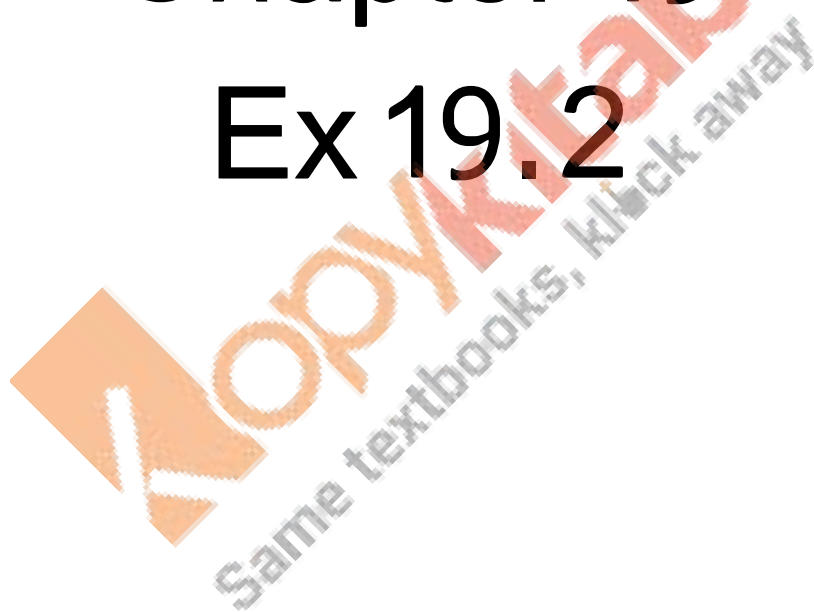


RD Sharma
Solutions
Class 12 Maths
Chapter 19
Ex 19.2



Indefinite Integrals Ex 19.2 Q1

$$\begin{aligned}\int (3x\sqrt{5} + 4\sqrt{x} + 5) dx \\&= \int 3x\sqrt{5} dx + \int 4\sqrt{x} dx + \int 5 dx \\&= \int 3x^{\frac{3}{2}} dx + 4 \int x^{\frac{1}{2}} dx + \int 5 dx \\&= \frac{x^{\frac{3}{2}+1}}{\frac{3}{2}+1} + \frac{4x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + 5x + c \\&= \frac{6}{5}x^{\frac{5}{2}} + \frac{8}{3}x^{\frac{3}{2}} + 5x + c\end{aligned}$$

Indefinite Integrals Ex 19.2 Q2

$$\begin{aligned}\int \left(2^x + \frac{5}{x} - \frac{1}{x^3} \right) dx \\&= \int 2^x dx + 5 \int \frac{1}{x} dx - \int \frac{1}{x^3} dx \\&= \frac{2^x}{\log 2} + 5 \log x - \frac{3}{2}x^{\frac{2}{3}} + c\end{aligned}$$

Indefinite Integrals Ex 19.2 Q3

$$\begin{aligned}\int \left(\sqrt{x} (ax^2 + bx + c) \right) dx \\&= \int \sqrt{x} \times ax^2 dx + \int \sqrt{x} \times bx dx + \int c\sqrt{x} dx \\&= \int ax^{\frac{5}{2}} dx + \int bx^{\frac{3}{2}} dx + \int cx^{\frac{1}{2}} dx \\&= \frac{ax^{\frac{5}{2}+1}}{\frac{5}{2}+1} + \frac{bx^{\frac{3}{2}+1}}{\frac{3}{2}+1} + \frac{cx^{\frac{1}{2}+1}}{\frac{1}{2}+1} + d \\&= \frac{2ax^{\frac{7}{2}}}{7} + \frac{2bx^{\frac{5}{2}}}{5} + \frac{2cx^{\frac{3}{2}}}{3} + d\end{aligned}$$

Indefinite Integrals Ex 19.2 Q4

$$\begin{aligned}\int (2-3x)(3+2x)(1-2x) dx \\&= \int (6+4x-9x-6x^2)(1-2x) dx \\&= \int (-6x^2-5x+6)(1-2x) dx \\&= \int (-6x^2+12x^3-5x+10x^2+6-12x) dx \\&= \int (4x^2+12x^3-17x+6) dx \\&= \int (12x^3+4x^2-17x+6) dx \\&= \frac{12}{4}x^4 + \frac{4}{3}x^3 - \frac{17}{2}x^2 + 6x + c \\&= 3x^4 + \frac{4}{3}x^3 - \frac{17}{2}x^2 + 6x + c\end{aligned}$$

Indefinite Integrals Ex 19.2 Q5

$$\begin{aligned}
 & \int \left(\frac{m}{x} + \frac{x}{m} + m^x + x^m + mx \right) dx \\
 &= m \int \frac{1}{x} dx + \frac{1}{m} \int x dx + \int m^x dx + \int x^m dx + m \int x dx \\
 &= m \log|x| + \frac{x^2}{2m} + \frac{m^x}{\log m} + \frac{x^{m+1}}{m+1} + \frac{mx^2}{2} + c
 \end{aligned}$$

Indefinite Integrals Ex 19.2 Q6

$$\begin{aligned}
 & \int \left(\sqrt{x} - \frac{1}{\sqrt{x}} \right)^2 dx \\
 &= \int \left(x + \frac{1}{x} - 2 \right) dx \\
 &= \int x dx + \int \frac{1}{x} dx - 2 \int 1 dx \\
 &= \frac{x^2}{2} + \log|x| - 2x + C
 \end{aligned}$$

Indefinite Integrals Ex 19.2 Q7

$$\begin{aligned}
 & \int \frac{(1+x)^3}{\sqrt{x}} dx \\
 &= \int \frac{1+x^3+3x^2+3x}{\sqrt{x}} dx \\
 &= \int \frac{1}{\sqrt{x}} dx + \int \frac{x^3}{\sqrt{x}} dx + \int \frac{3x^2}{\sqrt{x}} dx + \int \frac{3x}{\sqrt{x}} dx \\
 &= \int x^{-\frac{1}{2}} dx + \int x^{\frac{5}{2}} dx + 3 \int x^{\frac{3}{2}} dx + 3 \int x^{\frac{1}{2}} dx \\
 &= \frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + \frac{x^{\frac{5}{2}+1}}{\frac{5}{2}+1} + \frac{3x^{\frac{3}{2}+1}}{\frac{3}{2}+1} + 3 \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + c \\
 &= \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + \frac{x^{\frac{7}{2}}}{\frac{7}{2}} + \frac{3x^{\frac{5}{2}}}{\frac{5}{2}} + 3 \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + c \\
 &= 2x^{\frac{1}{2}} + \frac{2}{7}x^{\frac{7}{2}} + \frac{6}{5}x^{\frac{5}{2}} + \frac{6}{3}x^{\frac{3}{2}} + c \\
 &= 2x^{\frac{1}{2}} + \frac{2}{7}x^{\frac{7}{2}} + \frac{6}{5}x^{\frac{5}{2}} + 2x^{\frac{3}{2}} + c \\
 \therefore \int \frac{(1+x)^3}{\sqrt{x}} dx &= 2x^{\frac{1}{2}} + \frac{2}{7}x^{\frac{7}{2}} + \frac{6}{5}x^{\frac{5}{2}} + 2x^{\frac{3}{2}} + c
 \end{aligned}$$

Indefinite Integrals Ex 19.2 Q8

$$\begin{aligned}
 & \int \left\{ x^2 + e^{\log x} + \left(\frac{e}{2} \right)^x \right\} dx \\
 &= \int x^2 dx + \int e^{\log x} dx + \int \left(\frac{e}{2} \right)^x dx \\
 &= \frac{x^3}{3} + \int x dx + \int \left(\frac{e}{2} \right)^x dx \\
 &= \frac{x^3}{3} + \frac{x^2}{2} + \frac{1}{\log \left(\frac{e}{2} \right)} \times \left(\frac{e}{2} \right)^x + c
 \end{aligned}$$

Indefinite Integrals Ex 19.2 Q9

$$\begin{aligned}
 & \int (x^e + e^x + e^e) dx \\
 &= \int x^e dx + \int e^x dx + \int e^e dx \\
 &= \frac{x^{e+1}}{e+1} + e^x + e^e x + c \quad [\because e \text{ is constant}]
 \end{aligned}$$

$$\therefore \int (x^e + e^x + e^e) dx = \frac{x^{e+1}}{e+1} + e^x + e^e x + c$$

Indefinite Integrals Ex 19.2 Q10

$$\begin{aligned}
 \int \sqrt{x} \left(x^3 - \frac{2}{x} \right) dx &= \int x^{\frac{7}{2}} dx - 2 \int x^{-\frac{1}{2}} dx \\
 &= \frac{x^{\frac{7}{2}+1}}{\frac{7}{2}+1} - 2 \frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + c \\
 &= \frac{x^{\frac{9}{2}}}{\frac{9}{2}} - \frac{2x^{\frac{-1}{2}}}{\frac{-1}{2}} + c \\
 &= \frac{2}{9} x^{\frac{9}{2}} - 4x^{\frac{-1}{2}} + c
 \end{aligned}$$

$$\therefore \int \sqrt{x} \left(x^3 - \frac{2}{x} \right) dx = \frac{2}{9} x^{\frac{9}{2}} - 4\sqrt{x} + c$$

Indefinite Integrals Ex 19.2 Q11

$$\int \frac{1}{\sqrt{x}} \left(1 + \frac{1}{x} \right) dx$$

$$= \int \left(\frac{1}{\sqrt{x}} + \frac{1}{\sqrt{x} \times x} \right) dx$$

$$= \int x^{-\frac{1}{2}} + \int x^{-\frac{3}{2}} dx$$

$$= 2x^{\frac{1}{2}} - 2x^{-\frac{1}{2}} + C$$

$$= 2\sqrt{x} - \frac{2}{\sqrt{x}} + C$$

$$\therefore \int \frac{1}{\sqrt{x}} \left(1 + \frac{1}{x} \right) dx = 2\sqrt{x} - \frac{2}{\sqrt{x}} + C$$

Indefinite Integrals Ex 19.2 Q12

$$\int \frac{x^6 + 1}{x^2 + 1} dx$$

$$= \int \frac{(x^2)^3 + (1)^3}{x^2 + 1} dx$$

$$= \int \frac{(x^2 + 1)(x^4 + 1 - x^2)}{x^2 + 1} dx$$

$$= \int (x^4 - x^2 + 1) dx$$

$$= \int x^4 dx - \int x^2 dx + \int 1 dx$$

$$= \frac{x^5}{5} - \frac{x^3}{3} + x + C$$

Indefinite Integrals Ex 19.2 Q13

$$\int \frac{x^{-\frac{1}{3}} + \sqrt{x} + 2}{\sqrt[3]{x}} dx$$

$$= \int \frac{x^{-\frac{1}{3}}}{x^{\frac{1}{3}}} dx + \int \frac{x^{\frac{1}{2}}}{x^{\frac{1}{3}}} dx + \int \frac{2}{x^{\frac{1}{3}}} dx$$

$$= \int x^{-\frac{2}{3}} dx + \int x^{\frac{1}{6}} dx + 2 \int x^{-\frac{1}{3}} dx$$

$$= 3x^{\frac{1}{3}} + \frac{6}{7}x^{\frac{7}{6}} + 3x^{\frac{2}{3}} + C$$

Indefinite Integrals Ex 19.2 Q14

$$\int \frac{(1 + \sqrt{x})^2}{\sqrt{x}} dx$$

$$= \int \frac{1 + x + 2\sqrt{x}}{x^{\frac{1}{2}}} dx$$

$$= \int x^{-\frac{1}{2}} + \int x^{\frac{1}{2}} dx + 2 \int dx$$

$$= 2\sqrt{x} + \frac{2}{3}x^{\frac{3}{2}} + 2x + c$$

$$\therefore \int \frac{(1 + \sqrt{x})^2}{\sqrt{x}} dx = 2\sqrt{x} + 2x + \frac{2}{3}x^{\frac{3}{2}} + c$$

Indefinite Integrals Ex 19.2 Q15

$$\int \sqrt{x} (3 - 5x) dx$$

$$= 3 \int \sqrt{x} dx - 5 \int x^{\frac{3}{2}} dx$$

$$= \frac{3x^{\frac{3}{2}}}{\frac{3}{2}} - 5 \frac{x^{\frac{5}{2}}}{\frac{5}{2}} + c$$

$$= 2x^{\frac{3}{2}} - 2x^{\frac{5}{2}} + c$$

Indefinite Integrals Ex 19.2 Q16

$$\int \frac{(x+1)(x-2)}{\sqrt{x}} dx$$

$$= \int \frac{x^2 - 2x + x - 2}{x^{\frac{1}{2}}} dx$$

$$= \int \frac{x^2 - x - 2}{x^{\frac{1}{2}}} dx$$

$$= \int \frac{x^2}{x^{\frac{1}{2}}} dx - \int \frac{x}{x^{\frac{1}{2}}} dx - 2 \int \frac{1}{x^{\frac{1}{2}}} dx$$

$$= \frac{2x^{\frac{5}{2}}}{5} - \frac{2x^{\frac{3}{2}}}{3} - 4x^{\frac{1}{2}} + c$$

$$\therefore \int \frac{(x+1)(x-2)}{\sqrt{x}} dx = \frac{2}{5}x^{\frac{5}{2}} - \frac{2}{3}x^{\frac{3}{2}} - 4x^{\frac{1}{2}} + c$$

$$= \frac{2}{5}x^{\frac{5}{2}} - \frac{2x^{\frac{3}{2}}}{3} - 4\sqrt{x} + c$$

Indefinite Integrals Ex 19.2 Q17

$$\int \frac{x^5 + x^{-2} + 2}{x^2} dx$$

$$\begin{aligned} &= \int \left(\frac{x^5}{x^2} + \frac{x^{-2}}{x^2} + \frac{2}{x^2} \right) dx \\ &= \int x^3 dx + \int x^{-4} + 2 \int x^{-2} dx \\ &= \frac{x^4}{4} + \frac{x^{-3}}{-3} + \frac{2x^{-1}}{-1} + c \\ &= \frac{x^4}{4} - \frac{x^{-3}}{3} - \frac{2}{x} + c \end{aligned}$$

Indefinite Integrals Ex 19.2 Q18

$$\begin{aligned} &\int (3x + 4)^2 dx \\ &= \int (9x^2 + 16 + 24x) dx \\ &= 9 \int x^2 dx + 16 \int dx + 24 \int x dx \\ &= 9 \frac{x^3}{3} + 16x + 24 \frac{x^2}{2} + c \\ &= 3x^3 + 16x + 12x^2 + c \\ \therefore \int (3x + 4)^2 &= 3x^3 + 12x^2 + 16x + c \end{aligned}$$

Indefinite Integrals Ex 19.2 Q19

$$\begin{aligned} &\int \frac{2x^4 + 7x^3 + 6x^2}{x^2 + 2x} dx \\ &= \int \frac{x(2x^3 + 7x^2 + 6x)}{x(x+2)} dx \\ &= \int \frac{2x^3 + 7x^2 + 6x}{x+2} dx \\ &= \int \frac{2x^3 + 4x^2 + 3x^2 + 6x}{(x+2)} dx \\ &= \int \frac{2x^2(x+2) + 3x(x+2)}{(x+2)} dx \\ &= \int \frac{(x+2)(2x^2 + 3x)}{x+2} dx \\ &= \int (2x^2 + 3x) dx \\ &= \int 2x^2 dx + \int 3x dx \\ &= \frac{2}{3}x^3 + \frac{3}{2}x^2 + c \end{aligned}$$

Indefinite Integrals Ex 19.2 Q20

$$\begin{aligned}
 & \int \frac{5x^4 + 12x^3 + 7x^2}{x^2 + x} dx \\
 &= \int \frac{5x^4 + 7x^3 + 5x^3 + 7x^2}{x^2 + x} dx \\
 &= \int \frac{5x^3 + 7x^2 + 5x^2 + 7x}{x + 1} dx \\
 &= \int \frac{5x^2(x+1) + 7x(x+1)}{x+1} dx \\
 &= \int (5x^2 + 7x) dx \\
 &= \frac{5x^3}{3} + \frac{7x^2}{2} + C
 \end{aligned}$$

Indefinite Integrals Ex 19.2 Q21

$$\begin{aligned}
 & \int \frac{\sin^2 x}{1 + \cos x} dx \\
 &= \int \frac{1 - \cos^2 x}{1 + \cos x} dx \\
 &= \int \frac{(1 - \cos x)(1 + \cos x)}{(1 + \cos x)} dx \\
 &= \int (1 - \cos x) dx \\
 &= x - \sin x + C
 \end{aligned}$$

Indefinite Integrals Ex 19.2 Q22

$$\begin{aligned}
 & \int (\sec^2 x + \operatorname{cosec}^2 x) dx \\
 &= \int \sec^2 x dx + \int \operatorname{cosec}^2 x dx \\
 &= \tan x - \cot x + C
 \end{aligned}$$

$$\therefore \int (\sec^2 x + \operatorname{cosec}^2 x) dx = \tan x - \cot x + C$$

Indefinite Integrals Ex 19.2 Q23

Evaluate the integral as follows

$$\begin{aligned}
 \int \frac{\sin^3 x - \cos^3 x}{\sin^2 x \cos^2 x} dx &= \int \left(\frac{\sin^3 x}{\sin^2 x \cos^2 x} - \frac{\cos^3 x}{\sin^2 x \cos^2 x} \right) dx \\
 &= \int (\sin x \sec^2 x - \cos x \operatorname{cosec}^2 x) dx \\
 &= \int (\tan x \sec x - \cot x \operatorname{cosec} x) dx \\
 &= \sec x + \operatorname{cosec} x + C
 \end{aligned}$$

Indefinite Integrals Ex 19.2 Q24

$$I \int \frac{5 \cos^3 x + 6 \sin^3 x}{2 \sin^2 x \cos^2 x} dx$$

Now,

$$\begin{aligned} I &= \int \frac{5 \cos^3 x + \sin^3 x}{2 \sin^2 x \cos^2 x} dx \\ &= \int \frac{5 \cos^3 x}{2 \sin^2 x \cos^2 x} dx + \int \frac{6 \sin^3 x}{2 \sin^2 x \cos^2 x} dx \\ &= \frac{5}{2} \int \frac{\cos x}{\sin^2 x} dx + 3 \int \frac{\sin x}{\cos^2 x} dx \\ &= \frac{5}{2} \int \cot x \operatorname{cosec} x dx + 3 \int \sec x \tan x dx \\ &= + \frac{-5}{2} \operatorname{cosec} x + 3 \sec x + c \end{aligned}$$

$$\therefore I = \frac{-5}{2} \operatorname{cosec} x + 3 \sec x + c$$

Indefinite Integrals Ex 19.2 Q25

$$\int (\tan x + \cot x)^2 dx$$

$$\begin{aligned} &= \int (\tan^2 x + \cot^2 x + 2 \tan x \cot x) dx \\ &= \int \left(\sec^2 x - 1 + \operatorname{cosec}^2 x - 1 + \frac{2 \times 1}{\cot x} \cot x \right) dx \\ &= \int (\sec^2 x + \operatorname{cosec}^2 x) dx \\ &= \int \sec^2 x dx + \int \operatorname{cosec}^2 x \\ &= \tan x - \cot x + c \end{aligned}$$

$$\therefore \int (\tan x + \cot x)^2 = \tan x - \cot x + c$$

Indefinite Integrals Ex 19.2 Q26

$$\int \frac{1 - \cos 2x}{1 + \cos 2x} dx$$

$$\begin{aligned} &= \int \frac{2 \sin^2 x}{2 \cos^2 x} dx \\ &= \int \tan^2 x dx \\ &= \int (\sec^2 x - 1) dx \\ &= \int \sec^2 x dx - \int dx \\ &= \tan x - x + c \end{aligned}$$

$$\therefore \int \frac{1 - \cos 2x}{1 + \cos 2x} dx = \tan x - x + c$$

Indefinite Integrals Ex 19.2 Q27

$$\int \frac{\cos x}{1 - \cos x} dx$$

$$= \int \frac{\cos x (1 + \cos x)}{(1 - \cos x)(1 + \cos x)} dx$$

$$= \int \frac{\cos x + \cos^2 x}{1 - \cos^2 x} dx$$

$$= \int \frac{\cos x + \cos^2 x}{\sin^2 x} dx$$

$$= \int \frac{\cos x}{\sin^2 x} dx + \int \frac{\cos^2 x}{\sin^2 x} dx$$

$$= \int \cot x \times \operatorname{cosec} x dx + \int (\operatorname{cosec}^2 x - 1) dx$$

$$= -\operatorname{cosec} x - \cot x - x + c$$

$$\therefore \int \frac{\cos x}{1 - \cos x} \times dx = -\operatorname{cosec} x - \cot x - x + c$$

Indefinite Integrals Ex 19.2 Q28

$$\int \frac{\cos^2 x - \sin^2 x}{\sqrt{1 + \cos 4x}} dx$$

$$= \int \frac{\cos^2 x - \sin^2 x}{\sqrt{2 \cos^2 2x}} dx$$

$$= \frac{1}{\sqrt{2}} \int \frac{\cos^2 x - \sin^2 x}{\cos 2x} dx$$

$$= \frac{1}{\sqrt{2}} \int \frac{\cos^2 x - \sin^2 x}{\cos^2 x - \sin^2 x} dx$$

$$[\because \cos 2x = \cos^2 x - \sin^2 x]$$

$$= \frac{1}{\sqrt{2}} \int 1 \times dx$$

$$= \frac{x}{\sqrt{2}} + c$$

$$\therefore \int \frac{\cos^2 x - \sin^2 x}{\sqrt{1 + \cos 4x}} \times dx = \frac{x}{\sqrt{2}} + c$$

Indefinite Integrals Ex 19.2 Q29

$$\int \frac{1}{1 - \cos x} dx$$

$$= \int \frac{1}{1 - \cos x} \times \frac{1 + \cos x}{1 + \cos x} \times dx$$

$$= \int \frac{1 + \cos x}{1 - \cos^2 x} \times dx$$

$$= \int \frac{1 + \cos x}{\sin^2 x} \times dx$$

$$= \int \frac{1}{\sin^2 x} dx + \int \frac{\cos x}{\sin^2 x} dx$$

$$= \int \operatorname{cosec}^2 x dx + \int \cot x \times \operatorname{cosec} x dx$$

$$= -\cot x - \operatorname{cosec} x + c$$

$$\therefore \int \frac{1}{1 - \cos x} dx = -\cot x - \operatorname{cosec} x + c.$$

Indefinite Integrals Ex 19.2 Q30

$$\int \frac{1}{1 - \sin x} dx$$

$$= \int \frac{1}{1 - \sin x} \times \frac{1 + \sin x}{1 + \sin x} \times dx$$

$$= \int \frac{1 + \sin x}{1 - \sin^2 x} \times dx$$

$$= \int \frac{1 + \sin x}{\cos^2 x} \times dx$$

$$= \int \left(\frac{1}{\cos^2 x} + \frac{\sin x}{\cos^2 x} \right) \times dx$$

$$=$$

$$= \int \frac{1}{\cos^2 x} dx + \int \frac{\sin x}{\cos^2 x} \times dx$$

$$= \int \sec^2 x dx + \int \tan x \sec x dx$$

$$= \tan x + \sec x + c$$

$$\therefore \int \frac{1}{1 - \sin x} \times dx = \tan x + \sec x + c.$$

Indefinite Integrals Ex 19.2 Q31

$$\int \frac{\tan x}{\sec x + \tan x} \times dx$$

$$\begin{aligned}
 &= \int \frac{\tan x}{\sec x + \tan x} \times \frac{\sec x - \tan x}{\sec x - \tan x} \times dx \\
 &= \int \frac{\tan x (\sec x - \tan x)}{\sec^2 x - \tan^2 x} \times dx \\
 &= \int (\tan x \sec x - \tan^2 x) dx \\
 &= \int \sec x \tan x dx - \int (\sec^2 x - 1) dx \\
 &= \int \sec x \tan x dx - \int \sec^2 x dx + \int dx \\
 &= \sec x - \tan x + x + c
 \end{aligned}$$

$$\therefore \int \frac{\tan x}{\sec x + \tan x} \times dx = \sec x - \tan x + x + c.$$

Indefinite Integrals Ex 19.2 Q32

$$\int \frac{\operatorname{cosec} x}{\operatorname{cosec} x - \cot x} \times dx$$

$$\begin{aligned}
 &= \int \frac{\operatorname{cosec} x}{\operatorname{cosec} x - \cot x} \times \frac{\operatorname{cosec} x + \cot x}{\operatorname{cosec} x + \cot x} \times dx \\
 &= \int \frac{\operatorname{cosec} x (\operatorname{cosec} x + \cot x)}{\operatorname{cosec}^2 x - \cot^2 x} \times dx \\
 &= \int (\operatorname{cosec}^2 x + \operatorname{cosec} x \cot x) dx \\
 &= \int \operatorname{cosec}^2 x dx + \int \operatorname{cosec} x dx \\
 &= -\cot x - \operatorname{cosec} x + c
 \end{aligned}$$

$$\therefore \int \frac{\operatorname{cosec} x}{\operatorname{cosec} x - \cot x} \times dx = -\cot x - \operatorname{cosec} x + c.$$

Indefinite Integrals Ex 19.2 Q33

$$\int \frac{1}{1 + \cos 2x} \times dx$$

$$\begin{aligned}
 &= \int \frac{1}{2 \cos^2 x} \times dx \\
 &= \frac{1}{2} \int \sec^2 x \times dx \\
 &= \frac{1}{2} \times \tan x + c \\
 &= \frac{\tan x}{2} + c
 \end{aligned}$$

$$\therefore \int \frac{1}{1 + \cos 2x} \times dx = \frac{1}{2} \tan x + c.$$

Indefinite Integrals Ex 19.2 Q34

$$\int \frac{1}{1 - \cos 2x} dx$$

$$= \int \frac{1}{2 \sin^2 x} \times dx$$

$$= \frac{1}{2} \int \operatorname{cosec}^2 x \times dx$$

$$= \frac{-1}{2} \times \cot x + c$$

$$= \frac{-1 \cot x}{2} + c$$

$$\therefore \int \frac{1}{1 - \cos 2x} = \frac{-1}{2} \cot x + c.$$

Indefinite Integrals Ex 19.2 Q35

$$\int \tan^{-1} \left[\frac{\sin 2x}{1 + \cos 2x} \right] dx$$

$$= \int \tan^{-1} \left[\frac{2 \sin x \cos x}{2 \cos^2 x} \right] dx$$

$$= \int \tan^{-1} \left[\frac{\sin x}{\cos x} \right] dx$$

$$= \int \tan^{-1} (\tan x) dx$$

$$= \int x dx$$

$$= \frac{x^2}{2} + c$$

$$\therefore \int \tan^{-1} \left[\frac{\sin 2x}{1 + \cos 2x} \right] dx = \frac{x^2}{2} + c.$$

Indefinite Integrals Ex 19.2 Q36

$$\int \cos^{-1} (\sin x) dx$$

$$= \int \cos^{-1} \left[\cos \left(\frac{\pi}{2} - x \right) \right] dx$$

$$= \int \left(\frac{\pi}{2} - x \right) dx$$

$$= \frac{\pi}{2} \int dx - \int x dx$$

$$= \frac{\pi}{2} \times x - \frac{x^2}{2} + c$$

$$\therefore \int \cos^{-1} (\sin x) dx = \frac{\pi}{2} \times x - \frac{x^2}{2} + c.$$

Indefinite Integrals Ex 19.2 Q37

$$\int \cos^{-1}(\sin x) dx$$

$$= \int \cot^{-1} \left[\frac{\sin 2x}{1 - \cos 2x} \right] dx$$

$$= \int \cot^{-1} \left(\frac{\cos x}{\sin x} \right) dx$$

$$= \int \cot^{-1}(\cot x) dx$$

$$= \int x dx$$

$$= \frac{x^2}{2} + c$$

$$\therefore \int \cot^{-1} \left[\frac{\sin 2x}{1 - \cos 2x} \right] dx = \frac{x^2}{2} + c.$$

Indefinite Integrals Ex 19.2 Q38

$$\int \sin^{-1} \left(\frac{2 \tan x}{1 + \tan^2 x} \right) dx$$

$$= \int \sin^{-1}(\sin 2x) dx$$

$$\left[\because \sin 2x = \frac{2 \tan x}{1 + \tan^2 x} \right]$$

$$= \int 2x dx$$

$$= 2 \int x dx$$

$$= \frac{2x^2}{2} + c$$

$$= x^2 + c$$

$$\therefore \int \sin^{-1} \left(\frac{2 \tan x}{1 + \tan^2 x} \right) dx = x^2 + c.$$

Indefinite Integrals Ex 19.2 Q39

$$\int \frac{(x^3 + 8)(x - 1)}{x^2 - 2x + 4} dx$$

$$= \int \frac{(x + 2)(x^2 - 2x + 4)(x - 1)}{x^2 - 2x + 4} dx$$

$$= \int (x + 2)(x - 1) dx$$

$$= \int (x^2 - x + 2x - 2) dx$$

$$= \int (x^2 + x - 2) dx$$

$$= \frac{x^3}{3} + \frac{x^2}{2} - 2x + c$$

$$\therefore \int \frac{(x^3 + 8)(x - 1)}{x^2 - 2x + 4} dx = \frac{x^3}{3} + \frac{x^2}{2} - 2x + c.$$

Indefinite Integrals Ex 19.2 Q40

$$\int (a \tan x + b \cot x)^2 dx$$

$$\begin{aligned} &= \int (a^2 \tan^2 x + b^2 \cot^2 x + 2ab \tan x \cot x) dx \\ &= \int [a^2 (\sec^2 x - 1) + b^2 (\operatorname{cosec}^2 x - 1) + 2ab] dx \\ &= \int [a^2 \sec^2 x - a^2 + b^2 \operatorname{cosec}^2 x - b^2 + 2ab] dx \\ &= a^2 \tan x - a^2 x - b^2 \cot x - b^2 x + 2abx + c \\ &= a^2 \tan x + -b^2 \cot x - (a^2 + b^2 - 2ab)x + c \end{aligned}$$

$$\therefore \int (a \tan x + b \cot x)^2 dx = a^2 \tan x - b^2 \cot x - (a^2 + b^2 - 2ab)x + c.$$

Indefinite Integrals Ex 19.2 Q41

$$\int \frac{x^3 - 3x^2 + 5x - 7 + x^2 a^x}{2x^2} dx$$

$$\begin{aligned} &= \frac{1}{2} \int \frac{x^3}{x^2} dx - \frac{3}{2} \int \frac{x^2}{x^2} dx + \frac{5}{2} \int \frac{x}{x^2} dx - \frac{7}{2} \int \frac{1}{x^2} dx + \frac{1}{2} \int \frac{x^2 a^x}{x^2} dx \\ &= \frac{1}{2} \times \frac{x^2}{2} - \frac{3}{2} x + \frac{5}{2} \log x - \frac{7}{2} x^{-1} + \frac{1}{2} \frac{a^x}{\log a} + c \\ &= \frac{1}{2} \left[\frac{x^2}{2} - 3x + 5 \log x + \frac{7}{x} + \frac{a^x}{\log a} \right] + c \end{aligned}$$

$$\therefore \int \frac{x^3 - 3x^2 + 5x - 7 + x^2 a^x}{2x^2} dx = \frac{1}{2} \left[\frac{x^2}{2} - 3x + 5 \log x + \frac{7}{x} + \frac{a^x}{\log a} \right] + c$$

Indefinite Integrals Ex 19.2 Q42

$$\begin{aligned} \frac{\cos x}{1 + \cos x} &= \frac{\cos^2 \frac{x}{2} - \sin^2 \frac{x}{2}}{2 \cos^2 \frac{x}{2}} \left[\cos x = \cos^2 \frac{x}{2} - \sin^2 \frac{x}{2} \text{ and } \cos x = 2 \cos^2 \frac{x}{2} - 1 \right] \\ &= \frac{1}{2} \left[1 - \tan^2 \frac{x}{2} \right] \end{aligned}$$

$$\begin{aligned} \therefore \int \frac{\cos x}{1 + \cos x} dx &= \frac{1}{2} \int \left(1 - \tan^2 \frac{x}{2} \right) dx \\ &= \frac{1}{2} \int \left(1 - \sec^2 \frac{x}{2} + 1 \right) dx \\ &= \frac{1}{2} \int \left(2 - \sec^2 \frac{x}{2} \right) dx \\ &= \frac{1}{2} \left[2x - \frac{\tan \frac{x}{2}}{\frac{1}{2}} \right] + C \\ &= x - \tan \frac{x}{2} + C \end{aligned}$$

Indefinite Integrals Ex 19.2 Q43

$$\frac{1 - \cos x}{1 + \cos x} = \frac{2 \sin^2 \frac{x}{2}}{2 \cos^2 \frac{x}{2}} \quad \left[2 \sin^2 \frac{x}{2} = 1 - \cos x \text{ and } 2 \cos^2 \frac{x}{2} = 1 + \cos x \right]$$

$$= \tan^2 \frac{x}{2}$$

$$= \left(\sec^2 \frac{x}{2} - 1 \right)$$

$$\therefore \int \frac{1 - \cos x}{1 + \cos x} dx = \int \left(\sec^2 \frac{x}{2} - 1 \right) dx$$

$$= \left[\frac{\tan \frac{x}{2}}{\frac{1}{2}} - x \right] + C$$

$$= 2 \tan \frac{x}{2} - x + C$$

Indefinite Integrals Ex 19.2 Q44

$$\int \left\{ 3 \sin x - 4 \cos x + \frac{5}{\cos^2 x} - \frac{6}{\sin^2 x} + \tan^2 x - \cot^2 x \right\} dx$$

$$= 3 \int \sin x dx - 4 \int \cos x dx + 5 \int \sec^2 x dx - 6 \int \operatorname{cosec}^2 x dx + \int \tan^2 x dx - \int \cot^2 x dx$$

$$= 3 \int \sin x dx - 4 \int \cos x dx + 5 \int \sec^2 x dx - 6 \int \operatorname{cosec}^2 x dx + \int (\sec^2 x - 1) dx - \int (\operatorname{cosec}^2 x - 1) dx$$

$$= 3 \int \sin x dx - 4 \int \cos x dx + 5 \int \sec^2 x dx - 7 \int \operatorname{cosec}^2 x dx$$

$$= -3 \cos x - 4 \sin x + 5 \tan x + 7 \cot x + c$$

Indefinite Integrals Ex 19.2 Q45

It is given that $f'(x) = x - \frac{1}{x^2}$

$$\therefore \int f'(x) = \int \left(x - \frac{1}{x^2} \right) dx$$

$$\Rightarrow f(x) = \int x dx - \int \frac{1}{x^2} dx$$

$$= \frac{x^2}{2} + x^{-1} + c$$

$$= \frac{x^2}{2} + \frac{1}{x} + c$$

$$\Rightarrow f(x) = \frac{x^2}{2} + \frac{1}{x} + c$$

Now,

$$f(1) = \frac{1}{2}$$

[given]

$$\Rightarrow \frac{1^2}{2} + \frac{1}{1} + c = \frac{1}{2}$$

$$\Rightarrow c = -1$$

Putting $c = -1$ in (i), we get

$$f(x) = \frac{x^2}{2} + \frac{1}{x} - 1.$$

It is given that $f'(x) = x + b$

$$\therefore \int f'(x) = \int (x + b) dx$$

$$\Rightarrow f(x) = \frac{x^2}{2} + bx + c \quad \text{---(i)}$$

Since,

$$f(1) = 5$$

$$\therefore \frac{1^2}{2} + b \times 1 + c = 5$$

$$\Rightarrow \frac{1}{2} + b + c = 5$$

$$\Rightarrow b + c = \frac{9}{2} \quad \text{---(ii)}$$

$$\text{and, } f(2) = 13$$

$$\Rightarrow \frac{(2)^2}{2} + b \times 2 + c = 13$$

$$\Rightarrow 2 + 2b + c = 13$$

$$\Rightarrow 2b + c = 11 \quad \text{---(iii)}$$

Subtracting equation (ii) from equation (iii), we get

$$b = 11 - \frac{9}{2}$$

$$\Rightarrow b = \frac{13}{2}$$

Putting $b = \frac{13}{2}$ in equation (ii), we get

$$\frac{13}{2} + c = \frac{9}{2}$$

$$\Rightarrow c = \frac{9}{2} - \frac{13}{2}$$

$$\Rightarrow c = \frac{9 - 13}{2} = \frac{-4}{2} = -2$$

Putting $b = \frac{13}{2}$ and $c = -2$ in equation (i), we get

$$f(x) = \frac{x^2}{2} + \frac{13}{2}x - 2$$

$$\therefore f(x) = \frac{x^2}{2} + \frac{13}{2}x - 2$$

We have,

$$f'(x) = 8x^3 - 2x$$

$$\Rightarrow f(x) = \int f'(x) dx = \int (8x^3 - 2x) dx$$

$$\begin{aligned}\Rightarrow f(x) &= \int (8x^3 - 2x) dx \\ &= \int 8x^3 dx - \int 2x dx \\ &= \frac{8x^4}{4} - \frac{2x^2}{2} + c \\ &= 2x^4 - x^2 + c\end{aligned}$$

$$\Rightarrow f(x) = 2x^4 - x^2 + c \quad \text{--- (i)}$$

Since, $f(2) = 8$

$$\therefore f(2) = 2(2)^4 - (2)^2 + c = 8$$

$$\Rightarrow 32 - 4 + c = 8$$

$$\Rightarrow 28 + c = 8$$

$$\Rightarrow c = -20$$

Putting $c = -20$ in equation (i), we get

$$f(x) = 2x^4 - x^2 - 20$$

Hence, $f(x) = 2x^4 - x^2 - 20$.

We have,

$$f(x) = \int f'(x) dx$$

$$\Rightarrow f(x) = \int (a \sin x + b \cos x) dx$$
$$= -a \cos x + b \sin x + c$$

$$\therefore f(x) = -a \cos x + b \sin x + c \quad \text{--- (i)}$$

Since,

$$f'(0) = 4$$

$$\therefore f'(0) = a \sin 0 + b \cos 0 = 4$$

$$\Rightarrow a \times 0 + b \times 1 = 4$$

$$\Rightarrow b = 4$$

Now,

$$f(0) = 3$$

$$\therefore f(0) = -a \cos 0 + b \sin 0 + c = 3$$

$$\Rightarrow -a + 0 + c = 3$$

$$\Rightarrow c - a = 3 \quad \text{--- (ii)}$$

and, $f\left(\frac{\pi}{2}\right) = 5$

$$\therefore f\left(\frac{\pi}{2}\right) = -a \cos\left(\frac{\pi}{2}\right) + b \sin\left(\frac{\pi}{2}\right) + c = 5$$

$$\Rightarrow -a \times 0 + b \times 1 + c = 5$$

$$\Rightarrow b + c = 5$$

$$\Rightarrow 4 + c = 5$$

$$[\because b = 4]$$

$$\Rightarrow c = 5 - 4$$

$$\Rightarrow c = 1$$

Putting $c = 1$ in equation (ii), we get

$$1 - a = 3$$

$$\Rightarrow -a = 3 - 1$$

$$\Rightarrow -a = 2$$

$$\Rightarrow a = -2$$

Putting $a = -2$, $b = 4$ and $c = 1$ in equation (i), we get

$$f(x) = -(-2) \cos x + 4 \sin x + 1$$

$$\Rightarrow f(x) = 2 \cos x + 4 \sin x + 1$$

Hence, $f(x) = 2 \cos x + 4 \sin x + 1$

We have,

$$f(x) = \sqrt{x} + \frac{1}{\sqrt{x}}$$

$$\begin{aligned}\Rightarrow \int f(x) &= \int \left(\sqrt{x} + \frac{1}{\sqrt{x}} \right) dx \\ &= \int x^{\frac{1}{2}} dx + \int x^{-\frac{1}{2}} dx \\ &= \frac{2}{3} x^{\frac{3}{2}} + 2x^{\frac{1}{2}} + c\end{aligned}$$

Hence, the primitive or anti-derivative of $f(x) = \frac{2}{3} x^{\frac{3}{2}} + 2x^{\frac{1}{2}} + c$.