RD Sharma Solutions Class 12 Maths Chapter 19 Ex 19.29

Indefinite Integrals Ex 19.29 Q1

Let $I = \int (x+1)\sqrt{x^2 - x + 1}\,dx$ ---(1)

Let $x + 1 = \lambda \frac{d}{dx}(x^2 - x + 1) + \mu$

$= \lambda(2x - 1) + \mu$

Equating similar terms, we get,

$2\lambda = 1 \Rightarrow \quad \lambda = \frac{1}{2}$

$-\lambda + \mu = 1$

$\Rightarrow \quad \mu = 1 + \lambda = 1 + \frac{1}{2} = \frac{3}{2} \therefore \quad \mu = \frac{3}{2}$

So,

$I = \int \left(\frac{1}{2}(2x-1) + \frac{3}{2}\right)\sqrt{x^2 - x + 1}\,dx$

$= \frac{1}{2}\int (2x-1)\sqrt{x^2 - x + 1}\,dx + \frac{3}{2}\int \sqrt{x^2 - x + 1}\,dx$

Let $x^2 - x + 1 = t$

$\Rightarrow \quad (2x-1)\,dx = dt$

$= \frac{1}{2}\int \sqrt{t}\,dt + \frac{3}{2}\int \sqrt{\left(x - \frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}\,dx$

$= \frac{1}{2}\frac{t^{\frac{3}{2}}}{\frac{3}{2}} + \frac{3}{2}\left\{\frac{\left(x - \frac{1}{2}\right)}{2}\sqrt{x^2 - x + 1} + \frac{3}{8}\log\left|\left(x - \frac{1}{2}\right) + \sqrt{x^2 - x + 1}\right|\right.$

$\Rightarrow \quad I = \frac{1}{3}t^{\frac{3}{2}} + \frac{3}{8}(2x-1)\sqrt{x^2 - x + 1} + \frac{9}{16}.\log\left|\left(x - \frac{1}{2}\right) + \sqrt{x^2 - x + 1}\right| + c$

Hence,

$I = \frac{1}{3}(x^2 - x + 1)^{\frac{3}{2}} + \frac{3}{8}(2x-1)\sqrt{x^2 - x + 1} + \frac{9}{16}\log\left|\left(x - \frac{1}{2}\right) + \sqrt{x^2 - x + 1}\right| + c$

Indefinite Integrals Ex 19.29 Q2

Let $I = \int (x+1)\sqrt{2x^2+3}\,dx$

Let $x + 1 = \lambda \frac{d}{dx}\left(2x^2+3\right) + \mu$

$= \lambda\left(4x\right) + \mu$

Equating similar terms, we get,

$4\lambda = 1 \quad \Rightarrow \quad \lambda = \frac{1}{4}$

$\mu = 1$

$\therefore \quad I = \int \frac{1}{4}\left(4x\right)\sqrt{2x^2+3}\,dx + \int 1.\sqrt{2x^2+3}\,dx$

Let $2x^2 + 3 = t$

$\Rightarrow \quad 4x\,dx = dt$

$I = \frac{1}{4}\int \sqrt{t}\,dt + \sqrt{2}\int \sqrt{x^2+\frac{3}{2}}\,dx$

$= \frac{1}{4}.\frac{t^{\frac{3}{2}}}{\frac{3}{2}} + \sqrt{2}\left\{\frac{x}{2}\sqrt{x^2+\frac{3}{2}} + \frac{3}{4}\log\left|x+\sqrt{x^2+\frac{3}{2}}\right| + c\right.$

Hence,

$I = \frac{1}{6}\left(2x^2+3\right)^{\frac{3}{2}} + \frac{x}{2}\sqrt{2x^2+3} + \frac{3}{2\sqrt{2}}\log\left|x+\sqrt{x^2+\frac{3}{2}}\right| + c$

Indefinite Integrals Ex 19.29 Q3

Let $I = \int (2x-5)\sqrt{2+3x-x^2}\,dx$

Let $2x-5 = \lambda \frac{d}{dx}\left(2+3x-x^2\right)+\mu$

$= \lambda\left(3-2x\right)+\mu$

Equating similar terms, we get,

$-2\lambda = 2 \quad \Rightarrow \quad \lambda = -1$

$3\lambda+\mu = -5 \quad \Rightarrow \quad \mu = -5-3\lambda = -2$

$\therefore \quad \mu = -2$

So,

$I = \int \left(-1\left(3-2x\right)-2\right)\sqrt{2+3x-x^2}\,dx$

$= -\int \left(3-2x\right)\sqrt{2+3x-x^2}\,dx - 2\int \sqrt{2+3x-x^2}\,dx$

Let $2+3x-x^2 = t$

$\Rightarrow \quad \left(3-2x\right)dx = dt$

$I = -\int \sqrt{t}\,dt - 2\int \sqrt{\frac{17}{4}-\left(\frac{9}{4}-3x-x^2\right)}\,dx$

$= -\frac{t^{\frac{3}{2}}}{\frac{3}{2}} - 2\int \sqrt{\left(\frac{\sqrt{17}}{2}\right)^2-\left(x-\frac{3}{2}\right)^2}\,dx$

$\Rightarrow \quad I = -\frac{2}{3}\left(2+3x-x^2\right)^{\frac{3}{2}} - 2\left\{\frac{\left(x-\frac{3}{2}\right)}{2}\sqrt{2+3x-x^2}+\frac{17}{8}\sin^{-1}\left(\frac{x-\frac{3}{2}}{\frac{\sqrt{17}}{2}}\right)\right.+c$

Hence,

$I = -\frac{2}{3}\left(2+3x-x^2\right)^{\frac{3}{2}} - \frac{\left(2x-3\right)}{2}\sqrt{2+3x-x^2} - \frac{17}{8}\sin^{-1}\left(\frac{2x-3}{\sqrt{17}}\right)+c$

Indefinite Integrals Ex 19.29 Q4

Let $I = \int (x+2)\sqrt{x^2+x+1}dx$

Let $x+2 = \lambda \frac{d}{dx}\left(x^2+x+1\right)+\mu$

$= \lambda\left(2x+1\right)+\mu$

Equating similar terms, we get,

$2\lambda = 1 \quad \Rightarrow \quad \lambda = \frac{1}{2}$

$\lambda+\mu = 2 \quad \Rightarrow \quad \mu = 2-\lambda = \frac{3}{2}$

$\therefore \quad \mu = \frac{3}{2}$

$\therefore \quad I = \int\left(\frac{1}{2}(2x+1)+\frac{3}{2}\right)\sqrt{x^2+x+1}dx$

$= \frac{1}{2}\int(2x+1)\sqrt{x^2+x+1}+\frac{3}{2}\int\sqrt{x^2+x+1}dx$

Let $x^2+x+1 = t$

$(2x+1)dx = dt$

$\therefore \quad I = \frac{1}{2}\int\sqrt{t}dt+\frac{3}{2}\int\sqrt{\left(x+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}dx$

$\Rightarrow \quad I = \frac{1}{2}\frac{t^{\frac{3}{2}}}{\frac{3}{2}}+\frac{3}{2}\left\{\frac{\left(x+\frac{1}{2}\right)}{2}\sqrt{x^2+x+1}+\frac{3}{8}\log\left|\left(x+\frac{1}{2}\right)+\sqrt{x^2+x+1}\right|+c\right.$

Hence,

$I = \frac{1}{3}\left(x^2+x+1\right)^{\frac{3}{2}}+\frac{3}{8}(2x+1)\sqrt{x^2+x+1}+\frac{9}{16}\log\left|\left(x+\frac{1}{2}\right)+\sqrt{x^2+x+1}\right|+c$

Indefinite Integrals Ex 19.29 Q5

Let $I = \int (4x+1)\sqrt{x^2 - x - 2}\,dx$

Let $4x + 1 = \lambda \frac{d}{dx}\left(x^2 - x - 2\right) + \mu$

$= \lambda(2x - 1) + \mu$

Equating similar terms, we get,

$2\lambda = 4 \quad \Rightarrow \quad \lambda = 2$

$-\lambda + \mu = 1 \quad \Rightarrow \quad \mu = 3$

So,

$I = \int \left(2(2x-1) + 3\right)\sqrt{x^2 - x - 2}\,dx$

$= 2\int (2x-1)\sqrt{x^2 - x - 2}\,dx + 3\int \sqrt{x^2 - x - 2}\,dx$

Let $x^2 - x - 2 = t$

$(2x - 1)\,dx = dt$

$\therefore \quad I = 2\int \sqrt{t}\,dt + 3\int \sqrt{\left(x - \frac{1}{2}\right)^2 - \left(\frac{3}{2}\right)^2}\,dx$

$\Rightarrow \quad I = 2\frac{t^{\frac{3}{2}}}{\frac{3}{2}} + 3\left\{\frac{\left(x - \frac{1}{2}\right)}{2}\sqrt{x^2 - x - 2} - \frac{9}{8}\log\left|\left(x - \frac{1}{2}\right) + \sqrt{x^2 - x - 2}\right| + c\right.$

Hence,

$I = \frac{4}{3}\left(x^2 - x - 2\right)^{\frac{3}{2}} + \frac{3}{4}(2x - 1)\sqrt{x^2 - x - 2} - \frac{27}{8}\log\left|\left(x - \frac{1}{2}\right) + \sqrt{x^2 - x - 2}\right| + c$

Indefinite Integrals Ex 19.29 Q6

Let $I = \int (x-2)\sqrt{2x^2-6x+5}dx$

Let $x - 2 = \lambda \frac{d}{dx}\left(2x^2 - 6x + 5\right) + \mu$

$= \lambda\left(4x-6\right) + \mu$

Equating similar terms, we get,

$4\lambda = 1 \quad \Rightarrow \quad \lambda = \frac{1}{4}$

$-6\lambda + \mu = -2 \quad \Rightarrow \quad \mu = -2 + 6\lambda = -\frac{2}{4} = -\frac{1}{2}$

$\therefore \quad \mu = -\frac{1}{2}$

So,

$I = \int\left(\frac{1}{4}\left(4x-6\right) + \left(-\frac{1}{2}\right)\right)\sqrt{2x^2-6x+5}dx$

$= \frac{1}{4}\int\left(4x-6\right)\sqrt{2x^2-6x+5} - \frac{1}{2}\int\sqrt{2x^2-6x+5}dx$

Let $2x^2 - 6x + 5 = t$

$\left(4x-6\right)dx = dt$

$\therefore \quad I = \frac{1}{4}\int\sqrt{t}dt - \frac{\sqrt{2}}{2}\int\sqrt{x^2-3x+\frac{5}{2}}dx$

$\Rightarrow \quad I = \frac{1}{4}\frac{t^{\frac{3}{2}}}{\frac{3}{2}} - \frac{1}{\sqrt{2}}\int\sqrt{\left(x-\frac{3}{2}\right)^2 + \left(\frac{1}{2}\right)^2}dx$

$= \frac{1}{6}\left(2x^2-6x+5\right)^{\frac{3}{2}} - \frac{1}{\sqrt{2}}\left\{\frac{\left(x-\frac{3}{2}\right)}{2}\sqrt{x^2-3x+\frac{5}{2}} + \frac{1}{8}\log\left|\left(x-\frac{3}{2}\right) + \sqrt{x^2-3x+\frac{5}{2}}\right| + c\right.$

Hence,

$I = \frac{1}{6}\left(2x^2-6x+5\right)^{\frac{3}{2}} - \frac{1}{8}\left(2x-3\right)\sqrt{2x^2-6x+5} - \frac{1}{8\sqrt{2}}\log\left|\left(x-\frac{3}{2}\right) + \sqrt{x^2-3x+\frac{5}{2}}\right| + c$

Indefinite Integrals Ex 19.29 Q7

Let $I = \int (x+1)\sqrt{x^2+x+1}\,dx$

Let $x + 1 = \lambda \frac{d}{dx}\left(x^2 + x + 1\right) + \mu$

$= \lambda(2x+1) + \mu$

Equating similar terms, we get,

$2\lambda = 1 \Rightarrow \quad \lambda = \frac{1}{2}$

$\lambda + \mu = 1 \Rightarrow \quad \mu = \frac{1}{2}$

So,

$I = \int\left(\frac{1}{2}(2x+1) + \frac{1}{2}\right)\sqrt{x^2+x+1}\,dx$

$= \frac{1}{2}\int(2x+1)\sqrt{x^2+x+1}\,dx + \frac{1}{2}\int\sqrt{x^2+x+1}\,dx$

Let $x^2 + x + 1 = t$

$\Rightarrow \quad (2x+1)\,dx = dt$

$= \frac{1}{2}\int\sqrt{t}\,dt + \frac{1}{2}\int\sqrt{\left(x+\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}\,dx$

$= \frac{1}{2}\cdot\frac{t^{\frac{3}{2}}}{\frac{3}{2}} + \frac{1}{2}\left\{\frac{\left(x+\frac{1}{2}\right)}{2}\sqrt{x^2+x+1} + \frac{3}{8}\log\left|\left(x+\frac{1}{2}\right) + \sqrt{x^2+x+1}\right| + c\right.$

$\Rightarrow \quad I = \frac{1}{3}\left(x^2+x+1\right)^{\frac{3}{2}} + \frac{1}{8}(2x+1)\sqrt{x^2+x+1} + \frac{3}{16}\log\left|\left(x+\frac{1}{2}\right) + \sqrt{x^2+x+1}\right| + c$

Hence,

$I = \frac{1}{3}\left(x^2+x+1\right)^{\frac{3}{2}} + \frac{1}{8}(2x+1)\sqrt{x^2+x+1} + \frac{3}{16}\log\left|\left(x+\frac{1}{2}\right) + \sqrt{x^2+x+1}\right| + c$

Indefinite Integrals Ex 19.29 Q8

Let $I = \int (2x+3)\sqrt{x^2+4x+3}dx$

Let $(2x+3) = \lambda \frac{d}{dx}(x^2+4x+3) + \mu$

$= \lambda(2x+4) + \mu$

Equating similar terms, we get,

$\lambda = 1$ and $4\lambda + \mu = 3$

$\Rightarrow \quad \mu = -1$

So,

$I = \int ((2x+4) + (-1))\sqrt{x^2+4x+3}dx$

$= \int (2x+4)\sqrt{x^2+4x+3}dx - \int \sqrt{x^2+4x+3}dx$

Let $x^2+4x+3 = t$

$\Rightarrow \quad (2x+4)dx = dt$

$\therefore \quad I = \int \sqrt{t}dt - \int \sqrt{(x+2)^2 - 1}dx$

$= \frac{3}{2}t^{\frac{3}{2}} - \frac{(x+2)}{2}\sqrt{x^2+4x+3} + \frac{1}{2}\log\left|(x+2) + \sqrt{x^2+4x+3}\right| + c$

Hence,

$I = \frac{2}{3}(x^2+4x+3)^{\frac{3}{2}} - \left(\frac{x+2}{2}\right)\sqrt{x^2+4x+3} + \frac{1}{2}\log\left|(x+2) + \sqrt{x^2+4x+3}\right| + c$

Indefinite Integrals Ex 19.29 Q9

Let $I = \int (2x - 5)\sqrt{x^2 - 4x + 3}dx$

Let $(2x - 5) = \lambda \frac{d}{dx}(x^2 - 4x + 3) + \mu$

$= \lambda(2x - 4) + \mu$

Equating similar terms, we get,

$\lambda = 1$ and $-4\lambda + \mu = -5$

$\Rightarrow \quad \mu = -1$

So,

$I = \int ((2x - 4) - 1)\sqrt{x^2 - 4x + 3}dx$

$= \int (2x - 4)\sqrt{x^2 - 4x + 3}dx - \int \sqrt{x^2 - 4x + 3}dx$

Let $x^2 - 4x + 3 = t$

$\Rightarrow \quad 2x - 4dx = dt$

$I = \int \sqrt{t}dt - \int \sqrt{(x - 2)^2 - 1}dx$

$= \frac{t^{\frac{3}{2}}}{\frac{3}{2}} - \frac{(x - 2)}{2}\sqrt{x^2 - 4x + 3} + \frac{1}{2}\log\left|(x - 2) + \sqrt{x^2 - 4x + 3}\right| + c$

Thus,

$I = \frac{2}{3}(x^2 - 4x + 3)^{\frac{3}{2}} - \frac{1}{2}(x - 2)\sqrt{x^2 - 4x + 3} + \frac{1}{2}\log\left|(x - 2) + \sqrt{x^2 - 4x + 3}\right| + c$

Indefinite Integrals Ex 19.29 Q10

Let $I = \int x\sqrt{x^2+x}dx$

Let $x = \lambda \frac{d}{dx}\left(x^2+x\right)+\mu$

$= \lambda\left(2x+1\right)+\mu$

Equating similar terms, we get,

$2\lambda = 1 \qquad \Rightarrow \qquad \lambda = \frac{1}{2}$

$\lambda+\mu = 0 \qquad \Rightarrow \qquad \mu = -\frac{1}{2}$

So,

$I = \int\left(\frac{1}{2}\left(2x+1\right)-\frac{1}{2}\right)\sqrt{x^2+x}dx$

$= \frac{1}{2}\int\left(2x+1\right)\sqrt{x^2+x} - \frac{1}{2}\int\sqrt{x^2+x}dx$

Let $x^2+x = t$

$\Rightarrow \qquad \left(2x+1\right)dx = dt$

So,

$I = \frac{1}{2}\int\sqrt{t}dt - \frac{1}{2}\int\sqrt{\left(x+\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2}$

$I = \frac{1}{2}.\frac{t^{\frac{3}{2}}}{\frac{3}{2}} - \frac{1}{2}\left\{\frac{\left(x+\frac{1}{2}\right)}{2}\sqrt{x^2+x} - \frac{1}{8}\log\left|\left(x+\frac{1}{2}\right)+\sqrt{x^2+x}\right|+c\right.$

Hence,

$I = \frac{1}{3}\left(x^2+x\right)^{\frac{3}{2}} - \frac{1}{8}\left(x+\frac{1}{2}\right)\sqrt{x^2+x} + \frac{1}{16}\log\left|\left(x+\frac{1}{2}\right)+\sqrt{x^2+x}\right|+c$

Indefinite Integrals Ex 19.29 Q11

Consider the integral $I=\int (x-3)\sqrt{x^2+3x-18}\,dx$

Let us express $x-3=\lambda\frac{d}{dx}\left[x^2+3x-18\right]+\mu$

$\Rightarrow x-3=\lambda[2x+3]+\mu$

$\Rightarrow x-3=2\lambda x+3\lambda+\mu$

Comparing the coefficients, we have,

$2\lambda=1$ and $3\lambda+\mu=-3$

$\Rightarrow \lambda=\frac{1}{2}$ and $3\times\frac{1}{2}+\mu=-3$

$\Rightarrow \lambda=\frac{1}{2}$ and $\mu=-3-\frac{3}{2}$

$\Rightarrow \lambda=\frac{1}{2}$ and $\mu=-\frac{9}{2}$

Then

$x-3=\lambda[2x+3]+\mu$

Now the integral $I=\int (x-3)\sqrt{x^2+3x-18}\,dx$

$=\int\left[\frac{1}{2}[2x+3]-\frac{9}{2}\right]\sqrt{x^2+3x-18}\,dx$

$I=\frac{1}{2}\int[2x+3]\sqrt{x^2+3x-18}\,dx-\frac{9}{2}\int\sqrt{x^2+3x-18}\,dx$

$\Rightarrow I=I_1+I_2$

where, $I_1=\frac{1}{2}\int[2x+3]\sqrt{x^2+3x-18}\,dx$ and

$I_2=-\frac{9}{2}\int\sqrt{x^2+3x-18}\,dx$

Let us consider the integral, I_1 :

$I_1=\frac{1}{2}\int[2x+3]\sqrt{x^2+3x-18}\,dx$

Substituting, $x^2+3x-18=t$

$\Rightarrow (2x+3)dx=dt$

Thus,

$I_1=\frac{1}{2}\int\sqrt{t}\,dt$

$=\frac{1}{2}\times\frac{t^{\frac{1}{2}+1}}{\frac{1}{2}+1}+C$

$$= \frac{1}{2} \times \frac{t^{\frac{3}{2}}}{\frac{3}{2}} + C$$

$$= \frac{1}{2} \times \frac{2}{3} \times t^{\frac{3}{2}} + C$$

$$= \frac{1}{3} \times t^{\frac{3}{2}} + C$$

$$= \frac{1}{3} \times \left(x^2 + 3x - 18\right)^{\frac{3}{2}} + C$$

Now consider the integral

$$I_2 = -\frac{9}{2}\int \sqrt{x^2 + 3x - 18}\, dx$$

$$= -\frac{9}{2}\int \sqrt{x^2 + 2 \times \frac{3}{2}x + \left(\frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2 - 18}\, dx$$

$$= -\frac{9}{2}\int \sqrt{\left(x + \frac{3}{2}\right)^2 - \frac{9}{4} - 18}\, dx$$

$$= -\frac{9}{2}\int \sqrt{\left(x + \frac{3}{2}\right)^2 - \left(\frac{9}{4} + 18\right)}\, dx$$

$$= -\frac{9}{2}\int \sqrt{\left(x + \frac{3}{2}\right)^2 - \left(\frac{9 + 72}{4}\right)}\, dx$$

$$= -\frac{9}{2}\int \sqrt{\left(x + \frac{3}{2}\right)^2 - \left(\frac{81}{4}\right)}\, dx$$

$$= -\frac{9}{2}\int \sqrt{\left(x + \frac{3}{2}\right)^2 - \left(\frac{9}{2}\right)^2}\, dx$$

$$= -\frac{9}{2}\int \sqrt{\left(x + \frac{3}{2}\right)^2 - \left(\frac{9}{2}\right)^2}\, dx$$

We know that $\int \sqrt{x^2 - a^2}dx = \frac{1}{2}x\sqrt{x^2 - a^2} - \frac{1}{2}a^2\log\left|x + \sqrt{x^2 - a^2}\right| + C$

$$\therefore I_2 = -\frac{9}{2}\left\{\frac{1}{2}\left(x + \frac{3}{2}\right)\sqrt{\left(x + \frac{3}{2}\right)^2 - \left(\frac{9}{2}\right)^2} - \frac{1}{2}\left(\frac{9}{2}\right)^2 \log\left|\left(x + \frac{3}{2}\right) + \sqrt{\left(x + \frac{3}{2}\right)^2 - \left(\frac{9}{2}\right)^2}\right|\right\} + C$$

$$= -\frac{9}{4}\left\{\left(\frac{2x + 3}{2}\right)\sqrt{x^2 + 3x - 18} - \left(\frac{729}{4}\right)\log\left|\left(x + \frac{3}{2}\right) + \sqrt{x^2 + 3x - 18}\right|\right\} + C$$

$$= -\frac{9}{8}(2x + 3)\sqrt{x^2 + 3x - 18} + \frac{729}{16}\log\left|\left(x + \frac{3}{2}\right) + \sqrt{x^2 + 3x - 18}\right| + C$$

Thus, $I = = \frac{1}{3}\left(x^2 + 3x - 18\right)^{\frac{3}{2}} - \frac{9}{8}(2x + 3)\sqrt{x^2 + 3x - 18} + \frac{729}{16}\log\left|\left(x + \frac{3}{2}\right) + \sqrt{x^2 + 3x - 18}\right| + C$

Indefinite Integrals Ex 19.29 Q12

$$\int (x+3)\sqrt{3-4x-x^2}\,dx$$

$$\text{Let } x+3 = A\frac{d}{dx}(3-4x-x^2)+B$$

$$x+3 = A(-4-2x)+B$$

$$x+3 = -2Ax + B - 4A$$

$$-2A = 1, B-4A = 3$$

$$A = -\frac{1}{2},$$

$$B = 4\times\left(-\frac{1}{2}\right)+3 = 1$$

$$\int (x+3)\sqrt{3-4x-x^2}\,dx$$

$$x+3 = -\frac{1}{2}(-4-2x)+1$$

$$\int\left[-\frac{1}{2}(-4-2x)+1\right]\sqrt{3-4x-x^2}\,dx$$

$$= -\frac{1}{2}\int(-4-2x)\sqrt{3-4x-x^2}\,dx + \int\sqrt{3-4x-x^2}\,dx$$

$$= I_1 + I_2 \ldots\ldots(i)$$

$$I_1 = -\frac{1}{2}\int(-4-2x)\sqrt{3-4x-x^2}\,dx$$

$$\text{Let } z = 3-4x-x^2$$

$$dz = -4-2x$$

$$I_1 = -\frac{1}{2}\int\sqrt{z}\,dz$$

$$= -\frac{1}{2}\left[\frac{z^{\frac{1}{2}+1}}{\frac{1}{2}+1}\right]$$

$$= -\frac{1}{2}\left[\frac{z^{\frac{3}{2}}}{\frac{3}{2}}\right]$$

$$= -\left[\frac{(3-4x-x^2)^{\frac{3}{2}}}{3}\right]$$

$$I_2 = \int\sqrt{3-4x-x^2}\,dx$$

$$= \int\sqrt{3-(x^2+4x+4)+4}\,dx$$

$$= \int \sqrt{7 - (x^2 + 4x + 4)}\,dx$$

$$= \int \sqrt{\left(\sqrt{7}\right)^2 - (x+2)^2}\,dx$$

$$= \frac{(x+2)\sqrt{\left(\sqrt{7}\right)^2 - (x+2)^2}}{2} + \frac{1}{2}\left(\sqrt{7}\right)^2 \tan^{-1}\left(\frac{x+2}{\sqrt{7}}\right) + C$$

$$= \frac{(x+2)\sqrt{3 - 4x - x^2}}{2} + \frac{7}{2}\tan^{-1}\left(\frac{x+2}{\sqrt{7}}\right)$$

From(i),

$$= I_1 + I_2$$

$$= -\frac{1}{3}\left(3 - 4x - x^2\right)^{\frac{3}{2}} + \frac{1}{2}(x+2)\sqrt{3 - 4x - x^2} + \frac{7}{2}\tan^{-1}\left(\frac{x+2}{\sqrt{7}}\right) + C$$