RD Sharma Solutions Class 12 Maths Chapter 19 Ex 19.11

Indefinite Integrals Ex 19.11 Q1

Let $I = \int \tan^3 x \sec^2 x dx$ ---(i)

Let $\tan x = t$. Then

$$d(\tan x) = dt$$

$$\Rightarrow \quad \sec^2 x dx = dt$$

$$\Rightarrow \quad dx = \frac{dt}{\sec^2 x}$$

Puting $\tan x = t$ and $dx = \frac{dt}{\sec^2 x}$ in equation (i), we get

$$I = \int t^3 \sec^2 x \times \frac{dt}{\sec^2 x}$$

$$= \int t^3 dt$$

$$= \frac{t^{3+1}}{3+1} + c$$

$$= \frac{t^4}{4} + c$$

$$= \frac{(\tan x)^4}{4} + c$$

$$\therefore \quad I = \frac{(\tan x)^4}{4} + c$$

$$= \frac{1}{4} \times \tan^4 x + c.$$

Indefinite Integrals Ex 19.11 Q2

Let $I = \int \tan x \sec^4 x dx$. Then

$$I = \int \tan x \sec^2 x \sec^2 x dx$$
$$= \int \tan x \left(1 + \tan^2 x\right) \sec^2 x dx$$
$$\Rightarrow \quad I = \int \left(\tan x + \tan^3 x\right) \sec^2 x dx$$

Substituting $\tan x = t$ and $\sec^2 x dx = dt$, we get

$$I = \int \left(t + t^3\right) dt$$
$$= \frac{t^2}{2} + \frac{t^4}{4} + c$$
$$= \frac{\tan^2 x}{2} + \frac{\tan^4}{4} + c$$
$$\therefore \quad I = \frac{1}{2} \times \tan^2 x + \frac{1}{4} \times \tan^4 x + c.$$

Indefinite Integrals Ex 19.11 Q3

Let $I = \int \tan^5 x \sec^4 x dx$. Then

$$I = \int \tan^4 x \sec^2 x \sec^2 x dx$$
$$= \int \tan^5 x \left(1 + \tan^2 x\right) \sec^2 x dx$$
$$= \int \left(\tan^5 x + \tan^7 x\right) \sec^2 x dx$$

Substituting $\tan x = t$ and $\sec^2 x dx = dt$, we get

$$I = \int \left(t^5 + t^7\right) dt$$
$$= \frac{t^6}{6} + \frac{t^8}{8} + c$$
$$= \frac{(\tan x)^6}{6} + \frac{(\tan x)^8}{8} + c$$
$$\therefore \quad I = \frac{1}{6} \times \tan^6 x + \frac{1}{8} \times \tan^8 x + c.$$

Indefinite Integrals Ex 19.11 Q4

Let $I = \int \sec^6 x \tan x dx$. Then

$$I = \int \sec^5 x \left(\sec x \tan x\right) dx$$

Substituting $\sec x = t$ and $\sec x \tan x = dt$, we get

$$I = \int t^5 dt$$
$$= \frac{t^6}{6} + c$$
$$= \frac{(\sec x)^6}{6} + c$$
$$\therefore \quad I = \frac{1}{6} \sec^6 x + c$$

Indefinite Integrals Ex 19.11 Q5

Let $I = \int \tan^5 x dx$. Then

$$I = \int \tan^2 x \tan^3 x dx$$
$$= \int \left(\sec^2 x - 1\right) \tan^3 x dx$$
$$= \int \sec^2 x \tan^3 x dx - \int \tan^3 x dx$$
$$= \int \sec^2 x \tan^3 x dx - \int \left(\sec^2 x - 1\right) \tan x dx$$
$$= \int \sec^2 x \tan^3 x dx - \int \sec^2 x \tan x dx + \int \tan x dx$$

Substituting $\tan x = t$ and $\sec^2 x dx = dt$ in first two integral, we get

$$I = \int t^3 dt - \int t dt + \int \tan x dx$$
$$= \frac{t^4}{4} - \frac{t^2}{2} + \log\left|\sec x\right| + c$$
$$= \frac{\tan^4 x}{4} - \frac{\tan^2 x}{2} + \log\left|\sec x\right| + c$$

$$\therefore \quad I = \frac{\tan^4 x}{4} - \frac{\tan^2 x}{2} + \log\left|\sec x\right| + c$$

Indefinite Integrals Ex 19.11 Q6

Let $I = \int \sqrt{\tan x} \sec^4 x dx$. Then

$$I = \int \sqrt{\tan x} \sec^2 x \sec^2 x dx$$
$$= \int \sqrt{\tan x} \left(1 + \tan^2 x\right) \sec^2 x dx$$
$$= \int \tan x^{\frac{1}{2}} \left(1 + \tan^2 x\right) \sec^2 x dx$$
$$\Rightarrow \quad I = \int \left(\tan x^{\frac{1}{2}} + \tan x^{\frac{5}{2}}\right) \sec^2 x dx$$

Substituting $\tan x = t$ and $\sec^2 x dx = dt$, we get

$$I = \int \left(t^{\frac{1}{2}} + t^{\frac{5}{2}}\right) dt$$
$$= \frac{2}{3} t^{\frac{3}{2}} + \frac{2}{7} t^{\frac{7}{2}} + c$$
$$= \frac{2}{3} (\tan x)^{\frac{3}{2}} + \frac{2}{7} (\tan x)^{\frac{7}{2}} + c$$

$$\therefore \quad I = \frac{2}{3} \tan^{\frac{3}{2}} x + \frac{2}{7} \tan^{\frac{7}{2}} x + c$$

Indefinite Integrals Ex 19.11 Q7

Let $I = \int \sec^4 2x dx$. Then

$$I = \int \sec^2 2x \sec^2 2x dx$$

$$= \int \left(1 + \tan^2 2x\right) \sec^2 2x dx$$

$$= \int \left(\sec^2 2x + \sec^2 2x \tan^2 2x\right) dx$$

$$\Rightarrow \quad I = \int \sec^2 2x dx + \int \sec^2 2x \tan^2 2x dx$$

$$\Rightarrow \quad I = \int \sec^2 2x \tan^2 2x dx + \int \sec^2 2x dx$$

Substituting $\tan 2x = t$ and $\sec^2 2x dx = \frac{dt}{2}$ in first integral, we get

$$I = \int t^2 \frac{dt}{2} + \int \sec^2 2x dx$$

$$= \frac{1}{2} \times \frac{t^3}{3} + \frac{1}{2} \tan 2x + c$$

$$\Rightarrow \quad I = \frac{1}{6} \tan^3 2x + \frac{1}{2} \tan 2x + c$$

$$\therefore \quad I = \frac{1}{2} \tan 2x + \frac{1}{6} \tan^3 2x + c$$

Indefinite Integrals Ex 19.11 Q8

Let $I = \int cosec^4 3x dx$. Then

$$I = \int cosec^2 3x \, cosec^2 3x dx$$

$$= \int \left(1 + \cot^2 3x\right) cosec^2 3x dx$$

$$= \int \left(cosec^2 3x + \cot^2 3x \, cosec^2 3x\right) dx$$

$$\Rightarrow \quad I = \int cosec^2 3x dx + \int \cot^2 3x \, cosec^2 3x dx$$

Substituting $\cot 3x = t$ and $cosec^2 3x dx = -dt$ in 2nd integral, we get

$$I = \int cosec^2 3x dx - \int t^2 \frac{dt}{3}$$

$$= \frac{-1}{3} \cot 3x - \frac{t^3}{9} + c$$

$$= \frac{-1}{3} \cot 3x - \frac{\cot^3 3x}{9} + c$$

$$\therefore \quad I = \frac{-1}{3} \cot 3x - \frac{1}{9} \cot^3 3x + c$$

Indefinite Integrals Ex 19.11 Q9

Let $I = \int \cot^n x \operatorname{cosec}^2 x dx$. $n \neq -1$ ---(i)

Let $\cot x = t$. Then

$$d(\cot x) = dt$$

$$\Rightarrow \quad -\operatorname{cosec}^2 x dx = dt$$

$$\Rightarrow \quad \operatorname{cosec}^2 x dx = -dt$$

Putting $\cot x = t$ and $\operatorname{cosec}^2 x dx = -dt$ in equation (i), we get

$$I = \int t^n \times (-dt)$$

$$= -\frac{t^{n+1}}{n+1} + c$$

$$\Rightarrow \quad = -\frac{(\cot x)^{n+1}}{n+1} + c$$

Indefinite Integrals Ex 19.11 Q10

Let $I = \int \cot^5 x \operatorname{cosec}^4 x dx$. Then,

$$I = \int \cot^5 x \operatorname{cosec}^2 x \operatorname{cosec}^2 x dx$$

$$= \int \cot^5 x \left(1 + \cot^2 x\right) \operatorname{cosec}^2 x dx$$

$$\Rightarrow \quad I = \int \left(\cot^5 x + \cot^7 x\right) \operatorname{cosec}^2 x dx$$

Substituting $\cot x = t$ and $-\operatorname{cosec}^2 x\, dx = dt$, we get

$$I = \int \left(t^5 + t^7\right)(-dt)$$

$$= -\frac{t^6}{6} - \frac{t^8}{8} + c$$

$$= -\frac{\cot^6 x}{6} - \frac{\cot^8 x}{8} + c$$

$$\therefore \quad I = -\frac{\cot^6 x}{6} - \frac{\cot^8 x}{8} + c$$

Indefinite Integrals Ex 19.11 Q11

Let $I = \int \cot^5 x dx$. Then,

$$I = \int \cot^3 x \times \cot^2 x dx$$
$$= \int \cot^3 x \times \left(cosec^2 x - 1\right) dx$$
$$= \int \cot^3 x \, cosec^2 x dx - \int \cot^3 x dx$$
$$= \int \cot^3 x \, cosec^2 x dx - \int \left(cosec^2 x - 1\right) \cot dx$$
$$= \int \cot^3 x \, cosec^2 x dx - \int cosec^2 x \cot x dx + \int \cot x dx$$
$$\Rightarrow \quad I = \int \cot^3 x \, cosec^2 x dx - \int cosec^2 x \cot x dx + \int \cot x dx$$

Substituting $\cot x = t$ and $-cosec^2 x \, dx = dt$ in first two integral, we get

$$I = \int t^3 (-dt) - \int t \times (-dt) + \int \cot x dx$$
$$= -\frac{t^4}{4} + \frac{t^2}{2} + \log|\sin x| + c$$
$$= -\frac{1}{4}\cot^4 x + \frac{1}{2}\cot^2 x + \log|\sin x| + c$$
$$\therefore \quad I = -\frac{1}{4}\cot^4 x + \frac{1}{2}\cot^2 x + \log|\sin x| + c$$

Indefinite Integrals Ex 19.11 Q12

Let $I = \int \cot^6 x dx$. Then,

$$I = \int \cot^2 x \times \cot^4 x dx$$
$$= \int \left(cosec^2 x - 1\right) \times \cot^4 x dx$$
$$= \int \left(cosec^2 x \cot^4 x - \cot^4 x\right) dx$$
$$= \int cosec^2 x \times \cot^4 x dx - \int \cot^4 dx$$
$$= \int cosec^2 x \cot^4 x dx - \int \cot^2 x \left(cosec^2 - 1\right) dx$$
$$= \int cosec^2 x \cot^4 x dx - \int \cot^2 x \, cosec^2 x dx + \int \cot^2 x dx$$
$$\Rightarrow \quad I = \int cosec^2 x \cot^4 x dx - \int \cot^2 x \, cosec^2 x dx + \int \left(cosec^2 x - 1\right) dx$$

Substituting $\cot x = t$ and $-cosec^2 x dx = dt$ in first two integral, we get

$$I = \int t^4 (-dt) - \int t^2 (-dt) + \int cosec^2 x dx - \int dx$$
$$= -\frac{t^5}{5} + \frac{t^3}{3} - \cot x - x + c$$
$$= -\frac{\cot^5 x}{5} + \frac{\cot^3 x}{3} - \cot x - x + c$$
$$\therefore \quad I = -\frac{1}{5} \times \cot^5 x + \frac{1}{3} \times \cot^3 x - \cot x - x + c$$