac{t^2}{4} + c$

$= \frac{\left(\tan^{-1} x^2\right)^2}{4} + c$

$\therefore I = \frac{1}{4}\left(\tan^{-1} x^2\right)^2 + c$

Indefinite Integrals Ex 19.9 Q57

Let $\quad I = \int \frac{\left(\sin^{-1} x\right)^3}{\sqrt{1-x^2}} dx \text{ - - - - (i)}$

Let $\quad \sin^{-1} x = t \quad$ then,

$d\left(\sin^{-1} x\right) = dt$

$\Rightarrow \quad \frac{1}{\sqrt{1-x^2}} dx = dt$

Putting $\sin^{-1} x = t$ and $\frac{1}{\sqrt{1-x^2}} dx = dt$ in equation (i),

we get

$$I = \int t^3 dt$$

$$= \frac{t^4}{4} + c$$

$\therefore \quad I = \frac{1}{4}\left(\sin^{-1} x\right)^4 + c$

Indefinite Integrals Ex 19.9 Q58

Let $I = \int \frac{\sin(2+3\log x)}{x} dx$ - - - - (i)

Let $2+3\log x = t$ then,

$d(2+3\log x) = dt$

$\Rightarrow$ $3\frac{1}{x}dx = dt$

$\Rightarrow$ $\frac{dx}{x} = \frac{dt}{3}$

Putting $2+3\log x = t$ and $\frac{dx}{x} = \frac{dt}{3}$ in equation (i),

we get

$$I = \int \sin t \frac{dt}{3}$$

$$= \frac{1}{3}(-\cos t) + c$$

$$= -\frac{1}{3}\cos(2+3\log x) + c$$

$\therefore$ $I = -\frac{1}{3}\cos(2+3\log x) + c$

Indefinite Integrals Ex 19.9 Q59

Let $I = \int xe^{x^2}\,dx$ ----(i)

Let $x^2 = t$ then,

$d\left(x^2\right) = dt$

$\Rightarrow$ $2x\,dx = dt$

$\Rightarrow$ $x\,dx = \frac{dt}{2}$

Putting $x^2 = t$ and $x\,dx = \frac{dt}{2}$ in equation (i),

we get

$$I = \int e^t \frac{dt}{2}$$

$$= \frac{1}{2}e^t + c$$

$$= \frac{1}{2}e^{x^2} + c$$

$\therefore$ $I = \frac{1}{2}e^{x^2} + c$

Indefinite Integrals Ex 19.9 Q60

Let $I = \int \frac{e^{2x}}{1+e^x} dx ----(i)$

Let $1+e^x = t$ then,

$d(1+e^x) = dt$

$\Rightarrow$ $e^x\, dx = dt$

$\Rightarrow$ $dx = \frac{dt}{e^x}$

Putting $1+e^x = t$ and $dx = \frac{dt}{e^x}$ in equation (i),

we get

$$I = \int \frac{e^{2x}}{t} \times \frac{dt}{e^x}$$

$$= \int \frac{e^x}{t} dt$$

$$= \int \frac{t-1}{t} dt$$

$$= \int \left(\frac{t}{t} - \frac{1}{t}\right) dt$$

$$= t - \log|t| + c$$

$$= (1+e^x) - \log|1+e^x| + c$$

$\therefore$ $I = 1+e^x - \log|1+e^x| + c$

Indefinite Integrals Ex 19.9 Q61

Let $I = \int \frac{\sec^2 \sqrt{x}}{\sqrt{x}} dx$ ----(i)

Let $\sqrt{x} = t$ then,

$d\left(\sqrt{x}\right) = dt$

$\Rightarrow \frac{1}{2\sqrt{x}} dx = dt$

$\Rightarrow dx = 2\sqrt{x}\, dt$

$\Rightarrow dx = 2t\,dt \quad \left[\because \sqrt{x} = t\right]$

Putting $\sqrt{x} = t$ and $dx = 2t\,dt$ in equation (i), we get

$$I = \int \frac{\sec^2 t}{t} \times 2t\,dt$$
$$= 2\int \sec^2 t\,dt$$
$$= 2\tan t + c$$
$$= 2\tan\sqrt{x} + c$$

$\therefore I = 2\tan\sqrt{x} + c$

Indefinite Integrals Ex 19.9 Q62

$$\tan^3 2x \sec 2x = \tan^2 2x \tan 2x \sec 2x$$
$$= \left(\sec^2 2x - 1\right)\tan 2x \sec 2x$$
$$= \sec^2 2x \cdot \tan 2x \sec 2x - \tan 2x \sec 2x$$
$$\therefore \int \tan^3 2x \sec 2x\ dx = \int \sec^2 2x \tan 2x \sec 2x\ dx - \int \tan 2x \sec 2x\ dx$$
$$= \int \sec^2 2x \tan 2x \sec 2x\ dx - \frac{\sec 2x}{2} + C$$

Let $\sec 2x = t$

$\therefore 2\sec 2x \tan 2x\ dx = dt$

$$\therefore \int \tan^3 2x \sec 2x\ dx = \frac{1}{2}\int t^2 dt - \frac{\sec 2x}{2} + C$$
$$= \frac{t^3}{6} - \frac{\sec 2x}{2} + C$$
$$= \frac{\left(\sec 2x\right)^3}{6} - \frac{\sec 2x}{2} + C$$

Indefinite Integrals Ex 19.9 Q63

Let $I = \int \frac{x+\sqrt{x+1}}{x+2}dx$ - - - - - (i)

Let $x+1 = t^2$ then,

$d(x+1) = d(t^2)$

$\Rightarrow \quad dx = 2t\,dt$

Putting $x+1 = t^2$ and $dx = 2t\,dt$ in equation (i), we get

$$I = \int \frac{x+\sqrt{t^2}}{x+2}2t\,dt$$

$$= 2\int \frac{(t^2-1)+t}{(t^2-1)+2}t\,dt \quad [\because \quad x+1 = t^2]$$

$$= 2\int \frac{t^2+t-1}{t^2+1}t\,dt$$

$$= 2\int \frac{t^3+t^2-t}{t^2+1}dt$$

$$= 2\left[\int \frac{t^3}{t^2+1}dt + \int \frac{t^2}{t^2+1}dt - \int \frac{t}{t^2+1}dt\right]$$

$\therefore \quad I = 2\left[\int \frac{t^3}{t^2+1}dt + \int \frac{t^2}{t^2+1}dt - \int \frac{t}{t^2+1}dt\right]$ - - - - (ii)

Let $I_1 = \int \frac{t^3}{t^2+1}dt$

$I_2 = \int \frac{t^2}{t^2+1}dt$

and $I_3 = \int \frac{t}{t^2+1}dt$

Now, $I_1 = \int \frac{t^3}{t^2+1}dt$

$$= \int \left(t - \frac{t}{t^2+1}\right)dt$$

$$= \frac{t^2}{2} - \frac{1}{2}\log(t^2+1)$$

$\therefore \quad I = \frac{t^2}{2} - \frac{1}{2}\log(t^2+1) + c_1$ - - - - (iii)

Since, $I_2 = \int \frac{t^2}{t^2+1}dt$

$$= \int \frac{t^2+1-1}{t^2+1}dt$$

$$= \int \frac{t^2+1}{t^2+1}dt - \int \frac{1}{t^2+1}dt$$

$$= \int dt - \int \frac{1}{t^2+1}dt$$

$\Rightarrow \quad I_2 = t - \tan^{-1}(t^2) + c_2$ - - - - (iv) and, $\quad I_3 = \int \frac{t}{t^2+1} dt$

$= \frac{1}{2}\log\left(1+t^2\right) + c_3$ ------- (v)

Using equations (ii), (iii), (iv) and (v), we get

$$I = 2\left[\frac{t^2}{2} - \frac{1}{2}\log\left(t^2+1\right) + c_1 + t - \tan^{-1}\left(t^2\right) + c_2 - \frac{1}{2}\log\left(1+t^2\right) + c_3\right]$$

$$= 2\left[\frac{t^2}{2} + t - \tan^{-1}\left(t^2\right) - \log\left(1+t^2\right) + c_1 + c_2 + c_3\right]$$

$$= 2\left[\frac{t^2}{2} + t - \tan^{-1}\left(t^2\right) - \log\left(1+t^2\right) + c_4\right] \qquad \left[\text{Putting } c_1 + c_2 + c_3 = c_4\right]$$

$$= t^2 + 2t - 2\tan^{-1}\left(t^2\right) - 2\log\left(1+t^2\right) + 2c_4$$

$$= (x+1) + 2\sqrt{x+1} - 2\tan^{-1}\left(\sqrt{x+1}\right) - 2\log(1+x+1) + 2c_4$$

$$= (x+1) + 2\sqrt{x+1} - 2\tan^{-1}\left(\sqrt{x+1}\right) - 2\log(x+2) + c \qquad \left[\text{Putting } 2c_4 = c\right]$$

$\therefore \quad I = (x+1) + 2\sqrt{x+1} - 2\tan^{-1}\left(\sqrt{x+1}\right) - 2\log(x+2) + c$

Indefinite Integrals Ex 19.9 Q64

Let $\quad I = \int 5^{5^{5^x}} 5^{5^x} 5^x \, dx - - - - - \text{(i)}$

Let $\quad 5^{5^{5^x}} = t$ then,

$$d\left(5^{5^{5^x}}\right) = dt$$

$$\Rightarrow \quad 5^{5^{5^x}} 5^{5^x} 5^x (\log 5)^3 \, dx = dt$$

$$\Rightarrow \quad 5^{5^{5^x}} 5^{5^{5^x}} 5^x \, dx = \frac{dt}{(\log 5)^3}$$

Putting $5^{5^{5^x}} = t$ and $5^{5^{5^x}} 5^{5^x} 5^x \, dx = \dfrac{dt}{(\log 5)^3}$ in equation (i), we get

$$I = \int \frac{dt}{(\log 5)^3}$$

$$= \frac{1}{(\log 5)^3} \int dt$$

$$= \frac{t}{(\log 5)^3} + c$$

$$\therefore \quad I = \frac{5^{5^{5^x}}}{(\log 5)^3} + c$$

Indefinite Integrals Ex 19.9 Q65

Let $I = \int \frac{1}{x\sqrt{x^4 - 1}}\,dx$ ------ (i)

Let $x^2 = t$ then,

$d\left(x^2\right) = dt$

$\Rightarrow \quad 2x\,dx = dt$

$\Rightarrow \quad dx = \frac{dt}{2x}$

Putting $x^2 = t$ and $dx = \frac{dt}{2x}$ in equation (i),

we get

$$I = \int \frac{1}{x\sqrt{t^2 - 1}} \times \frac{dt}{2x}$$

$$= \frac{1}{2}\int \frac{1}{x^2\sqrt{t^2 - 1}}\,dt$$

$$= \frac{1}{2}\int \frac{1}{t\sqrt{t^2 - 1}}\,dt$$

$$= \frac{1}{2}\sec^{-1} t + c$$

$$= \frac{1}{2}\sec^{-1} x^2 + c$$

$\therefore \quad I = \frac{1}{2}\sec^{-1}\left(x^2\right) + c$

Indefinite Integrals Ex 19.9 Q66

Let $I = \int \sqrt{e^x - 1}\, dx$ ------ (i)

Let $e^x - 1 = t^2$ then,

$d\left(e^x - 1\right) = dt\left(t^2\right)$

$\Rightarrow \quad e^x\, dx = 2t\, dt$

$\Rightarrow \quad dx = \frac{2t}{e^x} dt$

$\Rightarrow \quad dx = \frac{2t}{t^2 + 1} dt \qquad \left[\because \quad e^x - 1 = t^2\right]$

Putting $e^x - 1 = t^2$ and $dx = \frac{2t\, dt}{t^2 + 1}$ in equation (i),

we get

$$I = \int \sqrt{t^2} \times \frac{2t\, dt}{t^2 + 1}$$

$$= 2\int \frac{t \times t}{t^2 + 1} dt$$

$$= 2\int \frac{t^2}{t^2 + 1} dt$$

$$= 2\int \frac{t^2 + 1 - 1}{t^2 + 1} dt$$

$$= 2\int \left[\frac{t^2 + 1}{t^2 + 1} - \frac{1}{t^2 + 1}\right] dt$$

$$= 2\int dt - 2\int \frac{1}{t^2 + 1} dt$$

$$= 2t - 2\tan^{-1}\left(t\right) + c$$

$$= 2\sqrt{\left(e^x - 1\right)} - 2\tan^{-1}\left(\sqrt{e^x - 1}\right) + c$$

$\therefore \quad I = 2\sqrt{e^x - 1} - 2\tan^{-1}\sqrt{e^x - 1} + c$

Indefinite Integrals Ex 19.9 Q67

$$I = \int \frac{1}{(x+1)\left(x^2+2x+2\right)}dx$$

$$= \int \frac{1}{(x+1)\left((x+1)^2+1\right)}dx$$

Let $x+1 = \tan u$

$\Rightarrow \quad dx = \sec^2 u \; du$

$$\therefore \quad I = \int \frac{\sec^2 u}{\tan u\left(\tan^2 u+1\right)}du$$

$$= \int \frac{\cos u}{\sin u}du$$

$$= \log\left|\sin u\right| + C$$

$$= \log\left|\frac{\tan u}{\sec^2 u}\right| + C$$

$$= \log\left|\frac{x+1}{\sqrt{x^2+2x+2}}\right| + C$$

Indefinite Integrals Ex 19.9 Q68

Let $I = \int \frac{x^5}{\sqrt{1+x^3}} dx \text{ - - - - (i)}$

Let $1+x^3 = t^2$ then,

$d\left(1+x^3\right) = d\left(t^2\right)$

$\Rightarrow \quad 3x^2 dx = dt\, 2t$

$\Rightarrow \quad dx = \frac{dt}{3x^2} 2t$

Putting $1+x^3 = t^2$ and $dx = \frac{2t}{3x^2} dt$ in equation (i),

we get

$$I = \int \frac{x^5}{\sqrt{t^2}} \times \frac{2t}{3x^2} dt$$

$$= \int \frac{x^5}{t} \times \frac{2t}{3x^2} dt$$

$$= \frac{2}{3} \int x^3 dt$$

$$= \frac{2}{3} \int \left(t^2 - 1\right) dt$$

$$= \frac{2}{3} \times \frac{t^3}{3} - \frac{2}{3} t + c$$

$$\therefore \quad I = \frac{2}{9}\left(1+x^3\right)^{\frac{3}{2}} - \frac{2}{3}\sqrt{1+x^3} + c$$

Indefinite Integrals Ex 19.9 Q69

Let $I = \int 4x^3\sqrt{5 - x^2}\,dx$ - - - - (i)

Let $5 - x^2 = t^2$ then,

$d\left(5 - x^2\right) = t^2$

$\Rightarrow$ $-2x\,dx = 2t\,dt$

$\Rightarrow$ $dx = \frac{-t}{x}dt$

Putting $5 - x^2 = t^2$ and $dx = \frac{-t}{x}dt$ in equation (i),

we get

$$I = \int 4x^3\sqrt{t^2} \times \frac{-t}{x}dt$$

$$= -4\int x^2 t \times t\,dt$$

$$= -4\int\left(5 - t^2\right)t^2 dt \quad \left[\because 5 - x^2 = t^2\right]$$

$$= -4\int\left(5t^2 - t^4\right)dt$$

$$= -20 \times \frac{t^3}{3} + 4\frac{t^5}{5} + c$$

$$= \frac{-20}{3} \times t^3 + \frac{4}{5} \times t^5 + c$$

$$= \frac{-20}{3} \times \left(5 - x^2\right)^{\frac{3}{2}} + \frac{4}{5} \times \left(5 - x^2\right)^{\frac{5}{2}} + c$$

$\therefore$ $I = \frac{-20}{3} \times \left(5 - x^2\right)^{\frac{3}{2}} + \frac{4}{5} \times \left(5 - x^2\right)^{\frac{5}{2}} + c$

Indefinite Integrals Ex 19.9 Q70

Let $I = \int \frac{1}{\sqrt{x} + x} dx$ - - - - (i)

Let $\sqrt{x} = t$ then,

$d\left(\sqrt{x}\right) = dt$

$\Rightarrow \quad \frac{1}{2\sqrt{x}} dx = dt$

$\Rightarrow \quad dx = 2\sqrt{x}\, dt$

Putting $\sqrt{x} = t$ and $2\sqrt{x}\, dt = dx$ in equation (i),
we get

$$I = \int \frac{1}{t + t^2} 2t \times dt \quad \left[\begin{array}{l}\because \sqrt{x} = t \\ \Rightarrow x = t^2\end{array}\right]$$

$$= \int \frac{2t}{t(1+t)} dt$$

$$= 2\int \frac{t}{(1+t)} dt$$

$$= 2\log|1+t| + c$$

$$= 2\log\left|1+\sqrt{x}\right| + c$$

$$\therefore \quad I = 2\log\left|1+\sqrt{x}\right| + c$$

Indefinite Integrals Ex 19.9 Q71

$$\frac{1}{x^2\left(x^4+1\right)^{\frac{3}{4}}}$$

Multiplying and dividing by x^{-3}, we obtain

$$\frac{x^{-3}}{x^2\cdot x^{-3}\left(x^4+1\right)^{\frac{3}{4}}}=\frac{x^{-3}\left(x^4+1\right)^{\frac{-3}{4}}}{x^2\cdot x^{-3}}$$

$$=\frac{\left(x^4+1\right)^{\frac{-3}{4}}}{x^5\cdot\left(x^4\right)^{-\frac{3}{4}}}$$

$$=\frac{1}{x^5}\left(\frac{x^4+1}{x^4}\right)^{-\frac{3}{4}}$$

$$=\frac{1}{x^5}\left(1+\frac{1}{x^4}\right)^{-\frac{3}{4}}$$

Let $\frac{1}{x^4}=t \Rightarrow -\frac{4}{x^5}dx=dt \Rightarrow \frac{1}{x^5}dx=-\frac{dt}{4}$

$$\therefore \int\frac{1}{x^2\left(x^4+1\right)^{\frac{3}{4}}}dx=\int\frac{1}{x^5}\left(1+\frac{1}{x^4}\right)^{-\frac{3}{4}}dx$$

$$=-\frac{1}{4}\int(1+t)^{-\frac{3}{4}}dt$$

$$=-\frac{1}{4}\left[\frac{(1+t)^{\frac{1}{4}}}{\frac{1}{4}}\right]+C$$

$$=-\frac{1}{4}\frac{\left(1+\frac{1}{x^4}\right)^{\frac{1}{4}}}{\frac{1}{4}}+C$$

$$=-\left(1+\frac{1}{x^4}\right)^{\frac{1}{4}}+C$$

Indefinite Integrals Ex 19.9 Q72

Let $I = \int \frac{\sin^5 x}{\cos^4 x} dx \text{ - - - - (i)}$

Let $\cos x = t$ then,

$d(\cos x) = dt$

$\Rightarrow$ $-\sin x \, dx = dt$

$\Rightarrow$ $dx = -\frac{dt}{\sin x}$

Putting $\cos x = t$ and $dx = -\frac{dt}{\sin x}$ in equation (i),

we get

$$I = \int \frac{\sin^5 x}{t^4} \times -\frac{dt}{\sin x}$$

$$= -\int \frac{\sin^4 x}{t^4} dt$$

$$= -\int \frac{\left(1-\cos^2 x\right)^2}{t^4} dt$$

$$= -\int \frac{\left(1-t^2\right)^2}{t^4} dt$$

$$= -\int \frac{1+t^4-2t^2}{t^4} dt$$

$$= -\int \left(\frac{1}{t^4} + \frac{t^4}{t^4} - \frac{2t^2}{t^4}\right) dt$$

$$= -\int \left(t^{-4} + 1 - 2t^{-2}\right) dt$$

$$= -\left[\frac{t^{-3}}{-3} + t - 2\frac{t^{-1}}{-1}\right] + c$$

$$= -\left[-\frac{1}{3} \times \frac{1}{t^3} + t + \frac{2}{t}\right] + c$$

$$= \frac{1}{3} \times \frac{1}{t^3} - t - \frac{2}{t} + c$$

$$= \frac{1}{3} \times \frac{1}{\cos^3 x} - \cos x - \frac{2}{\cos x} + c$$

$\therefore$ $I = -\cos x - \frac{2}{\cos x} + \frac{1}{3\cos^3 x} + c$