RD Sharma Solutions Class 12 Maths Chapter 19 Ex 19.6

Evaluate the integral as follows

$$\int \cos^4 2x dx = \int \left(\cos^2 2x\right)^2 dx$$

$$= \int \left(\frac{1}{2}(\cos 4x + 1)\right)^2 dx$$

$$= \int \left(\frac{1}{4}(\cos^2 4x) + \frac{1}{4} + \frac{\cos 4x}{2}\right) dx$$

$$= \int \left(\frac{1}{4}\left(\frac{1}{2}(\cos 8x + 1)\right) + \frac{1}{4} + \frac{\cos 4x}{2}\right) dx$$

$$= \int \frac{1}{8}\left(\cos 8x + \frac{3}{8} + \frac{\cos 4x}{2}\right) dx$$

$$= \frac{1}{64}\sin 8x + \frac{3}{8}x + \frac{1}{8}\sin 4x + C$$

Indefinite Integrals Ex 19.6 Q4

Let $I = \int \sin^2 bx dx$. Then,

$$I = \int \frac{1 - \cos 2bx}{2} x dx$$

$$= \frac{1}{2}\int dx - \frac{1}{2}\int \cos 2bx dx$$

$$= \frac{1}{2}x - \frac{1}{2}\frac{\sin(2bx)}{2b} + c$$

$$\therefore \quad I = \frac{x}{2} - \frac{\sin 2bx}{4b} + c$$

Indefinite Integrals Ex 19.6 Q5

Let $I = \int \sin^2 \frac{x}{2} dx$. Then,

$$I = \frac{1}{2} \int 2 \sin^2 \frac{x}{2} dx$$

$$= \frac{1}{2} \int (1 - \cos x) dx \qquad \left[\because \cos 2x = 1 - 2\sin^2 x\right]$$

$$= \frac{1}{2} \int dx - \frac{1}{2} \int \cos x dx$$

$$= \frac{1}{2} \times x - \frac{1}{2} \times \sin x + c$$

$$= \frac{1}{2} (x - \sin x) + c$$

$$\therefore \quad I = \frac{1}{2} (x - \sin x) + c.$$

Indefinite Integrals Ex 19.6 Q6

We have,

$$\int \cos^2 \frac{x}{2} dx = \frac{1}{2} \int 2 \cos^2 \frac{x}{2} dx$$

$$= \frac{1}{2} \int 1 + \cos x dx$$

$$= \frac{1}{2} \int dx + \frac{1}{2} \int \cos x dx$$

$$= \frac{1}{2} \times x + \frac{1}{2} \sin x + c$$

$$= \frac{1}{2} (x + \sin x) + c$$

$$\therefore \quad \int \cos^2 \frac{x}{2} = \frac{1}{2} (x + \sin x) + c.$$

Indefinite Integrals Ex 19.6 Q7

Let $I = \int \cos^2 nx dx$. Then,

$$I = \frac{1}{2}\int \ 2\cos^2 nx dx$$
$$= \frac{1}{2}\int[1+\cos 2nx]dx$$
$$= \frac{1}{2}\int\left[x+\frac{\sin 2nx}{2n}\right]+c$$
$$= \frac{x}{2}+\frac{1}{4n}\times\sin 2nx+c$$

$$\therefore \qquad I = \frac{x}{2}+\frac{1}{4n}\times\sin 2nx+c.$$

Indefinite Integrals Ex 19.6 Q8

Let $I = \int \sin\sqrt{1-\cos 2x}\ dx$. Then,

$$I = \int \sin x \times \sqrt{2\sin^2 x} \times dx$$
$$= \int \sin x \times \sqrt{2} \times \sin x dx$$
$$= \sqrt{2}\int \sin^2 x \times dx$$
$$= \frac{\sqrt{2}}{2}\int 2\sin^2 x \times dx$$
$$= \frac{\sqrt{2}}{2}\left[x-\frac{\sin 2x}{2}\right]+c$$
$$= \frac{\sqrt{2}x}{2}-\frac{\sqrt{2}}{4}\times\sin 2x+c$$
$$= \frac{1}{\sqrt{2}}\times x-\frac{\sin 2x}{2\sqrt{2}}+c$$

$$\therefore \qquad I = \frac{1}{\sqrt{2}}\times x-\frac{\sin 2x}{2\sqrt{2}}+c.$$