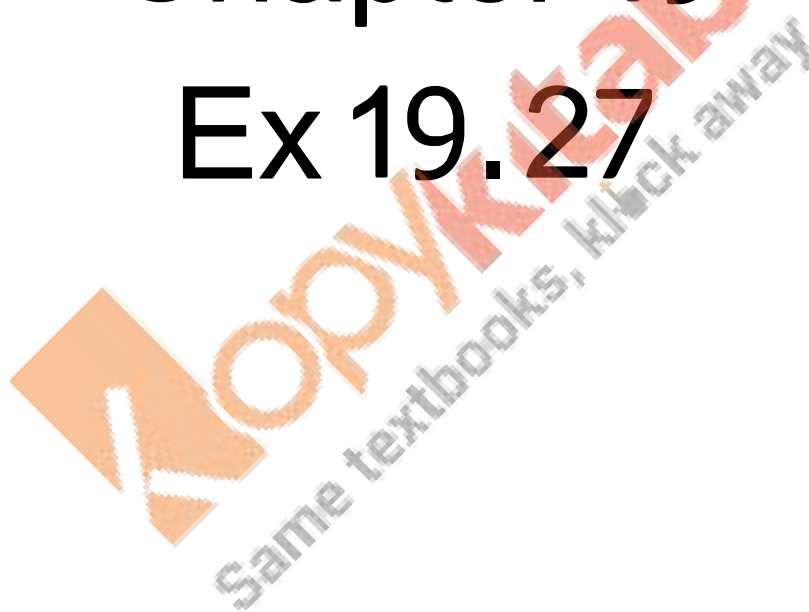


RD Sharma
Solutions Class
12 Maths
Chapter 19
Ex 19.27



Indefinite Integrals Ex 19.27 Q1

$$\text{Let } I = \int e^{ax} \cos bx dx$$

Integrating by parts,

$$\begin{aligned} I &= e^{ax} \frac{\sin bx}{b} - a \int e^{ax} \frac{\sin bx}{b} dx \\ &= \frac{1}{b} e^{ax} \sin bx - \frac{a}{b} \int e^{ax} \sin bx dx \\ &= \frac{1}{b} e^{ax} \sin bx - \frac{a}{b} \left[-e^{ax} \frac{\cos bx}{b} + \int a e^{ax} \frac{\cos bx}{b} dx \right] \\ &= \frac{1}{b} e^{ax} \sin bx + \frac{a}{b^2} e^{ax} \cos bx - \frac{a^2}{b^2} \int e^{ax} \cos bx dx \\ \Rightarrow I &= \frac{e^{ax}}{b^2} [b \sin bx + a \cos bx] - \frac{a^2}{b^2} I + c \\ \Rightarrow I \cdot \left\{ \frac{a^2 + b^2}{b^2} \right\} &= \frac{e^{ax}}{b^2} [b \cos bx + a \cos bx] + c \end{aligned}$$

Thus,

$$I = \frac{e^{ax}}{a^2 + b^2} [a \cos bx + a \cos bx] + c$$

Indefinite Integrals Ex 19.27 Q2

$$\text{Let } I = \int e^{ax} \sin(bx + c) dx$$

$$\begin{aligned} \Rightarrow & -e^{ax} \frac{\cos(bx + c)}{b} + \int ae^{ax} \frac{\cos(bx + c)}{b} dx \\ &= -\frac{1}{b} e^{ax} \cos(bx + c) + \frac{a}{b} \int e^{ax} \cos(bx + c) dx \\ &= -\frac{1}{b} e^{ax} \cos(bx + c) + \frac{a}{b} \left[\int e^{ax} \frac{\sin(bx + c)}{b} - \int ae^{ax} \frac{\sin(bx + c)}{b} dx \right] + c_1 \\ &= \frac{e^{ax}}{b^2} \{a \sin(bx + c) - b \cos(bx + c)\} - \frac{a^2}{b^2} \int e^{ax} \sin(bx + c) dx + c_1 \\ \Rightarrow & I = \frac{e^{ax}}{b^2} \{a \sin(bx + c) - b \cos(bx + c)\} - \frac{a^2}{b^2} I + c_1 \\ \Rightarrow & I = \left\{ \frac{a^2 + b^2}{b^2} \right\} - \frac{e^{ax}}{b^2} \{a \sin(bx + c) - b \cos(bx + c)\} + c_1 \\ \Rightarrow & I = \frac{e^{ax}}{a^2 + b^2} \{a \sin(bx + c) - b \cos(bx + c)\} + c_1 \end{aligned}$$

Let $\log x = t$

$$\Rightarrow \frac{1}{x} dx = dt$$

$$\Rightarrow dx = x dt$$

$$\Rightarrow dx = e^t dt$$

$$\therefore I = \int \cos(\log x) dx = \int e^t \cos t dt$$

We know that

$$\int e^{ax} \cos bx dx = \frac{e^{ax}}{a^2 + b^2} \{a \cos bx + b \sin bx\} + c$$

Here $a = 1$, $b = 1$

$$\text{So, } I = \frac{e^t}{2} \{\cos t + \sin t\} + c$$

Hence,

$$I = \int \cos(\log x) dx = \frac{e^{\log x}}{2} \{\cos(\log x) + \sin(\log x)\} + c$$

$$\Rightarrow I = \frac{x}{2} \{\cos(\log x) + \sin(\log x)\} + c$$

Indefinite Integrals Ex 19.27 Q4

$$\text{Let } I = \int e^{2x} \cos(3x + 4) dx$$

Integrating by parts

$$\begin{aligned} I &= e^{2x} \frac{\sin(3x + 4)}{3} - \int 2e^{2x} \frac{\sin(3x + 4)}{3} dx \\ &= \frac{1}{3} e^{2x} \sin(3x + 4) - \frac{2}{3} \int e^{2x} \sin(3x + 4) dx \\ &= \frac{1}{3} e^{2x} \sin(3x + 4) - \frac{2}{3} \left[-e^{2x} \frac{\cos(3x + 4)}{3} + \int 2e^{2x} \frac{\cos(3x + 4)}{3} dx \right] + c \\ I &= \frac{e^{2x}}{9} \{2 \cos(3x + 4) + 3 \sin(3x + 4)\} + c \end{aligned}$$

Hence,

$$I = \frac{e^{2x}}{13} \{2 \cos(3x + 4) + 3 \sin(3x + 4)\} + c$$

Indefinite Integrals Ex 19.27 Q5

$$\text{Let } I = \int e^{2x} \sin x \cos x dx$$

$$= \frac{1}{2} \int e^{2x} 2 \sin x \cos x dx$$

$$= \frac{1}{2} \int e^{2x} \sin 2x dx$$

We know that

$$\int e^{ax} \sin bx dx = \frac{e^{ax}}{a^2 + b^2} \{a \sin bx - b \cos bx\} + c$$

$$\Rightarrow \int e^{2x} \sin 2x dx = \frac{e^{2x}}{8} \{2 \sin 2x - 2 \cos 2x\} + c$$

$$\therefore I = \frac{1}{2} \cdot \frac{e^{2x}}{8} \{2 \sin 2x - 2 \cos 2x\} + c$$

$$\therefore I = \frac{e^{2x}}{8} \{\sin 2x - \cos 2x\} + c$$

Indefinite Integrals Ex 19.27 Q6

$$\text{Let } I = \int e^{2x} \sin x \, dx \quad \dots(1)$$

Integrating by parts, we obtain

$$I = \sin x \int e^{2x} dx - \int \left\{ \left(\frac{d}{dx} \sin x \right) \int e^{2x} dx \right\} dx$$

$$\Rightarrow I = \sin x \cdot \frac{e^{2x}}{2} - \int \cos x \cdot \frac{e^{2x}}{2} dx$$

$$\Rightarrow I = \frac{e^{2x} \sin x}{2} - \frac{1}{2} \int e^{2x} \cos x \, dx$$

Again integrating by parts, we obtain

$$I = \frac{e^{2x} \cdot \sin x}{2} - \frac{1}{2} \left[\cos x \int e^{2x} dx - \int \left\{ \left(\frac{d}{dx} \cos x \right) \int e^{2x} dx \right\} dx \right]$$

$$\Rightarrow I = \frac{e^{2x} \sin x}{2} - \frac{1}{2} \left[\cos x \cdot \frac{e^{2x}}{2} - \int (-\sin x) \frac{e^{2x}}{2} dx \right]$$

$$\Rightarrow I = \frac{e^{2x} \cdot \sin x}{2} - \frac{1}{2} \left[\frac{e^{2x} \cos x}{2} + \frac{1}{2} \int e^{2x} \sin x \, dx \right]$$

$$\Rightarrow I = \frac{e^{2x} \sin x}{2} - \frac{e^{2x} \cos x}{4} - \frac{1}{4} I$$

[From (1)]

$$\Rightarrow I = \frac{e^{2x} \sin x}{2} - \frac{e^{2x} \cos x}{4} - \frac{1}{4} I$$

$$\Rightarrow I + \frac{1}{4} I = \frac{e^{2x} \cdot \sin x}{2} - \frac{e^{2x} \cos x}{4}$$

$$\Rightarrow \frac{5}{4} I = \frac{e^{2x} \sin x}{2} - \frac{e^{2x} \cos x}{4}$$

$$\Rightarrow I = \frac{4}{5} \left[\frac{e^{2x} \sin x}{2} - \frac{e^{2x} \cos x}{4} \right] + C$$

$$\Rightarrow I = \frac{e^{2x}}{5} [2 \sin x - \cos x] + C$$

$$\text{Let } I = \int e^x \sin^2 x dx$$

$$= \frac{1}{2} \int e^x 2 \sin^2 x dx$$

$$= \frac{1}{2} \int e^x (1 - \cos 2x) dx$$

$$= \frac{1}{2} \int e^x dx - \frac{1}{2} \int e^x \cos 2x dx$$

$$\therefore \int e^{ax} \cos bx dx = \frac{e^{ax}}{a^2 + b^2} \{a \cos bx - b \sin bx\} + c$$

$$\therefore I = \frac{1}{2} \left[e^x - \frac{e^x}{5} \{ \cos 2x + 2 \sin 2x \} \right] + c$$

$$\therefore I = \frac{e^x}{2} - \frac{e^x}{10} \{ \cos 2x + 2 \sin 2x \} + c$$

Indefinite Integrals Ex 19.27 Q9

$$\text{Let } I = \int \frac{1}{x^3} \sin(\log x) dx$$

$$\text{Let } \log x = t$$

$$\Rightarrow \frac{1}{x} dx = dt$$

$$\Rightarrow dx = e^t dt$$

$$\therefore I = \int e^{-2t} \sin t dt$$

We know that,

$$\int e^{ax} \sin bx dx = \frac{e^{ax}}{a^2 + b^2} \{a \sin bx - b \cos bx\} + c$$

$$\therefore \int e^{-2t} \sin t dt = \frac{e^{-2t}}{5} \{-2 \sin t - \cos t\} + c$$

$$\therefore I = \frac{x^{-2}}{5} \{-2 \sin(\log x) - \cos(\log x)\} + c$$

Hence,

$$\int \frac{1}{x^3} \sin(\log x) dx = \frac{-1}{5x^2} \{2 \sin(\log x) + \cos(\log x)\} + c$$

Indefinite Integrals Ex 19.27 Q10

$$\text{Let } I = \int e^{2x} \cos^2 x dx$$

$$= \frac{1}{2} \int e^{2x} 2 \cos^2 x dx$$

$$= \frac{1}{2} \int e^{2x} (1 + \cos 2x) dx$$

$$= \frac{1}{2} \int e^{2x} dx + \frac{1}{2} \int e^{2x} \cos 2x dx$$

$$\therefore \int e^{ax} \cos bx dx = \frac{e^{ax}}{a^2 + b^2} \{a \cos bx - b \sin bx\} + c$$

$$\therefore I = \frac{1}{4} e^{2x} + \frac{1}{2} \frac{e^{2x}}{8} \{2 \cos 2x + 2 \sin 2x\} + c$$

Hence,

$$I = \frac{e^{2x}}{4} + \frac{e^{2x}}{16} \{2 \cos 2x + 2 \sin 2x\} + c$$

or

$$I = \frac{e^{2x}}{4} + \frac{e^{2x}}{8} \{\cos 2x + \sin 2x\} + c$$

Indefinite Integrals Ex 19.27 Q11

$$\text{Let } I = \int e^{-2x} \sin x dx$$

$$\therefore \int e^{ax} \sin bx = \frac{e^{ax}}{a^2 + b^2} \{a \sin bx - b \cos bx\} + c$$

$$\therefore I = \frac{e^{-2x}}{5} \{-2 \sin x - \cos x\} + c$$

Indefinite Integrals Ex 19.27 Q12

$$\text{Let } I = \int x^2 e^{x^3} \cos x^3 dx$$

$$\text{Let } x^3 = t$$

$$\Rightarrow 3x^2 dx = dt$$

$$\therefore I = \frac{1}{3} \int e^t \cos t dt$$

$$\because \int e^{ax} \cos bx dx = \frac{e^{ax}}{a^2 + b^2} \{a \cos bx + b \sin bx\} + c$$

$$\therefore I = \frac{1}{3} \left\{ \frac{e^t}{2} (\cos t + \sin t) \right\} + c$$

$$\therefore I = \frac{1}{3} \left\{ \frac{e^{x^3}}{2} (\cos x^3 + \sin x^3) \right\} + c$$