

RD Sharma
Solutions
Class 12 Maths
Chapter 19
Ex 19.5



Indefinite Integrals Ex 19.5 Q1

$$\text{Let } I = \int \frac{x+1}{\sqrt{2x+3}} dx$$

$$\text{Let } x+1 = \lambda(2x+3) + \mu$$

On equating the coefficients of like powers of x on both sides, we get

$$I = 2\lambda, \quad 3\lambda + \mu = 1$$

$$\Rightarrow \lambda = \frac{1}{2} \text{ and } 3 \times \frac{1}{2} + \mu = 1$$

$$\Rightarrow \lambda = \frac{1}{2} \text{ and } \mu = \frac{-1}{2}$$

Replacing $x+1$ by $\lambda(2x+3) + \mu$ in the given integral, we get

$$\begin{aligned} I &= \int \frac{\lambda(2x+3) + \mu}{\sqrt{2x+3}} dx \\ &= \int \frac{\lambda(2x+3)}{\sqrt{2x+3}} dx + \mu \int \frac{1}{\sqrt{2x+3}} dx \\ &= \lambda \int (2x+3)^{\frac{1}{2}} dx + \mu \int (2x+3)^{-\frac{1}{2}} dx \\ &= \lambda \frac{(2x+3)^{\frac{3}{2}}}{\frac{3}{2} \times 2} + \mu \frac{(2x+3)^{\frac{1}{2}}}{2 \times \frac{1}{2}} + c \\ &= \frac{1}{2} \times \frac{(2x+3)^{\frac{3}{2}}}{3} + \left(\frac{-1}{2}\right) \times (2x+3)^{\frac{1}{2}} + c \quad \left[\because \lambda = \frac{1}{2}, \mu = \frac{-1}{2} \right] \\ &= \frac{(2x+3)^{\frac{3}{2}}}{6} - \frac{(2x+3)^{\frac{1}{2}}}{2} + c \end{aligned}$$

$$\therefore I = \frac{1}{6} \times (2x+3)^{\frac{3}{2}} - \frac{1}{2} (2x+3)^{\frac{1}{2}} + c.$$

Indefinite Integrals Ex 19.5 Q2

Let $I = \int x\sqrt{x+2} dx$. Then,

$$I = \int \{(x+2) - 2\}x\sqrt{x+2} dx \quad \left[\because x = (x+2) - 2 \right]$$

$$\Rightarrow I = \int \left\{ (x+2)^{\frac{3}{2}} - 2(x+2)^{\frac{1}{2}} \right\} dx$$

$$\Rightarrow I = \frac{2}{5} (x+2)^{\frac{5}{2}} - \frac{4}{3} (x+2)^{\frac{3}{2}} + c$$

Indefinite Integrals Ex 19.5 Q3

Let $I = \int \frac{x-1}{\sqrt{x+4}} dx$. Then,

$$\begin{aligned} I &= \int \frac{x+4-4-1}{\sqrt{x+4}} dx \\ &= \int \frac{x+4-5}{\sqrt{x+4}} dx \\ &= \int \frac{x+4}{\sqrt{x+4}} dx - 5 \int \frac{1}{\sqrt{x+4}} dx \\ &= \int (x+4)^{\frac{1}{2}} dx - 5 \int (x+4)^{-\frac{1}{2}} dx \\ &= \frac{(x+4)^{\frac{3}{2}}}{\frac{3}{2}} - 5 \frac{(x+4)^{\frac{1}{2}}}{\frac{1}{2}} + C \\ &= \frac{2}{3} \times (x+4)^{\frac{3}{2}} - 10 (x+4)^{\frac{1}{2}} + C \end{aligned}$$

$$\therefore I = \frac{2}{3} \times (x+4)^{\frac{3}{2}} - 10 (x+4)^{\frac{1}{2}} + C$$

$$\text{Let } I = \int (x+2) \sqrt{3x+5} dx$$

Let $x+2 = \lambda(3x+5) + \mu$ on equating the coefficients of like powers of x on both sides, we get

$$3\lambda = 1 \quad \text{and} \quad 5\lambda + \mu = 2$$

$$\Rightarrow \lambda = \frac{1}{3} \quad \text{and} \quad 5 \times \frac{1}{3} + \mu = 2$$

$$\Rightarrow \lambda = \frac{1}{3} \quad \text{and} \quad \mu = \frac{1}{3}$$

$$\begin{aligned} \therefore I &= \int \left\{ \lambda(3x+5) + \mu \right\} \sqrt{3x+5} dx \\ &= \lambda \int (3x+5) \sqrt{3x+5} dx + \mu \int \sqrt{3x+5} dx \\ &= \lambda \int (3x+5)^{\frac{3}{2}} dx + \mu \int (3x+5)^{\frac{1}{2}} dx \\ &= \lambda \times \frac{(3x+5)^{\frac{5}{2}}}{\frac{5}{2} \times 3} + \mu \frac{(3x+5)^{\frac{3}{2}}}{3 \times \frac{3}{2}} + C \\ &= \frac{1}{3} \times \frac{2}{15} \times (3x+5)^{\frac{5}{2}} + \frac{1}{3} \times \frac{2}{9} (3x+5)^{\frac{3}{2}} + C \\ &= \frac{2}{45} \times (3x+5)^{\frac{5}{2}} + \frac{2}{27} \times (3x+5)^{\frac{3}{2}} + C \\ &= \frac{2}{9} \times (3x+5)^{\frac{3}{2}} \left[\frac{1}{5} \times (3x+5)^1 + \frac{1}{3} \right] + C \\ &= \frac{2}{9} \times (3x+5)^{\frac{3}{2}} \left[\frac{3(3x+5) + 5}{15} \right] + C \\ &= \frac{2}{9} \times (3x+5)^{\frac{3}{2}} \frac{(9x+15+5)}{15} + C \\ &= \frac{2}{135} \times (3x+5)^{\frac{3}{2}} (9x+20) + C \end{aligned}$$

$$\therefore I = \frac{2}{135} \times (9x+20) (3x+5)^{\frac{3}{2}} + C.$$

$$\text{Let } I = \int \frac{2x+1}{\sqrt{3x+2}} dx$$

Let $2x+1 = \lambda(3x+2) + \mu$ on equating the coefficients of like powers of x on both sides, we get

$$3\lambda = 2 \quad \text{and} \quad 2\lambda + \mu = 1$$

$$\Rightarrow \lambda = \frac{2}{3} \quad \text{and} \quad 2 \times \frac{2}{3} + \mu = 1$$

$$\Rightarrow \lambda = \frac{2}{3} \quad \text{and} \quad \mu = \frac{-1}{3}$$

$$\begin{aligned} \therefore I &= \int \frac{\lambda(3x+2) + \mu}{\sqrt{3x+2}} dx \\ &= \lambda \int \frac{3x+2}{\sqrt{3x+2}} dx + \mu \int \frac{1}{\sqrt{3x+2}} dx \\ &= \lambda \int (3x+2)^{\frac{1}{2}} dx + \mu \int (3x+2)^{-\frac{1}{2}} dx \\ &= \lambda \times \frac{(3x+2)^{\frac{3}{2}}}{\frac{3}{2} \times 3} + \mu \frac{(3x+2)^{\frac{1}{2}}}{\frac{1}{2} \times 3} + c \\ &= \frac{2}{3} \times \frac{2}{9} \times (3x+2)^{\frac{3}{2}} - \frac{1}{3} \times \frac{2}{3} (3x+2)^{\frac{1}{2}} + c \\ &= \frac{4}{27} \times (3x+2)^{\frac{3}{2}} - \frac{2}{9} \times (3x+2)^{\frac{1}{2}} + c \\ &= \frac{2}{9} \times \sqrt{3x+2} \left[\frac{2}{3} \times (3x+2) - 1 \right] + c \\ &= \frac{2}{9} \times \sqrt{3x+2} \left[\frac{6x+4-3}{3} \right] + c \\ &= \frac{2}{27} \times \sqrt{3x+2} (6x+1) + c \end{aligned}$$

$$\therefore I = \frac{2}{27} \times (6x+1) \sqrt{3x+2} + c.$$

$$\text{Let } I = \int \frac{3x+5}{\sqrt{7x+9}} dx$$

Let $3x+5 = \lambda(7x+9) + \mu$ on equating the coefficients of like powers of x on both sides, we get

$$7\lambda = 3 \quad \text{and} \quad 9\lambda + \mu = 5$$

$$\Rightarrow \lambda = \frac{3}{7} \quad \text{and} \quad 9 \times \frac{3}{7} + \mu = 5$$

$$\Rightarrow \lambda = \frac{3}{7} \quad \text{and} \quad \mu = \frac{8}{7}$$

$$\begin{aligned} \therefore I &= \int \frac{\lambda(7x+9) + \mu}{\sqrt{7x+9}} dx \\ &= \lambda \int \frac{7x+9}{\sqrt{7x+9}} dx + \mu \int \frac{1}{\sqrt{7x+9}} dx \\ &= \lambda \int (7x+9)^{\frac{1}{2}} dx + \mu \int (7x+9)^{-\frac{1}{2}} dx \\ &= \lambda \times \frac{(7x+9)^{\frac{3}{2}}}{\frac{3}{2} \times 7} + \mu \times \frac{(7x+9)^{\frac{1}{2}}}{\frac{1}{2} \times 7} + C \\ &= \frac{3}{7} \times \frac{2}{21} \times (7x+9)^{\frac{3}{2}} + \frac{8}{7} \times \frac{2}{7} (7x+9)^{\frac{1}{2}} + C \\ &= \frac{2}{49} \times (7x+9)^{\frac{3}{2}} + \frac{16}{49} \times (7x+9)^{\frac{1}{2}} + C \\ &= \frac{2}{49} \times (7x+9)^{\frac{1}{2}} [7x+9+8] + C \\ &= \frac{2}{49} \times (7x+9)^{\frac{1}{2}} [7x+17] + C \\ &= \frac{2}{49} \times (7x+17) \sqrt{7x+9} + C \end{aligned}$$

Let $I = \int \frac{x}{\sqrt{x+4}} dx$. Then,

$$\begin{aligned} I &= \int \frac{x+4-4}{\sqrt{x+4}} dx \\ &= \int \frac{x+4}{\sqrt{x+4}} dx - 4 \int \frac{1}{\sqrt{x+4}} dx \\ &= \int (x+4)^{\frac{1}{2}} dx - 4 \int (x+4)^{-\frac{1}{2}} dx \\ &= \frac{(x+4)^{\frac{3}{2}}}{\frac{3}{2}} - 4 \frac{(x+4)^{\frac{1}{2}}}{\frac{1}{2}} + C \\ &= \frac{2(x+4)^{\frac{3}{2}}}{3} - 8(x+4)^{\frac{1}{2}} + C \\ &= 2(x+4)^{\frac{1}{2}} \left[\frac{1}{3}(x+4) - 4 \right] + C \\ &= 2(x+4)^{\frac{1}{2}} \left[\frac{(x+4) - 12}{3} \right] + C \\ &= \frac{2}{3}(x+4)^{\frac{1}{2}} [x-8] + C \end{aligned}$$

$$\therefore I = \frac{2}{3} \times (x-8) \sqrt{x+4} + C.$$

Let $I = \int \frac{2-3x}{\sqrt{1+3x}} \times dx$. Then,

$$\begin{aligned} I &= \int \frac{2-3x-1+1}{\sqrt{1+3x}} \times dx \\ &= \int \frac{-3x-1+3}{\sqrt{1+3x}} \times dx \\ &= \int -\frac{(3x+1)}{\sqrt{1+3x}} \times dx + 3 \int \frac{1}{\sqrt{1+3x}} dx \\ &= -1 \int \frac{1+3x}{\sqrt{1+3x}} \times dx + 3 \int \frac{1}{\sqrt{1+3x}} dx \\ &= -1 \int (1+3x)^{\frac{1}{2}} dx + 3 \int (1+3x)^{-\frac{1}{2}} dx \\ &= -1 \times \frac{(1+3x)^{\frac{3}{2}}}{\frac{3}{2} \times 3} + 3 \times \frac{(1+3x)^{\frac{1}{2}}}{\frac{1}{2} \times 3} + c \\ &= -\frac{2}{9} \times (1+3x)^{\frac{3}{2}} + 2(1+3x)^{\frac{1}{2}} + c \\ &= 2(1+3x)^{\frac{1}{2}} \left[-\frac{1}{9}(1+3x)^1 + 1 \right] + c \\ &= 2(1+3x)^{\frac{1}{2}} \left[\frac{-1-3x+9}{9} \right] + c \\ &= 2(1+3x)^{\frac{1}{2}} \left[\frac{8-3x}{9} \right] + c \\ &= \frac{2}{9} \sqrt{1+3x} (8-3x) + c \end{aligned}$$

$$\therefore I = \frac{2}{9} (8-3x) \sqrt{1+3x} + c$$

$$\text{Let } I = \int 5x + 3\sqrt{2x-1} \, dx$$

Let $5x + 3 = \lambda(2x - 1) + \mu$ comparing both sides, we get

$$2\lambda = 5 \quad \text{and} \quad -\lambda + \mu = 3$$

$$\Rightarrow \quad \lambda = \frac{5}{2} \quad \text{and} \quad \frac{-5}{2} + \mu = 3$$

$$\Rightarrow \quad \lambda = \frac{5}{2} \quad \text{and} \quad \mu = \frac{11}{2}$$

$$\begin{aligned} \therefore I &= \int \{ \lambda(2x - 1) + \mu \} \sqrt{2x - 1} \, dx \\ &= \lambda \int (2x - 1) \sqrt{2x - 1} \, dx + \mu \int \sqrt{2x - 1} \, dx \\ &= \lambda \int (2x - 1)^{\frac{3}{2}} \, dx + \mu \int (2x - 1)^{\frac{1}{2}} \, dx \\ &= \lambda \frac{(2x - 1)^{\frac{5}{2}}}{\frac{5}{2} \times 2} + \mu \frac{(2x - 1)^{\frac{3}{2}}}{\frac{3}{2} \times 2} \\ &= \lambda \frac{(2x - 1)^{\frac{5}{2}}}{5} + \mu \frac{(2x - 1)^{\frac{3}{2}}}{3} + C \\ &= \frac{5}{2} \times \frac{(2x - 1)^{\frac{5}{2}}}{5} + \frac{11}{2} \times \frac{(2x - 1)^{\frac{3}{2}}}{3} + C \\ &= \frac{(2x - 1)^{\frac{5}{2}}}{2} + \frac{11}{6} \times (2x - 1)^{\frac{3}{2}} + C \\ &= \frac{1}{2} (2x - 1)^{\frac{3}{2}} \left[(2x - 1) + \frac{11}{3} \right] + C \\ &= \frac{1}{2} \times (2x - 1)^{\frac{3}{2}} \left[\frac{6x + 8}{3} \right] + C \\ &= \frac{1}{2} \times (2x - 1)^{\frac{3}{2}} \times 2 \frac{(3x + 4)}{3} + C \\ &= (2x - 1)^{\frac{3}{2}} \times \frac{(3x + 4)}{3} + C \\ &= \frac{1}{3} \times (3x + 4) (2x - 1)^{\frac{3}{2}} + C \end{aligned}$$

$$\therefore I = \frac{1}{3} \times (3x + 4) (2x - 1)^{\frac{3}{2}} + C.$$