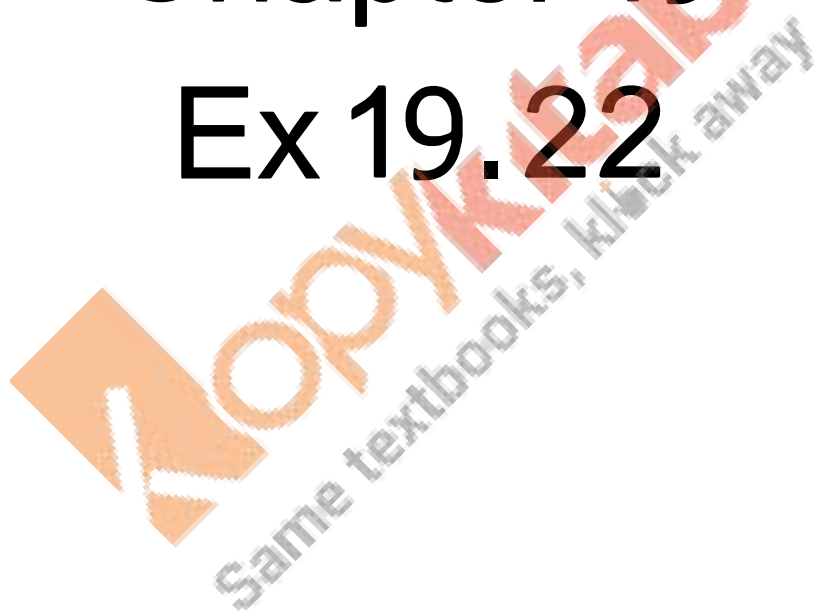


RD Sharma
Solutions
Class 12 Maths
Chapter 19
Ex 19.22



Indefinite Integrals Ex 19.22 Q1

$$\text{Let } I = \int \frac{1}{4 \cos^2 x + 9 \sin^2 x} dx$$

Dividing numerator and denominator by $\cos^2 x$

$$= \int \frac{\frac{1}{\cos^2 x}}{4 + 9 \tan^2 x} dx$$

$$I = \int \frac{\sec^2 x}{4 + 9 \tan^2 x} dx$$

$$\text{Let } \tan x = t$$

$$\sec^2 x dx = dt$$

$$I = \int \frac{dt}{4 + 9(t)^2}$$

$$= \int \frac{dt}{4 + (3t)^2}$$

$$\text{Let } 3t = u$$

$$3dt = du$$

$$I = \frac{1}{3} \int \frac{du}{(2)^2 + (u)^2}$$

$$= \frac{1}{3} \times \frac{1}{2} \times \tan^{-1} \left(\frac{u}{2} \right) + c$$

$$I = \frac{1}{6} \tan^{-1} \left(\frac{3t}{2} \right) + c$$

$$I = \frac{1}{6} \tan^{-1} \left(\frac{3 \tan x}{2} \right) + c$$

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Indefinite Integrals Ex 19.22 Q2

$$\text{Let } I = \int \frac{1}{4 \sin^2 x + 5 \cos^2 x} dx$$

Dividing numerator and denominator by $\cos^2 x$

$$I = \int \frac{\frac{1}{\cos^2 x}}{4 \tan^2 x + 5} dx$$

$$= \int \frac{\sec^2 x}{4 \tan^2 x + 5} dx$$

$$\text{Let } \tan x = t$$

$$\sec^2 x dx = dt$$

$$I = \int \frac{dt}{4 + 9(t)^2}$$

$$= \int \frac{dt}{4t^2 + 5}$$

$$\text{Let } 2t = u$$

$$2dt = du$$

$$I = \frac{1}{2} \int \frac{du}{(4)^2 + (\sqrt{5})^2}$$

$$= \frac{1}{2} \times \frac{1}{\sqrt{5}} \times \tan^{-1} \left(\frac{u}{\sqrt{5}} \right) + c$$

$$= \frac{1}{2\sqrt{5}} \tan^{-1} \left(\frac{2t}{\sqrt{5}} \right) + c$$

$$I = \frac{1}{2\sqrt{5}} \tan^{-1} \left(\frac{2 \tan x}{\sqrt{5}} \right) + c$$

Indefinite Integrals Ex 19.22 Q3

$$\begin{aligned}\text{Let } I &= \int \frac{2}{2 + \sin 2x} dx \\ &= \int \frac{2}{2 + 2 \sin x \cos x} dx\end{aligned}$$

Dividing numerator and denominator by $\cos^2 x$

$$\begin{aligned}I &= \int \frac{\frac{1}{\cos^2 x}}{\frac{1}{\cos^2 x} + \frac{\sin x \cos x}{\cos^2 x}} dx \\ &= \int \frac{\sec^2 x}{\sec^2 x + \tan x} dx \\ I &= \int \frac{\sec^2 x}{1 + \tan^2 x + \tan x} dx\end{aligned}$$

Let $\tan x = t$

$$\sec^2 x dx = dt$$

$$\begin{aligned}I &= \int \frac{dt}{t^2 + t + 1} \\ &= \int \frac{dt}{t^2 + 2t\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2 + 1} \\ I &= \int \frac{dt}{\left(t + \frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}\end{aligned}$$

$$= \frac{1}{\frac{\sqrt{3}}{2}} \tan^{-1} \left(\frac{t + \frac{1}{2}}{\frac{\sqrt{3}}{2}} \right) + c$$

$$= \frac{2}{\sqrt{3}} \tan^{-1} \left(\frac{2t + 1}{\sqrt{3}} \right) + c$$

$$I = \frac{2}{\sqrt{3}} \tan^{-1} \left(\frac{2 \tan x + 1}{\sqrt{3}} \right) + c$$

Indefinite Integrals Ex 19.22 Q4

$$\begin{aligned} \text{Let } I &= \int \frac{\cos x}{\cos 3x} dx \\ &= \int \frac{\cos x}{4 \cos^3 x - 3 \cos x} dx \end{aligned}$$

Dividing numerator and denominator by $\cos^3 x$

$$\begin{aligned} I &= \int \frac{\frac{\cos x}{\cos^3 x}}{\frac{4 \cos^3 x}{\cos^3 x} + \frac{3 \cos x}{\cos^3 x}} dx \\ &= \int \frac{\sec^2 x}{4 - 3 \sec^2 x} dx \\ &= \int \frac{\sec^2 x}{4 - 3(1 + \tan^2 x)} dx \\ &= \int \frac{\sec^2 x}{4 - 3 - 3 \tan^2 x} dx \\ &= \int \frac{\sec^2 x}{1 - 3 \tan^2 x} dx \end{aligned}$$

$$\begin{aligned} \text{Let } \tan x &= t \\ \sec^2 x dx &= dt \end{aligned}$$

$$\begin{aligned} I &= \int \frac{dt}{1 - 3t^2} \\ &= \int \frac{dt}{1 - (\sqrt{3}t)^2} \end{aligned}$$

$$\begin{aligned} \text{Let } \sqrt{3}t &= u \\ \sqrt{3}dt &= du \\ &= \int \frac{du}{(1)^2 - (u)^2} \end{aligned}$$

$$= \frac{1}{2\sqrt{3}i} \log \left| \frac{u+1}{1-u} \right| + c$$

$$= \frac{1}{2\sqrt{3}i} \log \left| \frac{\sqrt{3}t+1}{1-\sqrt{3}t} \right| + c$$

$$I = \frac{1}{2\sqrt{3}i} \log \left| \frac{1+\sqrt{3} \tan x}{1-\sqrt{3} \tan x} \right| + c$$

Indefinite Integrals Ex 19.22 Q5

Let $I = \int \frac{1}{1+3\sin^2 x} dx$

Dividing numerator and denominator by $\cos^2 x$

$$I = \int \frac{\frac{1}{\cos^2 x}}{\frac{1}{\cos^2 x} + \frac{3\sin^2 x}{\cos^2 x}} dx$$

$$= \int \frac{\sec^2 x}{\sec^2 x + 3\tan^2 x} dx$$

$$= \int \frac{\sec^2 x}{1 + \tan^2 x + 3\tan^2 x} dx$$

$$= \int \frac{\sec^2 x}{1 + 4\tan^2 x} dx$$

$$= \int \frac{\sec^2 x}{1 + (2\tan x)^2} dx$$

Let $2\tan x = t$

$$2\sec^2 x dx = dt$$

$$I = \frac{1}{2} \int \frac{dt}{1+t^2}$$

$$= \frac{1}{2} \tan^{-1} t + c$$

$$I = \frac{1}{2} \tan^{-1} (2 \tan x) + c$$

Indefinite Integrals Ex 19.22 Q6

Let $I = \int \frac{1}{3 + 2 \cos^2 x} dx$

Dividing numerator and denominator by $\cos^2 x$

$$I = \int \frac{\frac{1}{\cos^2 x}}{\frac{3}{\cos^2 x} + \frac{2 \cos^2 x}{\cos^2 x}} dx$$

$$= \int \frac{\sec^2 x}{3 \sec^2 x + 2} dx$$

$$= \int \frac{\sec^2 x}{3(1 + \tan^2 x) + 2} dx$$

$$= \int \frac{\sec^2 x}{3 + 3 \tan^2 x + 2} dx$$

$$= \int \frac{\sec^2 x}{5 + 3 \tan^2 x} dx$$

Let $\sqrt{3} \tan x = t$

$$\sqrt{3} \sec^2 x dx = dt$$

$$I = \frac{1}{\sqrt{3}} \int \frac{dt}{(\sqrt{5})^2 + t^2}$$

$$= \frac{1}{\sqrt{3} \times \sqrt{5}} \tan^{-1} \left(\frac{t}{\sqrt{5}} \right) + c$$

$$I = \frac{1}{\sqrt{15}} \tan^{-1} \left(\frac{\sqrt{3} \tan x}{\sqrt{5}} \right) + c$$