RD Sharma Solutions Class 12 Maths Chapter 19 Ex 19.19

Indefinite Integrals Ex 19.19 Q1

Let $I = \int \frac{x}{x^2 + 3x + 2} dx$

Let $x = \lambda \frac{d}{dx}\left(x^2 + 3x + 2\right) + \mu$

$= \lambda\left(2x + 3\right) + \mu$

$x = \left(2\lambda\right)x + \left(3\lambda + \mu\right)$

Comparing the coefficients of like powers of x,

$2\lambda = 1 \Rightarrow \quad \lambda = \frac{1}{2}$

$3\lambda + \mu = 0 \quad \Rightarrow \quad 3\left(\frac{1}{2}\right) + \mu = 0$

$\mu = -\frac{3}{2}$

so, $\quad I = \int \frac{\frac{1}{2}\left(2x + 3\right) - \frac{3}{2}}{x^2 + 3x + 2} dx$

$I = \frac{1}{2}\int \frac{2x + 3}{x^2 + 3x + 2} dx - \frac{3}{2}\int \frac{1}{x^2 + 3x + 2} dx$

$= \frac{1}{2}\int \frac{2x + 3}{x^2 + 3x + 2} dx - \frac{3}{2}\int \frac{1}{x^2 + 2x\left(\frac{3}{2}\right) + \left(\frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2 + 2} dx$

$I = \frac{1}{2}\int \frac{2x + 3}{x^2 + 3x + 2} dx - \frac{3}{2}\int \frac{1}{\left(x + \frac{3}{2}\right)^2 - \left(\frac{1}{2}\right)^2} dx$

$= \frac{1}{2}\log\left|x^2 + 3x + 2\right| - \frac{3}{2} \times \frac{1}{2\left(\frac{1}{2}\right)}\log\left|\frac{x + \frac{3}{2} - \frac{1}{2}}{x + \frac{3}{2} + \frac{1}{2}}\right| + c \qquad \left[\text{since, } \int \frac{1}{a^2 - x^2} dx = \frac{1}{2a}\log\left|\frac{x - a}{x + a}\right| + c\right]$

$I = \frac{1}{2}\log\left|x^2 + 3x + 2\right| - \frac{3}{2}\log\left|\frac{x + 1}{x + 2}\right| + c$

Indefinite Integrals Ex 19.19 Q2

Let $I = \int \frac{x+1}{x^2+x+3}dx$

Let $x+1 = \lambda \frac{d}{dx}\left(x^2+x+3\right)+\mu$

$$x+1 = \lambda\left(2x+1\right)+\mu$$

$$x+1 = \left(2\lambda\right)x+\left(\lambda+\mu\right)$$

Comparing the coefficients of like powers of x,

$$2\lambda = 1 \Rightarrow \quad \lambda = \frac{1}{2}$$

$$\lambda+\mu = 1 \quad \Rightarrow \quad \left(\frac{1}{2}\right)+\mu = 0$$

$$\mu = \frac{1}{2}$$

so, $$I = \int \frac{\frac{1}{2}\left(2x+1\right)+\frac{1}{2}}{x^2+x+3}dx$$

$$I = \frac{1}{2}\int \frac{2x+1}{x^2+x+3}dx+\frac{1}{2}\int \frac{1}{x^2+2x\left(\frac{1}{2}\right)+\left(\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2+3}dx$$

$$I = \frac{1}{2}\int \frac{2x+1}{x^2+x+3}dx+\frac{1}{2}\int \frac{1}{\left(x+\frac{1}{2}\right)^2+\left(\frac{11}{4}\right)}dx$$

$$I = \frac{1}{2}\int \frac{2x+1}{x^2+x+3}dx+\frac{1}{2}\int \frac{1}{\left(x+\frac{1}{2}\right)^2+\left(\frac{\sqrt{11}}{2}\right)^2}dx$$

$$= \frac{1}{2}\log\left|x^2+x+3\right|+\frac{1}{2}\times\frac{1}{\left(\frac{\sqrt{11}}{2}\right)}\tan^{-1}\left(\frac{x+\frac{1}{2}}{\frac{\sqrt{11}}{2}}\right)+c \qquad \left[\text{since, } \int \frac{1}{x^2+a^2}dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+c\right]$$

$$I = \frac{1}{2}\log\left|x^2+x+3\right|+\frac{1}{\sqrt{11}}\tan^{-1}\left(\frac{2x+1}{\sqrt{11}}\right)+c$$

Indefinite Integrals Ex 19.19 Q3

Let $I = \int \frac{x-3}{x^2+2x-4}dx$

Let $x - 3 = \lambda \frac{d}{dx}\left(x^2+2x-4\right)+\mu$

$= \lambda\left(2x+2\right)+\mu$

$x-3 = \left(2\lambda\right)x + \left(2\lambda+\mu\right)$

Comparing the coefficients of like powers of x,

$2\lambda = 1 \Rightarrow \quad \lambda = \frac{1}{2}$

$2\lambda + \mu = -3 \quad \Rightarrow \quad 2\left(\frac{1}{2}\right)+\mu = -3$

$\mu = -4$

so, $\quad I = \int \frac{\frac{1}{2}\left(2x+2\right)-4}{x^2+2x-4}dx$

$I = \frac{1}{2}\int \frac{2x+2}{x^2+2x-4}dx - 4\int \frac{1}{x^2+2x+\left(1\right)^2-\left(1\right)^2-4}dx$

$I = \frac{1}{2}\int \frac{2x+2}{x^2+2x-4}dx - 4\int \frac{1}{\left(x+1\right)^2-\left(\sqrt{5}\right)}dx$

$I = \frac{1}{2}\log\left|x^2+2x-4\right| - 4\times\frac{1}{2\sqrt{5}}\log\left|\frac{x+1-\sqrt{5}}{x+1+\sqrt{5}}\right|+c \quad \left[\text{since, } \int \frac{1}{x^2-a^2}dx = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right|+c\right]$

$I = \frac{1}{2}\log\left|x^2+2x-4\right| - \frac{2}{\sqrt{5}}\log\left|\frac{x+1-\sqrt{5}}{x+1+\sqrt{5}}\right|+c$

Indefinite Integrals Ex 19.19 Q4

Let $I = \int \frac{2x-3}{x^2+6x+13}dx$

Let $2x - 3 = \lambda \frac{d}{dx}\left(x^2+6x+13\right)+\mu$

$= \lambda\left(2x+6\right)+\mu$

$2x-3 = \left(2\lambda\right)x + \left(6\lambda+\mu\right)$

Comparing the coefficients of like powers of x,

$2\lambda = 2 \Rightarrow \quad \lambda = 1$

$6\lambda + \mu = -3 \quad \Rightarrow \quad 6\left(1\right)+\mu = -3$

$\mu = -9$

so, $\quad I = \int \frac{1\left(2x+6\right)-9}{x^2+6x+13}dx$

$I = \int \frac{2x+6}{x^2+6x+13}dx - 9\int \frac{1}{x^2+2x\left(3\right)+\left(3\right)^2-\left(3\right)^2+13}dx$

$I = \int \frac{2x+6}{x^2+6x+13}dx - 9\int \frac{1}{\left(x+3\right)^2+\left(2\right)^2}dx$

$= \log\left|x^2+6x+13\right| - 9\times\frac{1}{2}\tan^{-1}\left(\frac{x+3}{2}\right)+c \quad \left[\text{since, } \int \frac{1}{x^2+a^2}dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+c\right]$

$I = \log\left|x^2+6x+13\right| - \frac{9}{2}\tan^{-1}\left(\frac{x+3}{2}\right)+c$

Indefinite Integrals Ex 19.19 Q5

Let $I = \int \frac{x^2}{x^2+7x+10}dx$

$$= \int \left\{1 - \frac{7x+10}{x^2+7x+10}\right\}dx$$

$$I = x - \int \frac{7x+10}{x^2+7x+10}dx + c_1 \quad ----(i)$$

Let $I_1 = \int \frac{7x+10}{x^2+7x+10}dx$

Let $7x+10 = \lambda \frac{d}{dx}\left(x^2+7x+10\right) + \mu$

$$= \lambda\left(2x+7\right) + \mu$$

$$7x+10 = \left(2\lambda\right)x + 7\lambda + \mu$$

Comparing the coefficients of like powers of x,

$$7 = 2\lambda \quad \Rightarrow \quad \lambda = \frac{7}{2}$$

$$7\lambda + \mu = 10 \quad \Rightarrow \quad 7\left(\frac{7}{2}\right) + \mu = 10$$

$$\mu = -\frac{29}{2}$$

so, $I = \int \frac{\frac{1}{6}(6x-4) - \frac{1}{3}}{3x^2-4x+3}dx$

$$I = \frac{1}{6}\int \frac{6x-4}{3x^2-4x+3}dx - \frac{1}{9}\int \frac{1}{x^2 - \frac{4}{3}x + 1}dx$$

$$I = \frac{1}{6}\int \frac{6x-4}{3x^2-4x+3}dx - \frac{1}{9}\int \frac{1}{x^2 - 2x\left(\frac{2}{3}\right) + \left(\frac{2}{3}\right)^2 - \left(\frac{2}{3}\right)^2 + (2)^2}dx$$

$$I = \frac{1}{6}\int \frac{6x-4}{3x^2-4x+3}dx - \frac{1}{9}\int \frac{1}{\left(x-\frac{2}{3}\right)^2 + \left(\frac{\sqrt{5}}{3}\right)^2}dx$$

$$= \frac{1}{6}\log\left|3x^2-4x+3\right| - \frac{1}{9} \times \frac{1}{\left(\frac{\sqrt{5}}{3}\right)}\tan^{-1}\left(\frac{x-\frac{2}{3}}{\frac{\sqrt{5}}{3}}\right) + c \quad \left[\text{since, } \int \frac{1}{a^2+x^2}dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c\right]$$

$$I = \frac{1}{6}\log\left|3x^2-4x+3\right| - \frac{\sqrt{5}}{15}\tan^{-1}\left(\frac{3x-2}{\sqrt{5}}\right) + c$$

Indefinite Integrals Ex 19.19 Q6

We need to evaluate the integral $\int \frac{2x}{2+x-x^2}dx$

write the numerator in the following form

$$2x = \lambda\left\{\frac{d}{dx}\left(2+x-x^2\right)\right\}+\mu$$

i.e. $2x = \lambda\{-2x+1\}+\mu$

Equating the coefficients will give the values of λ, μ

$\lambda = -1, \mu = 1$

$$\int \frac{2x}{2+x-x^2}dx = \int \frac{\lambda\{-2x+1\}+\mu}{2+x-x^2}dx$$

$$= \int \frac{-1\{-2x+1\}+1}{2+x-x^2}dx$$

$$= \int \frac{-1\{-2x+1\}}{2+x-x^2}dx + \frac{1}{2+x-x^2}dx$$

$$= -\log\left|2+x-x^2\right| + \int \frac{1}{2+x-x^2}dx$$

$$= -\log\left|2+x-x^2\right| - \int \frac{1}{\left(x^2-x-2\right)}dx$$

$$= -\log\left|2+x-x^2\right| - \int \frac{1}{\left(x^2-x+\frac{1}{4}-2-\frac{1}{4}\right)}dx$$

$$= -\log\left|2+x-x^2\right| - \int \frac{1}{\left(x^2-x+\frac{1}{4}-\frac{9}{4}\right)}dx$$

$$= -\log\left|2+x-x^2\right| - \int \frac{1}{\left(\left(x-\frac{1}{2}\right)^2-\left(\frac{3}{2}\right)^2\right)}dx$$

$$= -\log\left|2+x-x^2\right| - \frac{1}{3}\log\left|\frac{\left(x-\frac{1}{2}\right)-\left(\frac{3}{2}\right)}{\left(x-\frac{1}{2}\right)+\left(\frac{3}{2}\right)}\right| + C$$

$$= -\log\left|2+x-x^2\right| - \frac{1}{3}\log\left|\frac{(x-2)}{(x+1)}\right| + C$$

Indefinite Integrals Ex 19.19 Q7

Let $I = \int \frac{1-3x}{3x^2+4x+2}dx$

Let $1-3x = \lambda \frac{d}{dx}\left(3x^2+4x+2\right)+\mu$

$= \lambda\left(6x+4\right)+\mu$

$1-3x = \left(6\lambda\right)x+\left(4\lambda+\mu\right)$

Comparing the coefficients of like powers of x,

$6\lambda = -3 \quad \Rightarrow \quad \lambda = -\frac{1}{2}$

$4\lambda+\mu = 1 \quad \Rightarrow \quad 4\left(-\frac{1}{2}\right)+\mu = 1$

$\mu = 3$

so, $I = \int \frac{-\frac{1}{2}\left(6x+4\right)+3}{3x^2+4x+2}dx$

$I = -\frac{1}{2}\int \frac{6x+4}{3x^2+4x+2}dx + 3\int \frac{1}{3x^2+4x+2}dx$

$I = -\frac{1}{2}\int \frac{6x+4}{3x^2+4x+2}dx + \frac{3}{3}\int \frac{1}{x^2+\frac{4}{3}x+\frac{2}{3}}dx$

$I = -\frac{1}{2}\int \frac{6x+4}{3x^2+4x+2}dx + \int \frac{1}{x^2+2x\left(\frac{2}{3}\right)+\left(\frac{2}{3}\right)^2-\left(\frac{2}{3}\right)^2+\frac{2}{3}}dx$

$= -\frac{1}{2}\int \frac{6x+4}{3x^2+4x+2}dx + \int \frac{1}{\left(x+\frac{2}{3}\right)^2+\frac{2}{9}}dx$

$I = -\frac{1}{2}\int \frac{6x+4}{3x^2+4x+2}dx + \int \frac{1}{\left(x+\frac{2}{3}\right)^2+\left(\frac{\sqrt{2}}{3}\right)^2}dx$

$= -\frac{1}{2}\log\left|3x^2+4x+2\right| + \frac{3}{\sqrt{2}}\tan^{-1}\left(\frac{x+\frac{2}{3}}{\frac{\sqrt{2}}{3}}\right)+c \qquad \left[\text{since, } \int \frac{1}{x^2+a^2}dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+c\right]$

$I = -\frac{1}{2}\log\left|3x^2+4x+2\right| + \frac{3}{\sqrt{2}}\tan^{-1}\left(\frac{3x+2}{\sqrt{2}}\right)+c$

Indefinite Integrals Ex 19.19 Q8

Let $I = \int \frac{2x+5}{x^2-x-2}dx$

Let $2x+5 = \lambda \frac{d}{dx}\left(x^2-x-2\right)+\mu$

$= \lambda\left(2x-1\right)+\mu$

$2x+5 = \left(2\lambda\right)x-\lambda+\mu$

Comparing the coefficients of like powers of x,

$2\lambda = 2 \quad \Rightarrow \quad \lambda = 1$

$-\lambda+\mu = 5 \quad \Rightarrow \quad -1+\mu = 5$

$\mu = 6$

so, $I = \int \frac{\left(2x-1\right)+6}{x^2-x-2}dx$

$$I = \int \frac{\left(2x-1\right)}{x^2-x-2}dx + 6\int \frac{1}{x^2-2x\left(\frac{1}{2}\right)+\left(\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2-2}dx$$

$$I = \int \frac{2x-1}{x^2-x-2}dx + 6\int \frac{1}{\left(x-\frac{1}{2}\right)^2-\frac{9}{4}}dx$$

$$I = \int \frac{2x-1}{x^2-x-2}dx + 6\int \frac{1}{\left(x-\frac{1}{2}\right)^2-\left(\frac{3}{2}\right)^2}dx$$

$$I = \log\left|x^2-x-2\right| + \frac{6}{2\left(\frac{3}{2}\right)}\log\left|\frac{x-\frac{1}{2}-\frac{3}{2}}{x-\frac{1}{2}+\frac{3}{2}}\right|+c \qquad \left[\text{since, } \int \frac{1}{x^2-a^2}dx = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right|+c\right]$$

$$I = \log\left|x^2-x-2\right| + 2\log\left|\frac{x-2}{x+1}\right|+c$$

Indefinite Integrals Ex 19.19 Q9

Let $I = \int \frac{ax^3 + bx}{x^4 + c^2} dx$

Let $ax^3 + bx = \lambda \frac{d}{dx}\left(x^4 + c^2\right) + \mu$

$$ax^3 + bx = \lambda\left(4x^3\right) + \mu$$

Comparing the coefficients of like powers c

$$4\lambda = a \quad \Rightarrow \quad \lambda = \frac{a}{4}$$

$$\mu = 0 \quad \Rightarrow \quad \mu = 0$$

so, $\quad I = \int \frac{\frac{a}{4}\left(4x^3\right) + bx}{x^4 + c^2} dx$

$$I = \frac{a}{4}\int \frac{4x^3}{x^4 + c^2} dx + b\int \frac{x}{\left(x^2\right)^2 + c^2} dx$$

$$I = \frac{a}{4}\int \frac{4x^3}{x^4 + c^2} dx + \frac{b}{2}\int \frac{2x}{\left(x^2\right)^2 + c^2} dx$$

$$= \frac{a}{4}\log\left|x^4 + c^2\right| + \frac{b}{2} I_1 \text{ ----(i)}$$

Now,

$$I_1 = \int \frac{2x}{\left(x^2\right)^2 + c^2} dx$$

Put $x^2 = t$

$\Rightarrow \quad 2x\,dx = dt$

$$I_1 = \int \frac{1}{(t)^2 + c^2} dx$$

$$= \frac{1}{c}\tan^{-1}\left(\frac{t}{c}\right) + c_1$$

$$I_1 = \frac{1}{c}\tan^{-1}\left(\frac{x^2}{c}\right) + c_1 \text{ ----- (ii)}$$

Using equation (ii) in equation (i),

$$I = \frac{a}{4}\log\left|x^4 + c^2\right| + \frac{b}{2c}\tan^{-1}\left(\frac{x^2}{c}\right) + k$$

K = Integration constant

Indefinite Integrals Ex 19.19 Q10

Let $I = \int \frac{x+2}{2x^2+6x+5} dx$

Let $x + 2 = \lambda \frac{d}{dx}\left(2x^2 + 6x + 5\right) + \mu$

$= \lambda\left(4x + 6\right) + \mu$

$x + 2 = \left(4\lambda\right)x + \left(6\lambda + \mu\right)$

Comparing the coefficients of like powers of x,

$4\lambda = 1 \quad \Rightarrow \quad \lambda = \frac{1}{4}$

$6\lambda + \mu = 2 \quad \Rightarrow \quad 6\left(\frac{1}{4}\right) + \mu = 2$

$\mu = \frac{1}{2}$

so, $I = \int \frac{\frac{1}{4}\left(4x+6\right) + \frac{1}{2}}{2x^2+6x+5} dx$

$I = \frac{1}{4}\int \frac{4x+6}{2x^2+6x+5} dx + \frac{1}{2}\int \frac{1}{2x^2+6x+5} dx$

$I = \frac{1}{4}\int \frac{4x+6}{2x^2+6x+5} dx + \frac{1}{4}\int \frac{1}{x^2+3x+\frac{5}{2}} dx$

$I = \frac{1}{4}\int \frac{4x+6}{2x^2+6x+5} dx + \frac{1}{4}\int \frac{1}{x^2+2x\left(\frac{3}{2}\right)+\left(\frac{3}{2}\right)^2-\left(\frac{3}{2}\right)^3+\frac{5}{2}} dx$

$= \frac{1}{4}\int \frac{4x+6}{2x^2+6x+5} dx + \frac{1}{4}\int \frac{1}{\left(x+\frac{3}{2}\right)^2+\frac{1}{4}} dx$

$I = \frac{1}{4}\int \frac{4x+6}{2x^2+6x+5} dx + \frac{1}{4}\int \frac{1}{\left(x+\frac{3}{2}\right)^2+\left(\frac{1}{2}\right)^2} dx$

$I = \frac{1}{4}\int \frac{4x+6}{2x^2+6x+5} dx + \frac{1}{4} \times \frac{1}{\frac{1}{2}} \tan^{-1}\left(\frac{x+\frac{3}{2}}{\frac{1}{2}}\right) + c\,dx \qquad \left[\text{since, } \int \frac{1}{x^2+a^2} dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c\right]$

$I = \frac{1}{4}\log\left|2x^2+6x+5\right| + \frac{1}{2}\tan^{-1}\left(2x+3\right) + c$

Indefinite Integrals Ex 19.19 Q11

Let $I=\int \frac{(3\sin x-2)\cos x}{5-\cos^2 x-4\sin x}dx$

$\therefore I=\int \frac{(3\sin x-2)\cos x}{5-(1-\sin^2 x)-4\sin x}dx$

$\Rightarrow I=\int \frac{(3\sin x-2)\cos x}{5-1+\sin^2 x-4\sin x}dx$

Substitute sinx=t

$\Rightarrow$ cosx dx = dt

Thus,

$I=\int \frac{(3t-2)}{4+t^2-4t}dt$

$\Rightarrow I=\int \frac{(3t-2)}{t^2-4t+4}dt$

$\Rightarrow I=\int \frac{(3t-2)}{(t-2)^2}dt$

Now let us separate the integrand into the simplest form using partial fractions.

$$\frac{(3t-2)}{(t-2)^2}=\frac{A}{(t-2)}+\frac{B}{(t-2)^2}$$

$$=\frac{A(t-2)+B}{(t-2)^2}$$

$$=\frac{At-2A+B}{(t-2)^2}$$

$\Rightarrow 3t-2=At-2A+B$

Comparing the coefficients, we have,

A=3

and

$-2A+B=-2$

Susbtituting the value of A=3 in the above equation, we have,

$\Rightarrow -2\times 3+B=-2$

$\Rightarrow -6+B=-2$

$\Rightarrow B=6-2$

$\Rightarrow B=4$

Thus, $I=\int \frac{(3t-2)}{(t-2)^2}dt$ becomes,

$$I=\int \frac{3}{(t-2)}dt+\int \frac{4}{(t-2)^2}dt$$

$$=3\log|t-2|-4\left(\frac{1}{t-2}\right)+C$$

$$=3\log|2-t|+4\left(\frac{1}{2-t}\right)+C$$

Now substituting t=sinx, we have,

$$I=3\log|2-\sin x|+4\left(\frac{1}{2-\sin x}\right)+C$$

Indefinite Integrals Ex 19.19 Q12

Let $I=\int \frac{5x-2}{1+2x+3x^2}dx$

Rewriting the numerator we have,

$5x-2 = A\frac{d}{dx}\left(1+2x+3x^2\right)+B$

$\Rightarrow 5x-2 = A\left(2+6x\right)+B$

$\Rightarrow 5x-2 = 6xA+2A+B$

Comparing the coefficients, we have,

$6A=5$ and $2A+B=-2$

$\Rightarrow A=\frac{5}{6}$

Substituting the value of A in $2A+B=-2, we have,$

$2\times\frac{5}{6}+B=-2$

$\Rightarrow \frac{10}{6}+B=-2$

$\Rightarrow B=-2-\frac{10}{6}$

$\Rightarrow B=\frac{-12-10}{6}$

$\Rightarrow B=\frac{-22}{6}$

$\Rightarrow B=\frac{-11}{3}$

$5x-2=\frac{5}{6}\left(2+6x\right)-\frac{11}{3}$

Thus, $I=\int \frac{5x-2}{1+2x+3x^2}dx$ becomes,

$I=\int \frac{\left[\frac{5}{6}\left(2+6x\right)-\frac{11}{3}\right]}{3x^2+2x+1}dx$

$=\frac{5}{6}\int \frac{\left(2+6x\right)}{3x^2+2x+1}dx-\frac{11}{3}\int \frac{dx}{3x^2+2x+1}$

$=\frac{5}{6}\log\left(3x^2+2x+1\right)-\frac{11}{3\times 3}\int \frac{dx}{x^2+\frac{2}{3}x+\frac{1}{3}}+C$

$=\frac{5}{6}\log\left(3x^2+2x+1\right)-\frac{11}{9}\int \frac{dx}{x^2+\frac{2}{3}x+\left(\frac{4}{3}\right)^2+\frac{1}{3}-\left(\frac{4}{3}\right)^2}+C$

$$= \frac{5}{6}\log\left(3x^2 + 2x + 1\right) - \frac{11}{9}\int \frac{dx}{\left(x + \frac{1}{3}\right)^2 + \left(\frac{\sqrt{2}}{3}\right)^2} + C$$

$$= \frac{5}{6}\log\left(3x^2 + 2x + 1\right) - \frac{11}{9} \times \frac{1}{\frac{\sqrt{2}}{3}} \tan^{-1}\left[\frac{\left(x + \frac{1}{3}\right)}{\frac{\sqrt{2}}{3}}\right] + C$$

$$= \frac{5}{6}\log\left(3x^2 + 2x + 1\right) - \frac{11}{9} \times \frac{3}{\sqrt{2}} \tan^{-1}\left[\frac{\left(\frac{3x + 1}{3}\right)}{\frac{\sqrt{2}}{3}}\right] + C$$

$$= \frac{5}{6}\log\left(3x^2 + 2x + 1\right) - \frac{11}{3\sqrt{2}} \tan^{-1}\left[\frac{3x + 1}{\sqrt{2}}\right] + C$$