RD Sharma Solutions Class 12 Maths Chapter 19 Ex 19.3

Indefinite Integrals Ex 19.3 Q1

Let $I = \int [2x - 3]^5 + \sqrt{3x + 2}dx$. Then,

$$I = \int (2x-3)^5 dx + \int (3x+2)^{\frac{1}{2}} dx$$

$$= \frac{(2x-3)^6}{2 \times 6} + \frac{(3x+2)^{\frac{3}{2}}}{3 \times \frac{3}{2}} + c$$

$$= \frac{(2x-3)^6}{12} + \frac{2}{9}(3x+2)^{\frac{3}{2}} + c$$

$$\therefore \quad I = \frac{(2x-3)^6}{12} + \frac{2}{9}(3x+2)^{\frac{3}{2}} + c$$

Indefinite Integrals Ex 19.3 Q2

Let $I = \int \left[\frac{1}{(7x-5)^3} + \frac{1}{\sqrt{5x-4}}\right] dx$. Then,

$$I = \int (7x-5)^{-3} dx + \int (5x-4)^{\frac{-1}{2}} dx$$

$$= \frac{(7x-5)^{-2}}{7 \times (-2)} + \frac{(5x-4)^{\frac{1}{2}}}{5 \times \frac{1}{2}} + c$$

$$= -\frac{(7x-5)^{6-2}}{14} + \frac{2}{5}\sqrt{(5x-4)} + c$$

$$\therefore \quad I = \frac{-1}{14}(7x-5)^{-2} + \frac{2}{5} \times \sqrt{5x-4} + c.$$

Indefinite Integrals Ex 19.3 Q3

Let $I = \int \frac{1}{2-3x} + \frac{1}{\sqrt{3x-2}} dx$. Then,

$$I = \int \frac{1}{2-3x} dx + \int \frac{1}{\sqrt{3x-2}} dx$$
$$= \frac{\log|2-3x|}{-3} + \frac{2}{3}(3x-2)^{\frac{1}{2}} c$$
$$= \frac{-1}{3} \times \log|2x-3| + \frac{2}{3} \times \sqrt{3x-2} + c$$

Indefinite Integrals Ex 19.3 Q4

Let $I = \int \frac{x+3}{(x+1)^4} dx$. Then,

$$I = \int \frac{x+1+2}{(x+1)^4} dx$$
$$= \int \frac{x+1}{(x+1)^4} \times dx + 2\int \frac{1}{(x+1)^4} \times dx$$
$$= \int \frac{1}{(x+1)^3} \times dx + 2\int \frac{1}{(x+1)^4} \times dx$$
$$= \int (x+1)^{-3} \times dx + 2\int (x-1)^{-4} dx$$
$$= \frac{(x+1)^{-2}}{-2} + 2\frac{(x+1)^{-3}}{-3} + c$$
$$= -\frac{1}{2} \times \frac{1}{(x+1)^2} - \frac{2}{3} \times \frac{1}{(x+1)^3} + c$$

$$\therefore \quad I = \frac{-1}{2(x+1)^2} - \frac{2}{3(x+1)^3} + c$$

Indefinite Integrals Ex 19.3 Q5

Let $I = \int \frac{1}{\sqrt{x+1} + \sqrt{x}} dx$. Then,

$$I = \int \frac{1}{\sqrt{x+1}+\sqrt{x}} \times \frac{\sqrt{x+1}-\sqrt{x}}{\sqrt{x+1}-\sqrt{x}} \times dx$$
$$= \int \frac{\sqrt{x+1}-\sqrt{x}}{\left(\sqrt{x+1}\right)^2 - \left(\sqrt{x}\right)^2} \times dx$$
$$= \int \frac{\sqrt{x+1}-\sqrt{x}}{x+1-x} \times dx$$
$$= \int \left(\sqrt{x+1}-\sqrt{x}\right) \times dx$$
$$= \int (x+1)^{\frac{1}{2}} dx - \int x^{\frac{1}{2}} dx$$
$$= \frac{2}{3}(x+1)^{\frac{3}{2}} - \frac{2}{3} x^{\frac{3}{2}} + c$$

$$\therefore \quad I = \frac{2}{3}(x+1)^{\frac{3}{2}} - \frac{2}{3} x^{\frac{3}{2}} + c.$$

Indefinite Integrals Ex 19.3 Q6

Let $I = \int \frac{1}{\sqrt{2x+3}+\sqrt{2x-3}}dx$. Then,

$$I = \int \frac{1}{\sqrt{2x+3}+\sqrt{2x-3}} \times \frac{\sqrt{2x+3}-\sqrt{2x-3}}{\sqrt{2x+3}-\sqrt{2x-3}} \times dx$$

$$= \int \frac{\sqrt{2x+3}-\sqrt{2x-3}}{\left(\sqrt{2x+3}\right)^2-\left(\sqrt{2x-3}\right)^2} \times dx$$

$$= \int \frac{\sqrt{2x+3}-\sqrt{2x-3}}{2x+3-2x+3} \times dx$$

$$= \frac{1}{6}\int (2x+3)^{\frac{1}{2}}dx - \frac{1}{6}\int (2x-3)^{\frac{1}{2}}dx$$

$$= \frac{1}{6} \times \frac{(2x+3)^{\frac{3}{2}}}{\frac{3}{2}\times 2} - \frac{1}{6}\frac{(2x-3)^{\frac{3}{2}}}{\frac{3}{2}\times 2} + c$$

$$= \frac{1}{18} \times (2x+3)^{\frac{3}{2}} - \frac{1}{18}(2x-3)^{\frac{3}{2}} + c$$

$$\therefore \qquad I = \frac{1}{18}(2x+3)^{\frac{3}{2}} - \frac{1}{18}(2x-3)^{\frac{3}{2}} + c.$$

Indefinite Integrals Ex 19.3 Q7

Let $I = \int \frac{2x}{(2x+1)^2}dx$. Then,

$$I = \int \frac{2x+1-1}{(2x+1)^2} \times dx$$

$$= \int \frac{2x+1}{(2x+1)^2} \times dx - \int \frac{1}{(2x+1)^2} \times dx$$

$$= \int \frac{1}{2x+1} \times dx - \int (2x+1)^{-2} \times dx$$

$$= \frac{1}{2}\log|2x+1| - \frac{(2x+1)^{-1}}{-1\times 2} + c$$

$$= \frac{1}{2}\log|2x+1| + \frac{1}{2} \times \frac{1}{2x+1} + c$$

$$\therefore \qquad I = \frac{1}{2}\log|2x+1| + \frac{1}{2(2x+1)} + c.$$

Indefinite Integrals Ex 19.3 Q8

Let $I = \int \frac{1}{\sqrt{x+a}+\sqrt{x+b}}dx$. Then,

$$I = \int \frac{1}{\sqrt{x+a}+\sqrt{x+b}} \times \frac{\sqrt{x+a}-\sqrt{x+b}}{\sqrt{x+a}-\sqrt{x+b}} \times dx$$
$$= \int \frac{\sqrt{x+a}-\sqrt{x+b}}{x+a-x-b} \times dx$$
$$= \int \frac{\sqrt{x+a}-\sqrt{x+b}}{a-b} \times dx$$
$$= \frac{1}{a-b}\left[\frac{2}{3}(x+a)^{\frac{3}{2}} - \frac{2}{3}(x+b)^{\frac{3}{2}}\right] + c$$
$$= \frac{2}{3(a-b)}\left[(x+a)^{\frac{3}{2}} - (x+b)^{\frac{3}{2}}\right] + c$$

$$\therefore \quad I = \frac{2}{3(a-b)}\left[(x+a)^{\frac{3}{2}} - (x+b)^{\frac{3}{2}}\right] + c.$$

Indefinite Integrals Ex 19.3 Q9

Let $I = \int \sin\sqrt{1+\cos 2x}\ dx$

$$I = \int \sin x \times \sqrt{2\cos^2 x} \times dx$$
$$= \int \sin x \times \sqrt{2} \times \cos x \times dx$$
$$= \sqrt{2}\int \sin x \times \cos x \times dx$$
$$= \frac{\sqrt{2}}{2}\int 2\sin x \times \cos x \times dx$$
$$= \frac{\sqrt{2}}{2}\int \sin 2x dx$$
$$= \frac{\sqrt{2}}{2} \times \frac{-\cos 2x}{2} + c$$
$$= \frac{-1}{2\sqrt{2}} \times \cos 2x + c$$

$$\therefore \quad I = \frac{-1}{2\sqrt{2}} \times \cos 2x + c$$

Indefinite Integrals Ex 19.3 Q10

Let $I = \int \frac{1+\cos x}{1-\cos x} dx$. Then,

$$I = \int \frac{2\cos^2\frac{x}{2}}{2\sin^2\frac{x}{2}} \times dx$$

$$= \int \frac{\cos^2\frac{x}{2}}{\sin^2\frac{x}{2}} \times dx$$

$$= \int \cot^2\frac{x}{2} \times dx$$

$$= \int \left(\operatorname{cosec}^2\frac{x}{2} - 1\right) dx$$

$$= \frac{-\cot\frac{x}{2}}{\frac{1}{2}} - x + c$$

$$= -2\cot\frac{x}{2} - x + c$$

Indefinite Integrals Ex 19.3 Q11

Let $I = \int \frac{1-\cos x}{1+\cos x} \times dx$. Then,

$$I = \int \frac{2\sin^2\frac{x}{2}}{2\cos^2\frac{x}{2}} \times dx$$

$$= \int \frac{\sin^2\frac{x}{2}}{\cos^2\frac{x}{2}} \times dx$$

$$= \int \tan^2\frac{x}{2} dx$$

$$= \int \left(\sec^2\frac{x}{2} - 1\right) dx$$

$$= \frac{\tan\frac{x}{2}}{\frac{1}{2}} - x + c$$

$$= 2\tan\frac{x}{2} - x + c$$

Indefinite Integrals Ex 19.3 Q12

Let $I = \int \frac{1}{1-\sin\frac{x}{2}}$. Then,

$$I = \int \frac{1}{1-\sin\frac{x}{2}} \times \frac{1+\sin\frac{x}{2}}{1+\sin\frac{x}{2}} dx$$

$$= \int \frac{1+\sin\frac{x}{2}}{1-\sin^2\frac{x}{2}} \times dx$$

$$= \int \frac{1+\sin\frac{x}{2}}{\cos^2\frac{x}{2}} \times dx$$

$$= \int \frac{1}{\cos^2\frac{x}{2}} dx + \int \frac{\sin\frac{x}{2}}{\cos^2\frac{x}{2}} dx$$

$$= \int \sec^2\frac{x}{2} dx + \int \sec\frac{x}{2} \tan\frac{x}{2} dx$$

$$= \frac{\tan\frac{x}{2}}{\frac{1}{2}2} + + \frac{\sec\frac{x}{2}}{\frac{1}{2}} + c$$

$$= 2\tan\frac{x}{2} + 2\sec\frac{x}{2} + c$$

$$\therefore \qquad I = 2\left(\tan\frac{x}{2} + \sec\frac{x}{2}\right) + c$$

Indefinite Integrals Ex 19.3 Q13

Let $I = \int \frac{1}{1+\cos 3x} \times dx$. Then,

$$I = \int \frac{1}{1+\cos 3x} \times \frac{1-\cos 3x}{1-\cos 3x} \times dx$$

$$= \int \frac{1-\cos 3x}{1-\cos^2 3x} \times dx$$

$$= \int \frac{1-\cos 3x}{\sin^2 3x} \times dx$$

$$= \int \left(\frac{1}{\sin^2 3x} - \frac{\cos 3x}{\sin^2 3x}\right) dx$$

$$= \int \left(\operatorname{cosec}^2 3x - \operatorname{cosec} 3x \cot 3x\right) dx$$

$$= \frac{-\cot 3x}{3} + \frac{\operatorname{cosec} 3x}{3} + c$$

$$= \frac{-1}{3} \times \frac{\cos 3x}{\sin 3x} + \frac{1}{3} \times \frac{1}{\sin 3x} + c$$

$$= \frac{1-\cos 3x}{3\sin 3x} + c$$

$$\therefore \qquad I = \frac{1-\cos 3x}{3\sin 3x} + c.$$

Indefinite Integrals Ex 19.3 Q14

$$\text{Consider } I = \int \left(e^{x}+1\right)^{2} e^{x} dx$$

$$\text{let } \left(e^{x}+1\right) = t \rightarrow e^{x} dx = dt$$

$$I = \int \left(e^{x}+1\right)^{2} e^{x} dx$$

$$= \int (t)^{2} dt$$

$$= \frac{t^{3}}{3} + C$$

$$= \frac{\left(e^{x}+1\right)^{3}}{3} + C$$

Indefinite Integrals Ex 19.3 Q15

Let $I = \int \left(e^{x} + \frac{1}{e^{x}}\right)^{2} dx$. Then,

$$I = \int \left(e^{x} + \frac{1}{e^{x}}\right)^{2} \times dx$$

$$= \int \left(e^{2x} + \frac{1}{e^{2x}} + 2\right) dx$$

$$= \frac{e^{2x}}{2} - \frac{1}{2} e^{-2x} + 2x + c$$

$$\therefore \quad I = \frac{1}{2} \times e^{2x} + 2x - \frac{1}{2} \times e^{-2x} + c$$

Indefinite Integrals Ex 19.3 Q16

Let $I = \int \frac{1+\cos 4x}{\cot x - \tan x} \times dx$. Then,

$$I = \int \frac{2\cos^2 2x}{\frac{\cos x}{\sin x} - \frac{\sin x}{\cos x}} dx$$

$$= \int \frac{2\cos^2 2x}{\frac{\cos^2 x - \sin^2 x}{\sin x \cos x}} dx$$

$$= \int \frac{2\cos^2 2x \times \sin x \cos x}{\cos^2 x - \sin^2 x} dx$$

$$= \int \frac{\cos^2 2x \times \sin 2x}{\cos^2 2x} dx$$

$$= \int \cos 2x \times \sin 2x \times dx$$

$$= \frac{1}{2} \int 2 \sin 2x \cos 2x dx$$

$$= \frac{1}{2} \int [\sin(2x + 2x) + \sin(2x - 2x)] dx$$

$$= \frac{1}{2} \int (\sin 4x + \sin 0) dx$$

$$= \frac{1}{2} \int (\sin 4x + 0) dx$$

$$= \frac{1}{2} \int \sin 4x$$

$$= -\frac{1}{2} \times \frac{\cos 4x}{4} + c$$

$$= -\frac{1}{8} \times \cos 4x + c$$

Indefinite Integrals Ex 19.3 Q17

Let $I = \int \frac{1}{\sqrt{x+3} - \sqrt{x+2}} dx$. Then,

$$I = \int \frac{1}{\sqrt{x+3} - \sqrt{x+2}} \times \frac{\sqrt{x+3} + \sqrt{x+2}}{\sqrt{x+3} + \sqrt{x+2}} dx$$

$$= \int \frac{\sqrt{x+3} + \sqrt{x+2}}{x+3-x-2} dx$$

$$= \int \left[(x+3)^{\frac{1}{2}} + (x+2)^{\frac{1}{2}} \right] dx$$

$$= \frac{(x+3)^{\frac{3}{2}}}{\frac{3}{2}} + \frac{(x+2)^{\frac{3}{2}}}{\frac{3}{2}} + c$$

$$= \frac{2}{3} \times (x+3)^{\frac{3}{2}} + \frac{2}{3}(x+2)^{\frac{3}{2}} + c$$

$$= \frac{2}{3} \left\{ (x+3)^{\frac{3}{2}} + (x+2)^{\frac{3}{2}} \right\} + c$$

$$\therefore \quad I = \frac{2}{3} \left\{ (x+3)^{\frac{3}{2}} + (x+2)^{\frac{3}{2}} \right\} + c$$

Indefinite Integrals Ex 19.3 Q18

$$\tan^2(2x-3) = \sec^2(2x-3) - 1$$

Let $2x - 3 = t$

$\Rightarrow 2dx = dt$

$$\Rightarrow \int \tan^2(2x-3)\,dx = \int \left[\left(\sec^2(2x-3)\right) - 1\right] dx$$

$$= \frac{1}{2}\int \left(\sec^2 t\right) dt - \int 1\,dx$$

$$= \frac{1}{2}\int \sec^2 t\,dt - \int 1\,dx$$

$$= \frac{1}{2}\tan t - x + C$$

$$= \frac{1}{2}\tan(2x-3) - x + C$$

Indefinite Integrals Ex 19.3 Q19

$$\text{Consider } I = \int \frac{1}{\cos^2 x(1-\tan x)^2} dx$$

$$= \int \frac{1}{\cos^2 x\left(1 - \frac{\sin x}{\cos x}\right)^2} dx$$

$$= \int \frac{1}{(\cos x - \sin x)^2} dx$$

$$= \int \frac{1}{1 - \sin 2x} dx$$

$$= \int \frac{1}{1 + \cos\left(\frac{\pi}{2} + 2x\right)} dx$$

$$= \int \frac{1}{2\cos^2\left(\frac{\pi}{4} + x\right)} dx$$

$$= \frac{1}{2}\int \sec^2\left(\frac{\pi}{4} + x\right) dx$$