

RD Sharma
Solutions
Class 12 Maths
Chapter 19
Ex 19.6



Indefinite Integrals Ex 19.16 Q1

$$\text{Let } I = \int \frac{\sec^2 x}{1 - \tan^2 x} dx$$

$$\text{Let } \tan x = t$$

$$\Rightarrow \sec^2 x \, dx = dt$$

$$I = \int \frac{dt}{(1)^2 - t^2}$$

$$= \frac{1}{2(1)} \log \left| \frac{1+t}{1-t} \right| + c \quad \left[\text{Since, } \int \frac{1}{a^2 - x^2} dx = \frac{1}{2a} \log \left| \frac{a+x}{a-x} \right| + c \right]$$

$$I = \frac{1}{2} \log \left| \frac{1 + \tan x}{1 - \tan x} \right| + c$$

Indefinite Integrals Ex 19.16 Q2

$$\text{Let } I = \int \frac{e^x}{1 + e^{2x}} dx$$

$$\text{Let } \tan e^x = t$$

$$\Rightarrow e^x \, dx = dt$$

$$\text{so, } I = \int \frac{dt}{1 + t^2}$$

$$= \tan^{-1}(t) + c \quad \left[\text{Since, } \int \frac{1}{1 + x^2} dx = \tan^{-1} x + c \right]$$

$$I = \tan^{-1}(e^x) + c$$

Indefinite Integrals Ex 19.16 Q3

$$\text{Let } I = \int \frac{\cos x}{\sin^2 x + 4 \sin x + 5} dx$$

$$\text{Let } \sin x = t$$

$$\Rightarrow \cos x \, dx = dt$$

$$\begin{aligned} \text{so, } I &= \int \frac{dx}{t^2 + 4t + 5} \\ &= \int \frac{dt}{t^2 + 2t(2) + (2)^2 - (2)^2 + 5} \\ &= \int \frac{dt}{(t+2)^2 + 1} \end{aligned}$$

$$\text{Again, Let } (t+2) = u$$

$$dt = du$$

$$\begin{aligned} I &= \int \frac{dt}{u^2 + 1} \\ &= \tan^{-1}(u) + c \quad \left[\text{Since, } \int \frac{1}{x^2 + 1} dx = \tan^{-1} x + c \right] \end{aligned}$$

$$I = \tan^{-1}(t+2) + c$$

$$I = \tan^{-1}(\sin x + 2) + c$$

$$\text{Let } I = \int \frac{e^x}{e^{2x} + 5e^x + 6} dx$$

$$\text{Let } e^x = t$$

$$\Rightarrow e^x dx = dt$$

$$\begin{aligned} \text{so, } I &= \int \frac{dt}{t^2 + 5t + 6} \\ &= \int \frac{dt}{t^2 + 2t\left(\frac{5}{2}\right) + \left(\frac{5}{2}\right)^2 - \left(\frac{5}{2}\right)^2 + 6} \\ &= \int \frac{dt}{\left(t + \frac{5}{2}\right)^2 - \frac{1}{4}} \end{aligned}$$

$$\text{Put } \left(t + \frac{5}{2}\right) = u$$

$$\Rightarrow dt = du$$

$$I = \int \frac{dt}{u^2 - \left(\frac{1}{2}\right)^2}$$

$$= \frac{1}{2\left(\frac{1}{2}\right)} \log \left| \frac{u - \frac{1}{2}}{u + \frac{1}{2}} \right| + c$$

$$\left[\text{Since, } \int \frac{1}{x^2 - a^2} dx = \frac{1}{2a} \left| \frac{x - a}{x + a} \right| + c \right]$$

$$I = \log \left| \frac{2u - 1}{2u + 1} \right| + c$$

$$I = \log \left| \frac{2\left(t + \frac{5}{2}\right) - 1}{2\left(t + \frac{5}{2}\right) + 1} \right| + c$$

$$I = \log \left| \frac{e^x + 2}{e^x + 3} \right| + c$$

$$\text{Let } I = \int \frac{e^{3x}}{4e^{6x} - 9} dx$$

$$\text{Let } e^{3x} = t$$

$$\Rightarrow 3e^{3x} dx = dt$$

$$\Rightarrow e^{3x} dx = \frac{dt}{3}$$

$$I = \frac{1}{3} \int \frac{dt}{4t^2 - 9}$$

$$= \frac{1}{12} \int \frac{dt}{t^2 - \frac{9}{4}}$$

$$= \frac{1}{12} \int \frac{dt}{t^2 - \left(\frac{3}{2}\right)^2}$$

$$= \frac{1}{12} \times \frac{1}{2\left(\frac{3}{2}\right)} \log \left| \frac{t - \frac{3}{2}}{t + \frac{3}{2}} \right| + c \quad \left[\text{Since, } \int \frac{1}{x^2 - a^2} dx = \frac{1}{2a} \left| \frac{x - a}{x + a} \right| + c \right]$$

$$I = \frac{1}{36} \log \left| \frac{2t - 3}{2t + 3} \right| + c$$

$$I = \frac{1}{36} \log \left| \frac{2e^{3x} - 3}{2e^{3x} + 3} \right| + c$$

Indefinite Integrals Ex 19.16 Q6

$$\text{Let } I = \int \frac{dx}{e^x + e^{-x}}$$

$$= \int \frac{dx}{e^x + \frac{1}{e^x}}$$

$$= \int \frac{e^x dx}{(e^x)^2 + 1}$$

$$\text{Let } e^x = t$$

$$\Rightarrow e^x dx = dt$$

$$I = \int \frac{dt}{t^2 + 1}$$

$$I = \tan^{-1} t + c \quad \left[\text{Since } \int \frac{1}{1 + x^2} dx = \tan^{-1} x + c \right]$$

$$I = \tan^{-1} (e^x) + c$$

Indefinite Integrals Ex 19.16 Q7

$$\text{Let } I = \int \frac{x}{x^4 + 2x^2 + 3} dx$$

$$\text{Let } x^2 = t$$

$$\Rightarrow 2x dx = dt$$

$$\Rightarrow x dx = \frac{dt}{2}$$

$$\begin{aligned} I &= \frac{1}{2} \int \frac{dt}{t^2 + 2t + 3} \\ &= \frac{1}{2} \int \frac{dt}{t^2 + 2t + 1 - 1 + 3} \\ &= \frac{1}{2} \int \frac{dt}{(t+1)^2 + 2} \end{aligned}$$

$$\text{put } t+1 = u$$

$$\Rightarrow dt = du$$

$$\begin{aligned} I &= \frac{1}{2} \int \frac{du}{u^2 + (\sqrt{2})^2} \\ &= \frac{1}{2} \times \frac{1}{\sqrt{2}} \tan^{-1} \left(\frac{u}{\sqrt{2}} \right) + c \\ I &= \frac{1}{2\sqrt{2}} \tan^{-1} \left(\frac{t+1}{\sqrt{2}} \right) + c \\ I &= \frac{1}{2\sqrt{2}} \tan^{-1} \left(\frac{x^2+1}{\sqrt{2}} \right) + c \end{aligned}$$

$$\left[\text{Since } \int \frac{1}{x^2 + a^2} dx = \frac{1}{a} \tan^{-1} \left(\frac{x}{a} \right) + c \right]$$

Indefinite Integrals Ex 19.16 Q8

$$\begin{aligned} \text{Let } I &= \int \frac{3x^5}{1+x^{12}} dx \\ &= \int \frac{3x^5}{1+(x^6)^2} dx \end{aligned}$$

$$\text{Let } x^6 = t$$

$$\Rightarrow 6x^5 dx = dt$$

$$\Rightarrow x^5 dx = \frac{dt}{6}$$

$$\begin{aligned} I &= \frac{3}{6} \int \frac{dt}{1+t^2} \\ &= \frac{1}{2} \tan^{-1}(t) + c \end{aligned}$$

$$\left[\text{Since } \int \frac{1}{x^2 + 1} dx = \tan^{-1} x + c \right]$$

$$I = \frac{1}{2} \tan^{-1}(x^6) + c$$

Indefinite Integrals Ex 19.16 Q9

$$\begin{aligned}\text{Let } I &= \int \frac{x^2}{x^6 - a^6} dx \\ &= \int \frac{x^2}{(x^3)^2 - (a^3)^2} dx\end{aligned}$$

$$\text{Let } x^3 = t$$

$$\Rightarrow 3x^2 dx = dt$$

$$\Rightarrow x^2 dx = \frac{dt}{3}$$

$$\begin{aligned}\text{so, } I &= \frac{1}{3} \int \frac{dt}{t^2 - (a^3)^2} \\ &= \frac{1}{3} \times \frac{1}{2a^3} \log \left| \frac{t - a^3}{t + a^3} \right|\end{aligned}$$

$$\left[\text{Since } \int \frac{1}{x^2 - a^2} dx = \frac{1}{2a} \log \left| \frac{x - a}{x + a} \right| + c \right]$$

$$I = \frac{1}{6a^3} \log \left| \frac{x^3 - a^3}{x^3 + a^3} \right| + c$$

Indefinite Integrals Ex 19.16 Q10

$$\begin{aligned}\text{Let } I &= \int \frac{x^2}{x^6 + (a^3)^2} dx \\ &= \int \frac{x^2}{(x^3)^2 + (a^3)^2} dx\end{aligned}$$

$$\text{Let } x^3 = t$$

$$\Rightarrow 3x^2 dx = dt$$

$$\Rightarrow x^2 dx = \frac{dt}{3}$$

$$\begin{aligned}\text{so, } I &= \frac{1}{3} \int \frac{dt}{t^2 + (a^3)^2} \\ &= \frac{1}{3} \times \frac{1}{(a^3)} \tan^{-1} \left(\frac{t}{a^3} \right) + c\end{aligned}$$

$$\left[\text{Since } \int \frac{1}{x^2 + a^2} dx = \frac{1}{a} \tan^{-1} \left(\frac{x}{a} \right) + c \right]$$

$$I = \frac{1}{3a^3} \tan^{-1} \left(\frac{x^3}{a^3} \right) + c$$

Indefinite Integrals Ex 19.16 Q11

$$\begin{aligned}\text{Let } I &= \int \frac{1}{x(x^6+1)} dx \\ &= \int \frac{x^5}{x^6(x^6+1)} dx\end{aligned}$$

$$\text{Let } x^6 = t$$

$$\Rightarrow 6x^5 dx = dt$$

$$\Rightarrow x^5 dx = \frac{dt}{6}$$

$$\begin{aligned}I &= \frac{1}{6} \int \frac{dt}{t(t+1)} \\ &= \frac{1}{6} \int \frac{dt}{t^2+t} \\ &= \frac{1}{6} \int \frac{dt}{t^2+2t\left(\frac{1}{2}\right)+\left(\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2} \\ &= \frac{1}{6} \int \frac{dt}{\left(t+\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2}\end{aligned}$$

$$\text{Let } t+\frac{1}{2} = u$$

$$\Rightarrow dt = du$$

$$\begin{aligned}I &= \frac{1}{6} \int \frac{du}{u^2-\left(\frac{1}{2}\right)^2} \\ &= \frac{1}{6} \times \frac{1}{2\left(\frac{1}{2}\right)} \log \left| \frac{u-\frac{1}{2}}{u+\frac{1}{2}} \right| + c\end{aligned}$$

$$\left[\text{Since } \int \frac{1}{x^2-a^2} dx = \frac{1}{2a} \log \left| \frac{x-a}{x+a} \right| + c \right]$$

$$I = \frac{1}{6} \log \left| \frac{t+\frac{1}{2}-\frac{1}{2}}{t+\frac{1}{2}+\frac{1}{2}} \right| + c$$

$$I = \frac{1}{6} \log \left| \frac{x^6}{x^6+1} \right| + c$$

$$\text{Let } I = \int \frac{x}{x^4 - x^2 + 1} dx$$

$$\text{Let } x^2 = t$$

$$\Rightarrow 2x dx = dt$$

$$\Rightarrow x dx = \frac{dt}{2}$$

$$\begin{aligned} \text{so, } I &= \frac{1}{2} \int \frac{dt}{t^2 - t + 1} \\ &= \frac{1}{2} \int \frac{dt}{t^2 - 2t \times \left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2 + 1} \\ &= \frac{1}{2} \int \frac{dt}{\left(t - \frac{1}{2}\right)^2 + \left(\frac{3}{4}\right)} \end{aligned}$$

$$\text{Let } t - \frac{1}{2} = u$$

$$\Rightarrow dt = du$$

$$\begin{aligned} I &= \frac{1}{2} \int \frac{du}{u^2 + \left(\frac{\sqrt{3}}{2}\right)^2} \\ &= \frac{1}{2} \times \frac{1}{\left(\frac{\sqrt{3}}{2}\right)} \tan^{-1} \left(\frac{u}{\frac{\sqrt{3}}{2}} \right) + c \quad \left[\text{Since } \int \frac{1}{x^2 + a^2} dx = \frac{1}{a} \tan^{-1} \left(\frac{x}{a} \right) + c \right] \end{aligned}$$

$$I = \frac{1}{\sqrt{3}} \tan^{-1} \left(\frac{t - \frac{1}{2}}{\frac{\sqrt{3}}{2}} \right) + c$$

$$I = \frac{1}{\sqrt{3}} \tan^{-1} \left(\frac{2x^2 - 1}{\sqrt{3}} \right) + c$$

$$\begin{aligned}\text{Let } I &= \int \frac{x}{3x^4 - 18x^2 + 11} dx \\ &= \frac{1}{3} \int \frac{x}{x^4 - 6x^2 + \frac{11}{3}} dx\end{aligned}$$

$$\text{Let } x^2 = t$$

$$\Rightarrow 2x dx = dt$$

$$\Rightarrow x dx = \frac{dt}{2}$$

$$\begin{aligned}I &= \frac{1}{3} \times \frac{1}{2} \int \frac{dt}{t^2 - 6t + \frac{11}{3}} \\ &= \frac{1}{6} \int \frac{dt}{t^2 - 2t(3) + (3)^2 - (3)^2 + \frac{11}{3}} \\ &= \frac{1}{6} \int \frac{dt}{(t-3)^2 - \left(\frac{16}{3}\right)}\end{aligned}$$

$$\text{Let } t-3 = u$$

$$\Rightarrow dt = du$$

$$\begin{aligned}I &= \frac{1}{6} \int \frac{du}{u^2 - \left(\frac{4}{\sqrt{3}}\right)^2} \\ &= \frac{1}{6} \times \frac{1}{2\left(\frac{4}{\sqrt{3}}\right)} \log \left| \frac{u - \frac{4}{\sqrt{3}}}{u + \frac{4}{\sqrt{3}}} \right| + c \quad \left[\text{Since } \int \frac{1}{x^2 - a^2} dx = \frac{1}{2a} \log \left| \frac{x-a}{x+a} \right| + c \right] \\ I &= \frac{\sqrt{3}}{48} \log \left| \frac{t-3-\frac{4}{\sqrt{3}}}{t-3+\frac{4}{\sqrt{3}}} \right| + c\end{aligned}$$

$$I = \frac{\sqrt{3}}{48} \log \left| \frac{x^2 - 3 - \frac{4}{\sqrt{3}}}{x^2 - 3 + \frac{4}{\sqrt{3}}} \right| + c$$

To evaluate the following integral follow the steps:

Let $e^x = t$ therefore $e^x dx = dt$

Now

$$\begin{aligned}\int \frac{e^x}{(1+e^x)(2+e^x)} dx &= \int \frac{dt}{(1+t)(2+t)} \\&= \int \frac{dt}{(1+t)} - \int \frac{dt}{(2+t)} \\&= \ln|1+t| - \ln|2+t| + c \\&= \ln \left| \frac{1+t}{2+t} \right| + c \\&= \ln \left| \frac{1+e^x}{2+e^x} \right| + c\end{aligned}$$