RD Sharma Solutions Class 12 Maths Chapter 19 Ex 19.26

Indefinite Integrals Ex 19.26 Q1

Let $I = \int e^x (\cos x - \sin x) dx$

$= \int e^x \cos x dx - \int e^x \sin x dx$

Integrating by parts

$$= e^x \cos x - \int e^x \left(\frac{d}{dx} \cos x\right) dx - \int e^x \sin x dx$$

$$= e^x \cos x + \int e^x \sin x dx - \int e^x \sin x dx$$

$$= e^x \cos x + c$$

$\therefore \quad \int e^x (\cos x - \sin x) dx = e^x \cos x + c$

Indefinite Integrals Ex 19.26 Q2

$I = \int e^x (x^{-2} - 2x^{-3}) dx$

$= \int e^x x^{-2} dx - 2\int e^x x^{-3} dx$

Integrating by parts

$$= e^x x^{-2} - \int e^x \left(\frac{d}{dx}(x^{-2})\right) dx - 2\int e^x x^{-3} dx$$

$$= e^x x^{-2} + 2\int e^x x^{-3} dx - 2\int e^x x^{-3} dx$$

$$= \frac{e^x}{x^2} + c$$

$$\int e^x \left(\frac{1}{x^2} - \frac{2}{x^3}\right) dx = \frac{e^x}{x^2} + c$$

Indefinite Integrals Ex 19.26 Q3

$$e^x\left(\frac{1+\sin x}{1+\cos x}\right)$$

$$=e^x\left(\frac{\sin^2\frac{x}{2}+\cos^2\frac{x}{2}+2\sin\frac{x}{2}\ \cos\frac{x}{2}}{2\cos^2\frac{x}{2}}\right)$$

$$=\frac{e^x\left(\sin\frac{x}{2}+\cos\frac{x}{2}\right)^2}{2\cos^2\frac{x}{2}}$$

$$=\frac{1}{2}e^x\cdot\left(\frac{\sin\frac{x}{2}+\cos\frac{x}{2}}{\cos\frac{x}{2}}\right)^2$$

$$=\frac{1}{2}e^x\left[\tan\frac{x}{2}+1\right]^2$$

$$=\frac{1}{2}e^2\left(1+\tan\frac{x}{2}\right)^2$$

$$=\frac{1}{2}e^x\left[1+\tan^2\frac{x}{2}+2\tan\frac{x}{2}\right]$$

$$=\frac{1}{2}e^x\left[\sec^2\frac{x}{2}+2\tan\frac{x}{2}\right]$$

$$\frac{e^x(1+\sin x)dx}{(1+\cos x)}=e^x\left[\frac{1}{2}\sec^2\frac{x}{2}+\tan\frac{x}{2}\right] \qquad \ldots(1)$$

Let $\tan\frac{x}{2}=f(x)\Rightarrow f'(x)=\frac{1}{2}\sec^2\frac{x}{2}$

It is known that, $\int e^x\{f(x)+f'(x)\}dx=e^xf(x)+C$

From equation (1), we obtain

$$\int\frac{e^x(1+\sin x)}{(1+\cos x)}dx=e^x\tan\frac{x}{2}+C$$

Indefinite Integrals Ex 19.26 Q4

Let $I = \int e^x \left(\cot x - \operatorname{cosec}^2 x\right) dx$

$$= \int e^x \cot x dx - \int e^x \operatorname{cosec}^2 x dx$$

Integrating by parts

$$= e^x \cot x - \int e^x \left(\frac{d}{dx} \cot x\right) dx - \int e^x \operatorname{cosec}^2 x dx$$

$$= e^x \cot x + \int e^x \operatorname{cosec}^2 x dx - \int e^x \operatorname{cosec}^2 x dx$$

$$= e^x \cot x + c$$

$$\int e^x \left(\cot x - \operatorname{cosec}^2 x\right) dx = e^x \cot x + c$$

Indefinite Integrals Ex 19.26 Q5

Let $I = \int e^x \frac{1}{2x} dx - \int e^x \frac{1}{2x^2} dx$

Integrating by parts

$$= \frac{e^x}{2x} - \int e^x \left(\frac{d}{dx}\left(\frac{1}{2x}\right)\right) dx - \int \frac{e^x}{2x^2} dx$$

$$= \frac{e^x}{2x} + \int \frac{e^x}{2x^2} dx - \int \frac{e^x}{2x^2} dx$$

$$= \frac{e^x}{2x} + c$$

Indefinite Integrals Ex 19.26 Q7

Let $I = \int e^x \left(\tan x - \log \cos x\right) dx$

$$= \int e^x \tan x dx - \int e^x \log \cos x dx$$

Integrating by parts

$$= \int e^x \tan x dx - \left\{e^x \log \cos x - \int e^x \left(\frac{d}{dx} \log \cos x\right) dx\right\}$$

$$= \int e^x \tan x dx - \left\{e^x \log \cos x + \int e^x \tan x dx\right\}$$

$$= \int e^x \tan x dx - e^x \log \cos x - \int e^x \tan x dx + c$$

$$= -e^x \log \cos x + c$$

$$= e^x \log \sec x + c \qquad \left[\because \quad \log \sec x = -\log \cos x\right]$$

Indefinite Integrals Ex 19.26 Q8

Let $I = \int e^x \left[\sec x + \log(\sec x + \tan x)\right] dx$

$$= \int e^x \sec x dx + \int e^x \log(\sec x + \tan x) dx$$

Integrating by parts

$$= \int e^x \sec x dx + e^x \log(\sec x + \tan x) - \int e^x \left\{\frac{d}{dx} \log(\sec x + \tan x)\right\} dx$$

$$= \int e^x \sec x dx + e^x \log(\sec x + \tan x) - \int e^x \sec x dx$$

$$= e^x \log(\sec x + \tan x) + c$$

Indefinite Integrals Ex 19.26 Q9

Let $I = \int e^x (\cot x + \log \sin x) dx$

$$= \int e^x \cot x dx + \int e^x \log \sin x dx$$

Integrating by parts

$$= \int e^x \log \sin x dx + \int e^x \cot x dx$$

$$= (\log \sin x) e^x - \int e^x \left(\frac{d}{dx} \log \sin x\right) dx + \int e^x \cot x dx$$

$$= e^x \log \sin x - \int e^x \cot x dx + \int e^x \cot x dx$$

$$= e^x \log \sin x + c$$

Indefinite Integrals Ex 19.26 Q10

Let $I = \int e^x \frac{x+1-2}{(x+1)^3}$

$$= \int e^x \left\{ \frac{1}{(x+1)^2} + \frac{-2}{(x+1)^3} \right\} dx$$

$$= \int e^x \frac{1}{(x+1)^2} dx + \int e^x \frac{(-2)}{(x+1)^3} dx$$

Integrating by parts

$$= e^x \frac{1}{(x+1)^2} - \int e^x \left(\frac{d}{dx}(x+1)^{-2} \right) dx + \int e^x \frac{(-2)}{(x+1)^3} dx$$

$$= e^x \frac{1}{(x+1)^2} - \int e^x \frac{(-2)}{(x+1)^3} dx + \int e^x \frac{(-2)}{(x+1)^3} dx$$

$$= e^x \frac{1}{(x+1)^2} + c$$

Indefinite Integrals Ex 19.26 Q11

Let $I = \int e^x \left(\frac{\sin 4x - 4}{2 \sin^2 2x} \right) dx$

$$= \int e^x \left\{ \frac{2 \sin 2x \cos 2x}{2 \sin^2 2x} - \frac{4}{2 \sin^2 2x} \right\} dx$$

$$= \int e^x \left(\cot 2x - 2 \operatorname{cosec}^2 2x \right) dx$$

$$= \int e^x \cot 2x dx - 2 \int e^x \operatorname{cosec}^2 2x dx$$

Integrating by parts

$$= e^x \cot 2x - \int e^x \frac{d}{dx} (\cot 2x) dx - 2 \int e^x \operatorname{cosec}^2 2x dx$$

$$= e^x \cot 2x + 2 \int e^x \operatorname{cosec}^2 2x - 2 \int e^x \operatorname{cosec}^2 2x dx$$

$$= e^x \cot 2x + c$$

Indefinite Integrals Ex 19.26 Q12

Let $I = \int \frac{2-x}{(1-x)^2} e^x dx$

$$= \int e^x \left\{ \frac{(1-x)+1}{(1-x)^2} \right\} dx$$

$$= \int e^x \left\{ \frac{1}{1-x} + \frac{1}{(1-x)^2} \right\} dx$$

Here, $f(x) = \frac{1}{1-x}$ and $f'(x) = \frac{1}{(1-x)^2}$

And we know that,

$$\int e^{ax} \left(af(x) + f'(x) \right) dx = e^{ax} f(x) + c$$

$$\therefore \quad \int e^x \left\{ \frac{1}{1-x} + \frac{1}{(1-x)^2} \right\} dx = e^x \cdot \frac{1}{1-x} + c$$

Hence,

$$I = \frac{e^x}{1-x} + c$$

Indefinite Integrals Ex 19.26 Q13

Let $I = \int e^x \frac{1+x}{(2+x)^2} dx$

$$= \int e^x \left(\frac{x+2-1}{(2+x)^2} \right) dx$$

$$= \int e^x \left\{ \frac{1}{x+2} - \frac{1}{(x+2)^2} \right\} dx$$

$$= \int e^x \frac{1}{x+2} dx - \int e^x \frac{1}{(x+2)^2} dx$$

Integrating by parts

$$= e^x \frac{1}{x+2} - \int e^x \left(\frac{d}{dx} \left(\frac{1}{x+2} \right) \right) dx - \int e^x \frac{1}{(x+2)^2} dx$$

$$= e^x \frac{1}{x+2} + \int e^x \frac{1}{(x+2)^2} dx - \int e^x \frac{1}{(x+2)^2} dx$$

$$= \frac{e^x}{x+2} + c$$

Indefinite Integrals Ex 19.26 Q14

Let $I = \int \frac{\sqrt{1-\sin x}}{1+\cos x} e^{-\frac{x}{2}} dx$

Put $= \frac{x}{2} = t$

$\Rightarrow \quad x = 2t$

$dx = 2dt$

$\therefore \quad \int \frac{\sqrt{1-\sin x}}{1+\cos x} e^{-\frac{x}{2}} dx$

$= 2\int \frac{\sqrt{1-\sin 2t}}{1+\cos 2t} e^{-t} dt \qquad \left[\because \sin^2 t + \cos^2 t = 1\right]$

$= 2\int \frac{\sqrt{\sin^2 t + \cos^2 t - 2\sin t \cos t}}{1+\cos 2t} e^{-t} dt$

$= 2\int \frac{\sqrt{(\cos t - \sin t)^2}}{2\cos^2 t} e^{-t} dt$

$= 2\int \frac{(\cos t - \sin t)}{2\cos^2 t} e^{-t} dt$

$= \int (\sec t - \tan t \sec t) e^{-t} dt$

$= \int \sec t\, e^{-t} dt - \int \tan t \sec t\, e^{-t} dt$

Integrating by parts

$= e^{-t} \sec t + \int e^{-t} \frac{d}{dt}(\sec t) dt - \int \tan t \sec t\, e^{-t} dt$

$= -e^{-t} \sec t + \int e^{-t} \sec t \tan t dt - \int \sec t \tan t\, e^{-t} dt$

$= -e^{-t} \sec t + c$

Putting the value of t

$= -e^{-\frac{x}{2}} \sec \frac{x}{2} + c$

Indefinite Integrals Ex 19.26 Q15

We have,

$$I = \int e^x \left(\log x + \frac{1}{x}\right) dx$$

We know that

$$\int e^{ax}\left(af(x) + f'(x)\right) dx = e^{ax} f(x) + c$$

Here $f(x) = \log x$ and $f'(x) = \frac{1}{x}$

$$\therefore \qquad \int e^x \left(\log x + \frac{1}{x}\right) dx = e^x \log x + c$$

Indefinite Integrals Ex 19.26 Q16

We have,

$$I = \int e^x \left(\log x + \frac{1}{x^2}\right) dx$$

$$= \int e^x \left(\log x + \frac{1}{x} - \frac{1}{x} + \frac{1}{x^2}\right) dx$$

$$= \int e^x \left(\log x - \frac{1}{x}\right) dx + \int e^x \left(\frac{1}{x} + \frac{1}{x^2}\right) dx$$

Integrating by parts

$$= e^x \left(\log x - \frac{1}{x}\right) - \int e^x \frac{d}{dx}\left(\log x - \frac{1}{x}\right) dx + \int e^x \left(\frac{1}{x} + \frac{1}{x^2}\right) dx$$

$$= e^x \left(\log x - \frac{1}{x}\right) - \int e^x \left(\frac{1}{x} + \frac{1}{x^2}\right) dx + \int e^x \left(\frac{1}{x} + \frac{1}{x^2}\right) dx$$

$$= e^x \left(\log x - \frac{1}{x}\right) + c$$

Indefinite Integrals Ex 19.26 Q18

$$\text{Let } I = \int e^x \left(\sin^{-1} x + \frac{1}{\sqrt{1-x^2}}\right) dx$$

$$I = \int e^x \sin^{-1} x + \int e^x \frac{1}{\sqrt{1-x^2}} dx$$

Integrating by parts

$$= e^x \sin^{-1} x - \int e^x \left(\frac{d}{dx}\left(\sin^{-1} x\right)\right) dx + \int e^x \frac{1}{\sqrt{1-x^2}} dx$$

$$= e^x \sin^{-1} x - \int e^x \frac{1}{\sqrt{1-x^2}} dx + \int e^x \frac{1}{\sqrt{1-x^2}} dx$$

$$= e^x \sin^{-1} x + c$$

Indefinite Integrals Ex 19.26 Q19

Let $I = \int e^{2x}\left(-\sin x + 2\cos x\right)dx$

$= -\int e^{2x}\sin x dx + 2\int e^{2x}\cos x dx$

Applying by parts in the 2[nd] integrand

$$\therefore \quad I = -\int e^{2x}\sin x dx + 2\left\{\frac{1}{2}e^{2x}\cos x + \int\frac{1}{2}e^{2x}\sin x dx\right\}$$

$$= -\int e^{2x}\sin x dx + e^{2x}\cos x + \int e^{2x}\sin x dx + c$$

$$= e^{2x}\cos x + c$$

Thus,

$I = e^{2x}\cos x + c$

Indefinite Integrals Ex 19.26 Q20

Let $I = \int e^{x}\left(\tan^{-1}x + \frac{1}{1+x^2}\right)dx$

Here, $f(x) = \tan^{-1}x$ and $f'(x) = \frac{1}{1+x^2}$

And we know that,

$\int e^{ax}\left(af(x) + f'(x)\right)dx = e^{ax}f(x) + c$

$$\therefore \quad \int e^{x}\left(\tan^{-1}x + \frac{1}{1+x^2}\right)dx = e^{x}\tan^{-1}x + c$$

Thus,

$I = e^{x}\tan^{-1}x + c$

Indefinite Integrals Ex 19.26 Q21

$$\text{Let } I = \int e^x \left(\frac{\sin x \cos x - 1}{\sin^2 x}\right) dx$$

$$= \int e^x \left(\cot x - \operatorname{cosec}^2 x\right) dx$$

$$= \int e^x \left(\cot x + \left(-\operatorname{cosec}^2 x\right)\right) dx$$

$$\because \quad \int e^{ax}\left(af(x) + f'(x)\right) dx = e^{ax} f(x) + c$$

Here $f(x) = \cot x$

$$\Rightarrow \quad f'(x) = -\operatorname{cosec}^2 x$$

$$\therefore \quad \int e^x \left(\cot x - \operatorname{cosec}^2 x\right) dx = e^x \cot x + c$$

Thus,

$$I = e^x \cot x + c$$

Indefinite Integrals Ex 19.26 Q23

Let $I = \int \frac{e^x (x-4)}{(x-2)^3} dx$

$= \int e^x \left\{ \frac{(x-2)-2}{(x-2)^3} \right\} dx$

$= \int e^x \left\{ \frac{1}{(x-2)^2} - \frac{2}{(x-2)^3} \right\} dx$

Here, $f(x) = \frac{1}{(x-2)^2}$ and $f'(x) = \frac{-2}{(x-2)^3}$

And we know that,

$\int e^{ax} \left(af(x) + f'(x) \right) dx = e^{ax} f(x) + c$

$\therefore \quad \int e^x \left\{ \frac{1}{(x-2)^2} - \frac{2}{(x-2)^3} \right\} dx = \frac{e^x}{(x-2)^2} + c$

$\therefore \quad I = \frac{e^x}{(x-2)^2} + c$

Indefinite Integrals Ex 19.26 Q24

Let $I=\int e^{2x}\left(\frac{1-\sin 2x}{1-\cos 2x}\right)dx$

We have, $\cos 2x = 1-2\sin^2 x$

$I=\int e^{2x}\left(\frac{1-\sin 2x}{1-\left(1-2\sin^2 x\right)}\right)dx$

$=\int e^{2x}\left(\frac{1-\sin 2x}{2\sin^2 x}\right)dx$

$=\int e^{2x}\left(\frac{\operatorname{cosec}^2 x}{2}-\frac{2\sin x\cos x}{2\sin^2 x}\right)dx$

$=\int e^{2x}\left(\frac{\operatorname{cosec}^2 x}{2}-\frac{\cos x}{\sin x}\right)dx$

$=\int e^{2x}\left(\frac{\operatorname{cosec}^2 x}{2}-\cot x\right)dx$

$=\frac{1}{2}\int e^{2x}\operatorname{cosec}^2 x\,dx-\int e^{2x}\cot x\,dx$

That is

$I=I_1+I_2$, where, $I_1=\frac{1}{2}\int e^{2x}\operatorname{cosec}^2 x\,dx$ and $I_2=-\int e^{2x}\cot x\,dx$

Consider $I_1=\frac{1}{2}\int e^{2x}\operatorname{cosec}^2 x\,dx$

Take e^{2x} as the first function and $\operatorname{cosec}^2 x$ as the second function.

So, $u=e^{2x}$; $du=2e^{2x}dx$

and

$\int \operatorname{cosec}^2 x\,dx=\int dv$

$\Rightarrow v=-\cot x$

$I_1=\frac{1}{2}\left[e^{2x}(-\cot x)-\int(-\cot x)2e^{2x}dx\right]$

$\Rightarrow I_1=\frac{1}{2}\left[e^{2x}(-\cot x)+2\int \cot x e^{2x}dx\right]$

$\Rightarrow I_1=\frac{1}{2}\left[e^{2x}(-\cot x)\right]+\int \cot x e^{2x}dx$

Thus,

$I=\frac{1}{2}\left[e^{2x}(-\cot x)\right]+\int \cot x e^{2x}dx-\int e^{2x}\cot x\,dx$

$=I=\frac{1}{2}\left[e^{2x}(-\cot x)\right]+C$