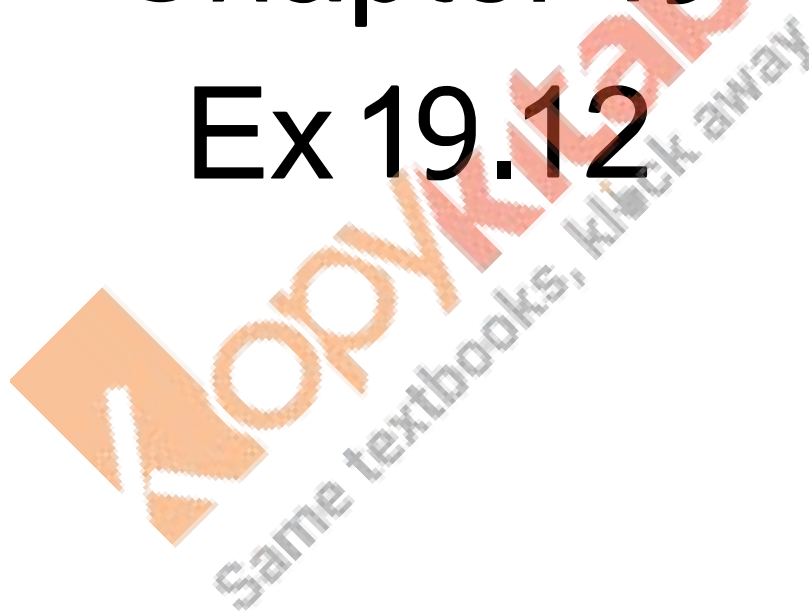


RD Sharma
Solutions
Class 12 Maths
Chapter 19
Ex 19.12



Indefinite Integrals Ex 19.12 Q1

$$\text{Let } I = \int \sin^4 x \cos^3 x dx$$

Here, power of $\cos x$ is odd, so we substitute

$$\sin x = t$$

$$\Rightarrow \cos x dx = dt$$

$$\Rightarrow dx = \frac{dt}{\cos x}$$

$$\therefore I = \int t^4 \cos^3 x \frac{dt}{\cos x}$$

$$= \int t^4 \cos^2 x dt$$

$$= \int t^4 (1 - \sin^2 x) dt$$

$$= \int t^4 (1 - t^2) dt$$

$$= \int (t^4 - t^6) dt$$

$$= \frac{t^5}{5} - \frac{t^7}{7} + C$$

$$\therefore I = \frac{1}{5} \times \sin^5 x - \frac{1}{7} \times \sin^7 x + C$$

Indefinite Integrals Ex 19.12 Q2

Let $I = \int \sin^5 x dx$. Then

$$\begin{aligned} I &= \int \sin^3 x \sin^2 x dx \\ &= \int \sin^3 x (1 - \cos^2 x) dx \\ &= \int (\sin^3 x - \sin^3 x \cos^2 x) dx \\ &= \int [\sin x (1 - \cos^2 x) - \sin^3 x \cos^2 x] dx \\ &= \int (\sin x - \sin x \cos^2 x - \sin^3 x \cos^2 x) dx \end{aligned}$$

$$\Rightarrow I = \int \sin x dx - \int \sin x \cos^2 x dx - \int \sin^3 x \cos^2 x dx$$

Putting $\cos x = t$ and $-\sin x dx = dt$ in 2nd and 3rd integrals, we get

$$\begin{aligned} I &= \int \sin x dx - \int t^2 (-dt) + \int \sin^2 x t^2 dt \\ &= \int \sin x dx + \int t^2 dt + \int (1 - \cos^2 x) t^2 dt \\ &= \int \sin x dx + \int t^2 dt + \int (1 - t^2) t^2 dt \\ &= -\cos x + \frac{t^3}{3} + \frac{t^3}{3} - \frac{t^5}{5} + c \\ &= -\cos x + \frac{2}{3} t^3 - \frac{1}{5} t^5 + c \\ &= -\cos x + \frac{2}{3} (\cos^3 x) - \frac{1}{5} (\cos^5 x) + c \end{aligned}$$

$$\therefore I = -\left[\cos x - \frac{2}{3} \cos^3 x + \frac{1}{5} \cos^5 x \right] + c$$

Indefinite Integrals Ex 19.12 Q3

Let $I = \int \cos^5 x dx$. Then

$$\begin{aligned} I &= \int \cos^2 x \cos^3 x dx \\ &= \int (1 - \sin^2 x) \cos^3 x dx \\ &= \int \cos^3 x dx - \int \sin^2 x \cos^3 x dx \\ &= \int \cos^2 x \cos x dx - \int \sin^2 x (1 - \sin^2 x) \cos x dx \\ &= \int (\cos x - \sin^2 x \cos x) dx - \int (\sin^2 x \cos x - \sin^4 x \cos x) dx \end{aligned}$$

$$\Rightarrow I = \int \cos x dx - 2 \int \sin^2 x \cos x dx + \int \sin^4 x \cos x dx$$

Putting $\sin x = t$ and $\cos x dx = dt$ in 2nd and 3rd integrals, we get

$$\begin{aligned} I &= \int \cos x dx - 2 \int t^2 dt + \int t^4 dt \\ &= \sin x - \frac{2}{3} t^3 + \frac{t^5}{5} + c \\ &= \sin x - \frac{2}{3} \sin^3 x + \frac{1}{5} \sin^5 x + c \end{aligned}$$

$$\therefore I = \sin x - \frac{2}{3} \sin^3 x + \frac{1}{5} \sin^5 x + c$$

Indefinite Integrals Ex 19.12 Q4

$$\text{Let } I = \int \sin^5 x \cos x \, dx \quad \text{---(i)}$$

Let $\sin x = t$. Then,

$$d(\sin x) = dt$$

$$\Rightarrow \cos x \, dx = dt$$

Putting $\sin x = t$ and $\cos x \, dx = dt$ in equation (i), we get

$$I = \int t^5 dt$$

$$= \frac{t^6}{6} + C$$

$$= \frac{\sin^6 x}{6} + C$$

$$\therefore I = \frac{1}{6} \sin^6 x + C$$

Indefinite Integrals Ex 19.12 Q5

$$\text{Let } I = \int \sin^3 x \cos^6 x \, dx$$

Here, power of $\sin x$ is odd, so we substitute

$$\cos x = t$$

$$\Rightarrow -\sin x \, dx = dt$$

$$\therefore I = \int \sin^2 x t^6 (-dt)$$

$$= -\int (1 - \cos^2 x) t^6 dt$$

$$= -\int (t^6 - t^8) dt$$

$$= -\frac{t^7}{7} + \frac{t^9}{9} + C$$

$$= -\frac{\cos^7 x}{7} + \frac{\cos^9 x}{9} + C$$

$$\therefore I = -\frac{1}{7} \cos^7 x + \frac{1}{9} \cos^9 x + C$$

Indefinite Integrals Ex 19.12 Q6

Let $I = \int \cos^7 x dx$. Then

$$I = \int \cos^6 x \cos x dx$$

$$= \int (\cos^2 x)^3 \cos x dx$$

$$= \int (1 - \sin^2 x)^3 \cos x dx$$

$$= \int [1 - \sin^6 x - 3 \sin^2 x + 3 \sin^4 x] \cos x dx$$

$$= \int [\cos x - \sin^6 x \cos x - 3 \sin^2 x \cos x + 3 \sin^4 x \cos x] dx + c$$

$$\Rightarrow I = \int \cos x dx - \int \sin^6 x \cos x dx - 3 \int \sin^2 x \cos x dx + 3 \int \sin^4 x \cos x dx$$

Putting $\sin x = t$ and $\cos x dx = dt$ in 2nd and 3rd and 4th integral, we get

$$I = \int \cos x dx - \int t^6 dt - 3 \int t^2 dt + 3 \int t^4 dt$$

$$= \sin x - \frac{t^7}{7} - \frac{3}{3} t^3 + \frac{3}{5} t^5 + c$$

$$= \sin x - \frac{1}{7} \sin^7 x - \sin^3 x + \frac{3}{5} \sin^5 x + c$$

$$\therefore I = \sin x - \sin^3 x + \frac{3}{5} \sin^5 x - \frac{1}{7} \sin^7 x + c$$

Indefinite Integrals Ex 19.12 Q7

$$\text{Let } I = \int x \cos^3 x^2 \sin x^2 dx$$

Let $\cos x^2 = t$. Then

$$d(\cos x^2) = dt$$

$$\Rightarrow -2x \sin x^2 dx = dt$$

$$\Rightarrow x \sin x^2 dx = -\frac{dt}{2}$$

$$\therefore I = \int t^3 \times \frac{-dt}{2}$$

$$= -\frac{t^4}{8} + c$$

$$= -\frac{1}{8} \cos^4 x^2 + c$$

$$\therefore I = -\frac{1}{8} \cos^4 x^2 + c$$

Indefinite Integrals Ex 19.12 Q8

Let $I = \int \sin^7 x dx$. Then

$$I = \int \sin^6 x \sin x dx$$

$$= \int (\sin^2 x)^3 \sin x dx$$

$$= \int (1 - \cos^2 x)^3 \sin x dx$$

$$= \int (1 - \cos^6 x + 3 \cos^4 x - 3 \cos^2 x) \sin x dx$$

$$\Rightarrow I = \int \sin x dx - \int \cos^6 x \sin x dx + 3 \int \cos^4 x \sin x dx - 3 \int \cos^2 x \sin x dx$$

Putting $\cos x = t$ and $-\sin x dx = dt$ in 2nd, 3rd and 4th integral, we get

$$I = \int \sin x dx - \int t^6 (-dt) + 3 \int t^4 (-dt) - 3 \int t^2 (-dt)$$

$$= -\cos x + \frac{t^7}{7} - \frac{3}{5} t^5 + \frac{3}{3} t^3 + c$$

$$= -\cos x + \frac{\cos^7 x}{7} - \frac{3}{5} \cos^5 x + \cos^3 x + c$$

$$\therefore I = -\cos x + \cos^3 x - \frac{3}{5} \cos^5 x + \frac{1}{7} \cos^7 x + c$$

Indefinite Integrals Ex 19.12 Q9

Let $I = \int \sin^3 x \cos^5 x dx$. Then

Let $\cos x = t$. Then

$$d(\cos x) = dt$$

$$\Rightarrow -\sin x dx = dt$$

$$\Rightarrow dx = \frac{-dt}{\sin x}$$

$$\therefore I = \int \sin^3 x t^5 \frac{-dt}{\sin x}$$

$$= -\int \sin^2 x t^5 dt$$

$$= -\int (1 - \cos^2 x) t^5 dt$$

$$= -\int (1 - t^2) t^5 dt$$

$$= -\int (t^5 - t^7) dt$$

$$= -\frac{t^6}{6} + \frac{t^8}{8} + c$$

$$= -\frac{1}{6} \cos^6 x + \frac{1}{8} \cos^8 x + c$$

$$\therefore I = -\frac{1}{6} \cos^6 x + \frac{1}{8} \cos^8 x + c$$

Indefinite Integrals Ex 19.12 Q10

$$\text{Let } I = \int \frac{1}{\sin^4 x \cos^2 x} dx \quad \text{---(i)}$$

$$\text{Then, } I = \int \sin^{-4} x \cos^{-2} x dx$$

Since $-4 - 2 = -6$, which is even integer. So, we divide both numerator and denominator by $\cos^6 x$.

$$\begin{aligned} \therefore I &= \int \frac{\frac{1}{\cos^6 x}}{\frac{\sin^4 x \cos^2 x}{\cos^6 x}} dx \\ &= \int \frac{\sec^6 x}{\frac{\sin^4 x}{\cos^4 x}} dx \\ &= \int \frac{\sec^6 x}{\tan^4 x} dx \\ &= \int \frac{\sec^4 x \times \sec^2 x}{\tan^4 x} dx \\ &= \int \frac{(\sec^2 x)^2 \times \sec^2 x}{\tan^4 x} dx \\ &= \int \frac{(1 + \tan^2 x)^2 \times \sec^2 x}{\tan^4 x} dx \\ \Rightarrow I &= \int \frac{(1 + \tan^4 x + 2 \tan^2 x) \times \sec^2 x}{\tan^4 x} dx \quad \text{---(ii)} \end{aligned}$$

Let $\tan x = t$. Then,

$$d(\tan x) = dt$$

$$\Rightarrow \sec^2 x dx = dt$$

$$\Rightarrow dx = \frac{dt}{\sec^2 x}$$

Putting $\tan x = t$ and $dx = \frac{dt}{\sec^2 x}$ in equation (i), we get

$$\begin{aligned} I &= \int \frac{(1 + t^4 + 2t^2)}{t^4} \times \sec^2 x \times \frac{dt}{\sec^2 x} \\ &= \int (t^{-4} + 1 + 2t^{-2}) dt \\ &= -\frac{t^{-3}}{3} + t - 2t^{-1} + c \\ &= -\frac{1}{3t^3} + t - \frac{2}{t} + c \\ &= -\frac{1}{3 \tan^3 x} + \tan x - \frac{2}{\tan x} + c \\ &= -\frac{1}{3} \times \cot^3 x + \tan x - 2 \times \cot x + c \\ \therefore I &= -\frac{1}{3} \times \cot^3 x - 2 \cot x + \tan x + c \end{aligned}$$

$$\text{Let } I = \int \frac{1}{\sin^3 x \cos^5 x} dx \quad \text{---(i)}$$

$$\text{Then, } I = \int \sin^{-3} x \cos^{-5} x dx$$

Since $-3-5 = -8$, which is even integer. So, we divide both numerator and denominator by $\cos^8 x$.

$$\begin{aligned} \therefore I &= \int \frac{\frac{1}{\cos^8 x}}{\frac{\sin^3 x \cos^5 x}{\cos^8 x}} dx \\ &= \int \frac{\sec^8 x}{\tan^3 x} dx \\ &= \int \frac{\sec^8 x}{\tan^3 x} \sec^2 x dx \\ &= \int \frac{(\sec^2 x)^3}{\tan^3 x} \sec^2 x dx \\ &= \int \frac{(1 + \tan^2 x)^3}{\tan^3 x} \times \sec^2 x dx \\ \Rightarrow I &= \int \frac{(1 + \tan^6 x + 3 \tan^4 x + 3 \tan^2 x) \times \sec^2 x}{\tan^3 x} dx \quad \text{---(ii)} \end{aligned}$$

Let $t = \tan x$. Then,

$$d(\tan x) = dt$$

$$\Rightarrow \sec^2 x dx = dt$$

$$\begin{aligned} \therefore I &= \int \frac{(1 + t^6 + 3t^4 + 3t^2)}{t^3} dt \\ &= \int (t^{-3} + t^3 + 3t + 3t^{-1}) dt \\ &= -\frac{t^{-2}}{2} + \frac{t^4}{4} + \frac{3}{2} t^2 + 3 \log t + c \\ &= -\frac{1}{2t^2} + \frac{t^4}{4} + \frac{3}{2} t^2 + 3 \log t + c \\ &= -\frac{1}{2} \times \frac{1}{\tan^2 x} + \frac{\tan^4 x}{4} + \frac{3}{2} \times \tan^2 x + 3 \log |\tan x| + c \\ \therefore I &= \frac{-1}{2 \tan^2 x} + 3 \log |\tan x| + \frac{3}{2} \tan^2 x + \frac{1}{4} \tan^4 x + c \end{aligned}$$

$$\text{Let } I = \int \frac{1}{\sin^3 x \cos x} dx \quad \text{--- (i)}$$

$$\text{Then, } I = \int \sin^{-3} x \cos^{-1} x dx$$

Since $-3-1 = -4$, which is even integer. So, we divide both numerator and denominator by $\cos^4 x$.

$$\begin{aligned} \therefore I &= \int \frac{\frac{1}{\cos^4 x}}{\frac{\sin^3 x \cos x}{\cos^4 x}} dx \\ &= \int \frac{\sec^4 x}{\tan^3 x} dx \\ &= \int \frac{\sec^4 x}{\tan^3 x} \sec^2 x dx \\ &= \int \frac{(\sec^2 x)^3}{\tan^3 x} \sec^2 x dx \\ &= \int \frac{(1 + \tan^2 x)}{\tan^3 x} \times \sec^2 x dx \end{aligned}$$

Putting $\tan x = t$ and $\sec^2 x dx = dt$, we get

$$\begin{aligned} I &= \int \frac{1+t^2}{t^3} dt \\ &= \int \left(t^{-3} + \frac{1}{t} \right) dt \\ &= -\frac{t^{-2}}{2} + \log|t| + c \\ &= -\frac{1}{2t^2} + \log|t| + c \\ &= -\frac{1}{2 \tan^2 x} + \log|\tan x| + c \end{aligned}$$

$$\begin{aligned}
 \frac{1}{\sin x \cos^3 x} &= \frac{\sin^2 x + \cos^2 x}{\sin x \cos^3 x} \\
 &= \frac{\sin x}{\cos^3 x} + \frac{1}{\sin x \cos x} \\
 &= \tan x \sec^2 x + \frac{1 \cos^2 x}{\sin x \cos x} \\
 &\quad \frac{\cos^2 x}{\cos^2 x} \\
 &= \tan x \sec^2 x + \frac{\sec^2 x}{\tan x}
 \end{aligned}$$

$$\therefore \int \frac{1}{\sin x \cos^3 x} dx = \int \tan x \sec^2 x dx + \int \frac{\sec^2 x}{\tan x} dx$$

$$\text{Let } \tan x = t \Rightarrow \sec^2 x dx = dt$$

$$\begin{aligned}
 \Rightarrow \int \frac{1}{\sin x \cos^3 x} dx &= \int t dt + \int \frac{1}{t} dt \\
 &= \frac{t^2}{2} + \log|t| + C \\
 &= \frac{1}{2} \tan^2 x + \log|\tan x| + C
 \end{aligned}$$