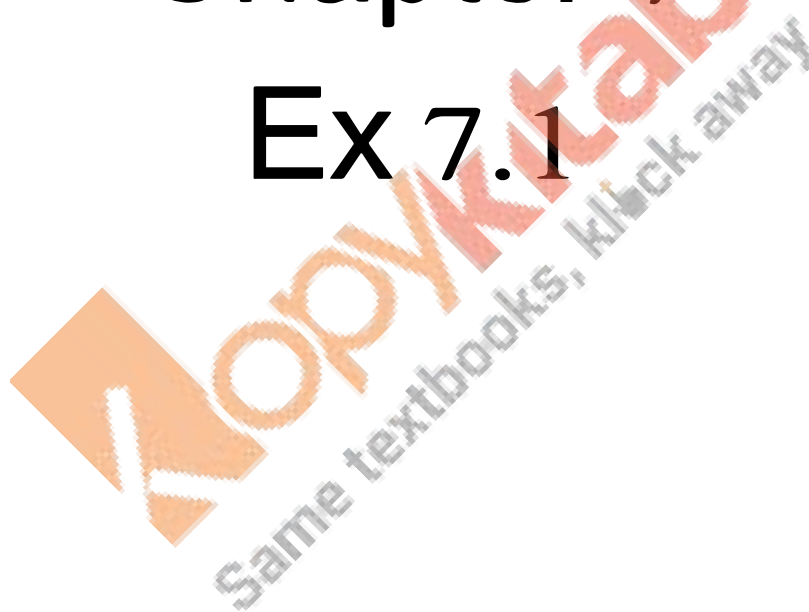


RD Sharma
Solutions
Class 11 Maths
Chapter 7
Ex 7.1



Trigonometric Ratios of Compound Angles Ex 7.1 Q1

We have,

$$\sin A = \frac{4}{5} \text{ and } \cos B = \frac{5}{13}$$

$$\begin{aligned}\therefore \cos A &= \sqrt{1 - \sin^2 A} \text{ and } \sin B = \sqrt{1 - \cos^2 B} \\ \Rightarrow \cos A &= \sqrt{1 - \left(\frac{4}{5}\right)^2} \text{ and } \sin B = \sqrt{1 - \left(\frac{5}{13}\right)^2} \\ \Rightarrow \cos A &= \sqrt{1 - \frac{16}{25}} \text{ and } \sin B = \sqrt{1 - \frac{25}{169}} \\ \Rightarrow \cos A &= \sqrt{\frac{25 - 16}{25}} \text{ and } \sin B = \sqrt{\frac{169 - 25}{169}} \\ \Rightarrow \cos A &= \sqrt{\frac{9}{25}} \text{ and } \sin B = \sqrt{\frac{144}{169}} \\ \Rightarrow \cos A &= \frac{3}{5} \text{ and } \sin B = \frac{12}{13}\end{aligned}$$

Now,

$$\begin{aligned}\sin(A + B) &= \sin A \cos B + \cos A \sin B \\ &= \frac{4}{5} \times \frac{5}{13} + \frac{3}{5} \times \frac{12}{13} \\ &= \frac{20}{65} + \frac{36}{65} \\ &= \frac{20 + 36}{65} \\ &= \frac{56}{65}\end{aligned}$$

We have,

$$\sin A = \frac{4}{5} \text{ and } \cos B = \frac{5}{13}$$

$$\begin{aligned}\therefore \cos A &= \sqrt{1 - \sin^2 A} \text{ and } \sin B = \sqrt{1 - \cos^2 B} \\ \Rightarrow \cos A &= \sqrt{1 - \left(\frac{4}{5}\right)^2} \text{ and } \sin B = \sqrt{1 - \left(\frac{5}{13}\right)^2} \\ \Rightarrow \cos A &= \sqrt{1 - \frac{16}{25}} \text{ and } \sin B = \sqrt{1 - \frac{25}{169}} \\ \Rightarrow \cos A &= \sqrt{\frac{25 - 16}{25}} \text{ and } \sin B = \sqrt{\frac{169 - 25}{169}} \\ \Rightarrow \cos A &= \sqrt{\frac{9}{25}} \text{ and } \sin B = \sqrt{\frac{144}{169}} \\ \Rightarrow \cos A &= \frac{3}{5} \text{ and } \sin B = \frac{12}{13}\end{aligned}$$

Now,

$$\begin{aligned}\cos(A + B) &= \cos A \cos B - \sin A \sin B \\ &= \frac{3}{5} \times \frac{5}{13} - \frac{4}{5} \times \frac{12}{13} \\ &= \frac{15}{65} - \frac{48}{65} \\ &= \frac{15 - 48}{65} \\ &= \frac{-33}{65}\end{aligned}$$

We have,

$$\sin A = \frac{4}{5} \text{ and } \cos B = \frac{5}{13}$$

$$\begin{aligned}\therefore \cos A &= \sqrt{1 - \sin^2 A} \text{ and } \sin B = \sqrt{1 - \cos^2 B} \\ \Rightarrow \cos A &= \sqrt{1 - \left(\frac{4}{5}\right)^2} \text{ and } \sin B = \sqrt{1 - \left(\frac{5}{13}\right)^2} \\ \Rightarrow \cos A &= \sqrt{1 - \frac{16}{25}} \text{ and } \sin B = \sqrt{1 - \frac{25}{169}} \\ \Rightarrow \cos A &= \sqrt{\frac{25 - 16}{25}} \text{ and } \sin B = \sqrt{\frac{169 - 25}{169}}\end{aligned}$$

$$\Rightarrow \cos A = \sqrt{\frac{9}{25}} \text{ and } \sin B = \sqrt{\frac{144}{169}}$$

$$\Rightarrow \cos A = \frac{3}{5} \text{ and } \sin B = \frac{12}{13}$$

Now,

$$\begin{aligned}\sin(A - B) &= \sin A \cos B - \cos A \sin B \\ &= \frac{4}{5} \times \frac{5}{13} - \frac{3}{5} \times \frac{12}{13} \\ &= \frac{20}{65} - \frac{36}{65} \\ &= \frac{20 - 36}{65} \\ &= -\frac{16}{65}\end{aligned}$$

We have,

$$\sin A = \frac{4}{5} \text{ and } \cos B = \frac{5}{13}$$

$$\begin{aligned}\therefore \cos A &= \sqrt{1 - \sin^2 A} \text{ and } \sin B = \sqrt{1 - \cos^2 B} \\ \Rightarrow \cos A &= \sqrt{1 - \left(\frac{4}{5}\right)^2} \text{ and } \sin B = \sqrt{1 - \left(\frac{5}{13}\right)^2} \\ \Rightarrow \cos A &= \sqrt{1 - \frac{16}{25}} \text{ and } \sin B = \sqrt{1 - \frac{25}{169}} \\ \Rightarrow \cos A &= \sqrt{\frac{25 - 16}{25}} \text{ and } \sin B = \sqrt{\frac{169 - 25}{169}} \\ \Rightarrow \cos A &= \sqrt{\frac{9}{25}} \text{ and } \sin B = \sqrt{\frac{144}{169}} \\ \Rightarrow \cos A &= \frac{3}{5} \text{ and } \sin B = \frac{12}{13}\end{aligned}$$

Now,

$$\begin{aligned}\cos(A - B) &= \cos A \cos B + \sin A \sin B \\ &= \frac{3}{5} \times \frac{5}{13} + \frac{4}{5} \times \frac{12}{13} \\ &= \frac{15}{65} + \frac{48}{65} \\ &= \frac{15 + 48}{65} \\ &= \frac{63}{65}\end{aligned}$$

Trigonometric Ratios of Compound Angles Ex 7.1 Q2

We have,

$$\sin A = \frac{12}{13} \text{ and } \sin B = \frac{4}{5}$$

$$\begin{aligned}\therefore \cos A &= -\sqrt{1 - \sin^2 A} \text{ and } \cos B = \sqrt{1 - \sin^2 B} \\ &\quad [\because \text{In the second quadrant } \cos \theta \text{ is negative}] \\ \Rightarrow \cos A &= -\sqrt{1 - \left(\frac{12}{13}\right)^2} \text{ and } \cos B = \sqrt{1 - \left(\frac{4}{5}\right)^2} \\ \Rightarrow \cos A &= -\sqrt{1 - \frac{144}{169}} \text{ and } \cos B = \sqrt{1 - \frac{16}{25}} \\ \Rightarrow \cos A &= -\sqrt{\frac{25}{169}} \text{ and } \cos B = \sqrt{\frac{9}{25}} \\ \Rightarrow \cos A &= -\frac{5}{13} \text{ and } \cos B = \frac{3}{5}\end{aligned}$$

Now,

(i)

$$\begin{aligned}\sin(A + B) &= \sin A \cos B + \cos A \sin B \\ &= \frac{12}{13} \times \frac{3}{5} - \frac{5}{13} \times \frac{4}{5} \\ &= \frac{36}{65} - \frac{20}{65} \\ &= \frac{16}{65}\end{aligned}$$

(ii)

$$\begin{aligned}\cos(A+B) &= \cos A \cos B - \sin A \sin B \\ &= \frac{-5}{13} \times \frac{3}{5} - \frac{12}{13} \times \frac{4}{5} \\ &= \frac{-15}{65} - \frac{48}{65} \\ &= \frac{-63}{65}\end{aligned}$$

We have,

$$\sin A = \frac{3}{5} \text{ and } \cos B = \frac{-12}{13}$$

$$\therefore \cos A = -\sqrt{1 - \sin^2 A} \text{ and } \sin B = \sqrt{1 - \cos^2 B}$$

[$\because$ In the second quadrant $\cos \theta$ is negative]

$$\Rightarrow \cos A = -\sqrt{1 - \left(\frac{3}{5}\right)^2} \text{ and } \sin B = \sqrt{1 - \left(\frac{-12}{13}\right)^2}$$

$$\Rightarrow \cos A = -\sqrt{1 - \frac{9}{25}} \text{ and } \sin B = \sqrt{1 - \frac{144}{169}}$$

$$\Rightarrow \cos A = -\sqrt{\frac{16}{25}} \text{ and } \sin B = \sqrt{\frac{25}{169}}$$

$$\Rightarrow \cos A = -\frac{4}{5} \text{ and } \sin B = \frac{5}{13}$$

Now,

$$\begin{aligned}\sin(A+B) &= \sin A \cos B + \cos A \sin B \\ &= \frac{3}{5} \times \left(\frac{-12}{13}\right) - \frac{4}{5} \times \frac{5}{13} \\ &= -\frac{36}{65} - \frac{20}{65} \\ &= -\frac{56}{65}\end{aligned}$$

$$\therefore \sin(A+B) = -\frac{56}{65}$$

Trigonometric Ratios of Compound Angles Ex 7.1 Q3

We have,

$$\cos A = -\frac{24}{25} \text{ and } \cos B = \frac{3}{5}$$

$$\therefore \sin A = -\sqrt{1 - \cos^2 A} \text{ and } \sin B = -\sqrt{1 - \cos^2 B}$$

[$\because$ In the 3rd and 4th quadrant $\sin \theta$ is negative]

$$\Rightarrow \sin A = -\sqrt{1 - \left(\frac{-24}{25}\right)^2} \text{ and } \sin B = -\sqrt{1 - \left(\frac{3}{5}\right)^2}$$

$$\Rightarrow \sin A = -\sqrt{1 - \frac{576}{625}} \text{ and } \sin B = -\sqrt{1 - \frac{9}{25}}$$

$$\Rightarrow \sin A = -\sqrt{\frac{49}{625}} \text{ and } \sin B = -\sqrt{\frac{16}{25}}$$

$$\Rightarrow \sin A = -\frac{7}{25} \text{ and } \sin B = -\frac{4}{5}$$

Now,

$$\begin{aligned}\text{(i)} \quad \sin(A+B) &= \sin A \cos B + \cos A \sin B \\ &= -\frac{7}{25} \times \frac{3}{5} - \frac{24}{25} \times \left(-\frac{4}{5}\right) \\ &= -\frac{21}{125} + \frac{96}{125} \\ &= \frac{75}{125} \\ &= \frac{3}{5}\end{aligned}$$

$$\begin{aligned}\text{(ii)} \quad \cos(A+B) &= \cos A \cos B - \sin A \sin B \\ &= -\frac{24}{25} \times \frac{3}{5} - \left(-\frac{7}{25}\right) \times \left(-\frac{4}{5}\right) \\ &= -\frac{72}{125} - \frac{28}{125} \\ &= \frac{-72-28}{125} \\ &= \frac{-100}{125} = -\frac{4}{5}\end{aligned}$$

Trigonometric Ratios of Compound Angles Ex 7.1 Q4

We have,

$$\tan A = \frac{3}{4}, \text{ and } \cos B = \frac{9}{41}$$

$$\therefore \sin B = \sqrt{1 - \cos^2 B}$$

$$\begin{aligned}
 \sin A &= \sqrt{1 - \cos^2 A} \\
 &= \sqrt{1 - \left(\frac{9}{41}\right)^2} \\
 &= \sqrt{1 - \frac{81}{1681}} \\
 &= \sqrt{\frac{1600}{1681}} \\
 &= \frac{40}{41}
 \end{aligned}$$

$$\therefore \tan B = \frac{\sin B}{\cos B} = \frac{\frac{41}{9}}{\frac{41}{41}} = \frac{40}{9}$$

Now,

$$\begin{aligned}
 \tan(A+B) &= \frac{\tan A + \tan B}{1 - \tan A \tan B} \\
 &= \frac{\frac{3}{4} + \frac{40}{9}}{1 - \frac{3}{4} \times \frac{40}{9}} \\
 &= \frac{\frac{27+160}{36}}{\frac{36-120}{36}} \\
 &= \frac{187}{-84} \\
 &= -\frac{187}{84}
 \end{aligned}$$

Trigonometric Ratios of Compound Angles Ex 7.1 Q5

We have,

$$\sin A = \frac{1}{2} \text{ and } \cos B = \frac{12}{13}$$

$$\therefore \cos A = -\sqrt{1 - \sin^2 A} \text{ and } \sin B = -\sqrt{1 - \cos^2 B}$$

[$\because$ cosine is negative in second quadrant and
sine is negative in fourth quadrant]

$$\Rightarrow \cos A = -\sqrt{1 - \left(\frac{1}{2}\right)^2} \text{ and } \sin B = -\sqrt{1 - \left(\frac{12}{13}\right)^2}$$

$$\Rightarrow \cos A = -\sqrt{1 - \frac{1}{4}} \text{ and } \sin B = -\sqrt{1 - \frac{144}{169}}$$

$$\Rightarrow \cos A = -\sqrt{\frac{3}{4}} \text{ and } \sin B = -\sqrt{\frac{25}{169}}$$

$$\Rightarrow \cos A = -\frac{\sqrt{3}}{2} \text{ and } \sin B = -\frac{5}{13}$$

$$\therefore \tan A = \frac{\sin A}{\cos A} = \frac{\frac{1}{2}}{-\frac{\sqrt{3}}{2}} = -\frac{1}{\sqrt{3}}$$

$$\text{and, } \tan B = \frac{\sin B}{\cos B} = \frac{-\frac{5}{13}}{\frac{12}{13}} = -\frac{5}{12}$$

$$\begin{aligned}
 \text{Now, } \tan(A-B) &= \frac{\tan A - \tan B}{1 + \tan A \tan B} \\
 &= \frac{-\frac{1}{\sqrt{3}} - \left(-\frac{5}{12}\right)}{1 + \left(-\frac{1}{\sqrt{3}}\right) \times \left(-\frac{5}{12}\right)} \\
 &= \frac{-\frac{1}{\sqrt{3}} + \frac{5}{12}}{1 + \frac{5}{12\sqrt{3}}} \\
 &= \frac{\frac{-12+5\sqrt{3}}{12\sqrt{3}}}{\frac{12\sqrt{3}+5}{12\sqrt{3}}} \\
 &= \frac{5\sqrt{3}-12}{5+12\sqrt{3}}
 \end{aligned}$$

$$\Rightarrow \tan(A-B) = \frac{5\sqrt{3}-12}{5+12\sqrt{3}}$$

Trigonometric Ratios of Compound Angles Ex 7.1 Q6

We have,

$$\sin A = \frac{1}{2} \text{ and } \cos B = \frac{\sqrt{3}}{2}$$

$$\therefore \cos A = -\sqrt{1 - \sin^2 A} \text{ and } \sin B = \sqrt{1 - \cos^2 B}$$

[$\because$ cosine is negative in second quadrant]

$$\Rightarrow \cos A = -\sqrt{1 - \left(\frac{1}{2}\right)^2} \text{ and } \sin B = \sqrt{1 - \left(\frac{\sqrt{3}}{2}\right)^2}$$

$$\Rightarrow \cos A = -\sqrt{1 - \frac{1}{4}} \text{ and } \sin B = \sqrt{1 - \frac{3}{4}}$$

$$\Rightarrow \cos A = -\sqrt{\frac{3}{4}} \text{ and } \sin B = \sqrt{\frac{1}{4}}$$

$$\Rightarrow \cos A = -\frac{\sqrt{3}}{2} \text{ and } \sin B = \frac{1}{2}$$

$$\therefore \tan A = \frac{\sin A}{\cos A} = \frac{\frac{1}{2}}{-\frac{\sqrt{3}}{2}} = \frac{-1}{\sqrt{3}}$$

$$\text{and, } \tan B = \frac{\sin B}{\cos B} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}}$$

Now,

$$\begin{aligned} \tan(A+B) &= \frac{\tan A + \tan B}{1 - \tan A \tan B} \\ &= \frac{\frac{-1}{\sqrt{3}} + \frac{1}{\sqrt{3}}}{1 - \left(\frac{-1}{\sqrt{3}}\right) \times \left(\frac{1}{\sqrt{3}}\right)} \\ &= 0 \end{aligned}$$

$$\therefore \tan(A+B) = 0$$

We have,

$$\sin A = \frac{1}{2} \text{ and } \cos B = \frac{\sqrt{3}}{2}$$

$$\therefore \cos A = -\sqrt{1 - \sin^2 A} \text{ and } \sin B = \sqrt{1 - \cos^2 B} \text{ [}\because \text{cosine is negative in second quadrant]}$$

$$\Rightarrow \cos A = -\sqrt{1 - \left(\frac{1}{2}\right)^2} \text{ and } \sin B = \sqrt{1 - \left(\frac{\sqrt{3}}{2}\right)^2}$$

$$\Rightarrow \cos A = -\sqrt{1 - \frac{1}{4}} \text{ and } \sin B = \sqrt{1 - \frac{3}{4}}$$

$$\Rightarrow \cos A = -\sqrt{\frac{3}{4}} \text{ and } \sin B = \sqrt{\frac{1}{4}}$$

$$\Rightarrow \cos A = -\frac{\sqrt{3}}{2} \text{ and } \sin B = \frac{1}{2}$$

$$\therefore \tan A = \frac{\sin A}{\cos A} = \frac{\frac{1}{2}}{-\frac{\sqrt{3}}{2}} = \frac{-1}{\sqrt{3}}$$

$$\text{and, } \tan B = \frac{\sin B}{\cos B} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}}$$

$$\text{Now, } \tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

$$\begin{aligned} &= \frac{\frac{-1}{\sqrt{3}} - \left(\frac{1}{\sqrt{3}}\right)}{1 + \left(\frac{-1}{\sqrt{3}}\right) \times \frac{1}{\sqrt{3}}} \\ &= \frac{\frac{-2}{\sqrt{3}}}{1 - \frac{1}{3}} \\ &= \frac{\frac{-2}{\sqrt{3}}}{\frac{3-1}{3}} \\ &= \frac{\frac{-2}{\sqrt{3}}}{\frac{2}{3}} \\ &= \frac{-3}{\sqrt{3}} \\ &= \frac{-\sqrt{3} \times \sqrt{3}}{\sqrt{3}} = -\sqrt{3} \end{aligned}$$

$$\therefore \tan(A-B) = -\sqrt{3}$$

Trigonometric Ratios of Compound Angles Ex 7.1 Q7

(i)

$$\begin{aligned} \sin 78^\circ \cos 18^\circ - \cos 78^\circ \sin 18^\circ & \quad [\sin(A-B) = \sin A \cos B - \cos A \sin B] \\ &= \sin(78^\circ - 18^\circ) \\ &= \sin 60^\circ \\ &= \frac{\sqrt{3}}{2} \end{aligned}$$

(ii)

$$\begin{aligned} \cos 47^\circ \cos 13^\circ - \sin 47^\circ \sin 13^\circ & \quad [\cos(A+B) = \cos A \cos B - \sin A \sin B] \\ &= \cos(47^\circ + 13^\circ) \\ &= \cos 60^\circ \end{aligned}$$

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$$= \frac{1}{2}$$

(iii)

$$\begin{aligned}\sin 36^\circ \cos 9^\circ + \cos 36^\circ \sin 9^\circ &= \sin(A+B) = \sin A \cos B + \cos A \sin B \\ &= \sin(36^\circ + 9^\circ) \\ &= \sin 45^\circ\end{aligned}$$

$$= \frac{1}{\sqrt{2}}$$

(iv)

$$\begin{aligned}\cos 80^\circ \cos 20^\circ + \sin 80^\circ \sin 20^\circ &= \cos(A-B) = \cos A \cos B + \sin A \sin B \\ &= \cos(80^\circ - 20^\circ) \\ &= \cos 60^\circ\end{aligned}$$

$$= \frac{1}{2}$$

Trigonometric Ratios of Compound Angles Ex 7.1 Q8

We have,

$$\cos A = \frac{-12}{13} \text{ and } \cot B = \frac{24}{7}$$

$$\therefore \sin A = \sqrt{1 - \cos^2 A} = \sqrt{1 - \left(\frac{-12}{13}\right)^2} = \sqrt{1 - \frac{144}{169}} = \sqrt{\frac{25}{169}} = \frac{5}{13}$$

and,

$$\begin{aligned}\operatorname{cosec} B &= -\sqrt{1 + \cot^2 B} \quad [\because \operatorname{cosec} \text{ is negative in third quadrant}] \\ &= -\sqrt{1 + \left(\frac{24}{7}\right)^2} = -\sqrt{1 + \frac{576}{49}} = -\sqrt{\frac{49 + 576}{49}} = -\sqrt{\frac{625}{49}} = -\frac{25}{7}\end{aligned}$$

$$\Rightarrow \sin B = \frac{-7}{25} \quad \left[\because \operatorname{cosec} B = \frac{1}{\sin B} \right]$$

Now,

$$\begin{aligned}\cos B &= -\sqrt{1 - \sin^2 B} \quad [\because \cos \theta \text{ is negative in third quadrant}] \\ &= -\sqrt{1 - \left(\frac{-7}{25}\right)^2} = -\sqrt{1 - \frac{49}{625}} = -\sqrt{\frac{625 - 49}{625}} = -\sqrt{\frac{576}{625}} = -\frac{24}{25}\end{aligned}$$

Now,

$$\begin{aligned}\sin(A+B) &= \sin A \cos B + \cos A \sin B \\ &= \frac{5}{13} \times \left(\frac{-24}{25}\right) + \left(\frac{-12}{13}\right) \times \left(\frac{-7}{25}\right) \\ &= \frac{-120}{325} + \frac{84}{325} \\ &= \frac{-120 + 84}{325} \\ &= \frac{-36}{325}\end{aligned}$$

We have,

$$\cos A = \frac{-12}{13} \text{ and } \cot B = \frac{24}{7}$$

$$\therefore \sin A = \sqrt{1 - \cos^2 A} = \sqrt{1 - \left(\frac{-12}{13}\right)^2} = \sqrt{1 - \frac{144}{169}} = \sqrt{\frac{25}{169}} = \frac{5}{13}$$

and,

$$\begin{aligned}\operatorname{cosec} B &= -\sqrt{1 + \cot^2 B} \quad [\because \operatorname{cosec} \text{ is negative in third quadrant}] \\ &= -\sqrt{1 + \left(\frac{24}{7}\right)^2} = -\sqrt{1 + \frac{576}{49}} = -\sqrt{\frac{49 + 576}{49}} = -\sqrt{\frac{625}{49}} = -\frac{25}{7}\end{aligned}$$

$$\Rightarrow \sin B = \frac{-7}{25} \quad \left[\because \operatorname{cosec} B = \frac{1}{\sin B} \right]$$

Now,

$$\begin{aligned}\cos B &= -\sqrt{1 - \sin^2 B} \quad [\because \cos \theta \text{ is negative in third quadrant}] \\ &= -\sqrt{1 - \left(\frac{-7}{25}\right)^2} = -\sqrt{1 - \frac{49}{625}} = -\sqrt{\frac{625 - 49}{625}} = -\sqrt{\frac{576}{625}} = -\frac{24}{25}\end{aligned}$$

Now,

$$\begin{aligned}\cos(A+B) &= \cos A \cos B - \sin A \sin B \\ &= \left(\frac{-12}{13}\right) \times \left(\frac{-24}{25}\right) - \left(\frac{5}{13}\right) \times \left(\frac{-7}{25}\right) \\ &= \frac{288}{325} + \frac{35}{325} \\ &= \frac{323}{325}\end{aligned}$$

We have,

$$\cos A = \frac{-12}{13} \text{ and } \cot B = \frac{24}{7}$$

$$\therefore \sin A = \sqrt{1 - \cos^2 A} = \sqrt{1 - \left(\frac{-12}{13}\right)^2} = \sqrt{1 - \frac{144}{169}} = \sqrt{\frac{25}{169}} = \frac{5}{13}$$

and,

$$\begin{aligned}\operatorname{cosec} B &= -\sqrt{1 + \cot^2 B} \quad [\because \operatorname{cosec} \text{ is negative in third quadrant}] \\ &= -\sqrt{1 + \left(\frac{24}{7}\right)^2} = -\sqrt{1 + \frac{576}{49}} = -\sqrt{\frac{49 + 576}{49}} = -\sqrt{\frac{625}{49}} = -\frac{25}{7}\end{aligned}$$

$$\Rightarrow \sin B = \frac{-7}{25} \quad \left[\because \operatorname{cosec} B = \frac{1}{\sin B} \right]$$

Now,

$$\begin{aligned}\cos B &= -\sqrt{1 - \sin^2 B} \quad [\because \cos \theta \text{ is negative in third quadrant}] \\ &= -\sqrt{1 - \left(\frac{-7}{25}\right)^2} = -\sqrt{1 - \frac{49}{625}} = -\sqrt{\frac{625 - 49}{625}} = -\sqrt{\frac{576}{625}} = -\frac{24}{25}\end{aligned}$$

Now,

$$\tan A = \frac{\sin A}{\cos A} = \frac{\frac{5}{13}}{\frac{-12}{13}} = \frac{-5}{12} \quad \left[\because \tan \theta = \frac{\sin \theta}{\cos \theta} \right]$$

$$\text{and, } \tan B = \frac{\sin B}{\cos B} = \frac{\frac{-7}{25}}{\frac{-24}{25}} = \frac{7}{24} \quad \left[\because \tan \theta = \frac{\sin \theta}{\cos \theta} \right]$$

$$\begin{aligned}\therefore \tan(A+B) &= \frac{\tan A + \tan B}{1 - \tan A \tan B} \\ &= \frac{\frac{-5}{12} + \frac{7}{24}}{1 - \left(\frac{-5}{12}\right) \times \frac{7}{24}} \\ &= \frac{\frac{-10 + 7}{24}}{1 + \frac{35}{288}} \\ &= \frac{\frac{-3}{24}}{\frac{288 + 35}{288}} \\ &= \frac{-3}{288 + 35}\end{aligned}$$

Trigonometric Ratios of Compound Angles Ex 7.1 Q9

$$\begin{aligned}\text{LHS: } \cos 105^\circ + \cos 15^\circ &= \cos(90^\circ + 15^\circ) + \cos(90^\circ - 75^\circ) \\ &= -\sin 15^\circ + \sin 75^\circ \quad \left[\begin{array}{l} \because \cos(90 + \theta) = -\sin \theta \\ \text{and } \cos(90 - \theta) = \sin \theta \end{array} \right] \\ &= \sin 75^\circ - \sin 15^\circ\end{aligned}$$

$$\therefore \cos 105^\circ + \cos 15^\circ = \sin 75^\circ - \sin 15^\circ$$

Hence proved.

Trigonometric Ratios of Compound Angles Ex 7.1 Q10

$$\begin{aligned}\text{LHS: } \frac{\tan A + \tan B}{\tan A - \tan B} &= \frac{\frac{\sin A}{\cos A} + \frac{\sin B}{\cos B}}{\frac{\sin A}{\cos A} - \frac{\sin B}{\cos B}} \quad \left[\because \tan \theta = \frac{\sin \theta}{\cos \theta} \right] \\ &= \frac{\frac{\sin A \cos B + \cos A \sin B}{\cos A \cos B}}{\frac{\sin A \cos B - \cos A \sin B}{\cos A \cos B}} \\ &= \frac{\sin A \cos B + \cos A \sin B}{\sin A \cos B - \cos A \sin B} \\ &= \sin(A+B) \quad \left[\because \sin(A+B) = \sin A \cos B + \cos A \sin B \right]\end{aligned}$$

$$= \frac{\sin(A+B)}{\sin(A-B)} \quad \left[\text{and, } \sin(A-B) = \sin A \cos B - \cos A \sin B \right]$$

$$\therefore \frac{\tan A + \tan B}{\tan A - \tan B} = \frac{\sin(A+B)}{\sin(A-B)}$$

Hence proved.

Trigonometric Ratios of Compound Angles Ex 7.1 Q11

$$\text{LHS: } \frac{\cos 11^\circ + \sin 11^\circ}{\cos 11^\circ - \sin 11^\circ}$$

Dividing numerator and denominator by $\cos 11^\circ$, we get

$$\begin{aligned} & \frac{\frac{\cos 11^\circ}{\cos 11^\circ} + \frac{\sin 11^\circ}{\cos 11^\circ}}{\frac{\cos 11^\circ}{\cos 11^\circ} - \frac{\sin 11^\circ}{\cos 11^\circ}} \\ &= \frac{1 + \tan 11^\circ}{1 - \tan 11^\circ} \quad \left[\because \tan \theta = \frac{\sin \theta}{\cos \theta} \right] \\ &= \frac{\tan 45^\circ + \tan 11^\circ}{1 - \tan 45^\circ \times \tan 11^\circ} \quad [\tan 45^\circ = 1] \\ &= \tan(45^\circ + 11^\circ) \quad \left[\because \tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} \right] \\ &= \tan 56^\circ \end{aligned}$$

$$\therefore \frac{\cos 11^\circ + \sin 11^\circ}{\cos 11^\circ - \sin 11^\circ} = \tan 56^\circ$$

Hence proved.

$$\text{LHS: } \frac{\cos 9^\circ + \sin 9^\circ}{\cos 9^\circ - \sin 9^\circ}$$

$$\begin{aligned} & \frac{\frac{\cos 9^\circ}{\cos 9^\circ} + \frac{\sin 9^\circ}{\cos 9^\circ}}{\frac{\cos 9^\circ}{\cos 9^\circ} - \frac{\sin 9^\circ}{\cos 9^\circ}} \quad \left[\text{Dividing numerator and denominator by } \cos 9^\circ \right] \\ &= \frac{1 + \tan 9^\circ}{1 - \tan 9^\circ} \quad \left[\because \tan \theta = \frac{\sin \theta}{\cos \theta} \right] \\ &= \frac{\tan 45^\circ + \tan 9^\circ}{1 - \tan 45^\circ \times \tan 9^\circ} \quad [\tan 45^\circ = 1] \\ &= \tan(45^\circ + 9^\circ) \quad \left[\because \tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} \right] \\ &= \tan 54^\circ \\ &= \text{RHS} \end{aligned}$$

$$\therefore \text{LHS} = \text{RHS}$$

Hence proved.

$$\text{LHS: } \frac{\cos 8^\circ - \sin 8^\circ}{\cos 8^\circ + \sin 8^\circ}$$

$$\begin{aligned} & \frac{\frac{\cos 8^\circ}{\cos 8^\circ} - \frac{\sin 8^\circ}{\cos 8^\circ}}{\frac{\cos 8^\circ}{\cos 8^\circ} + \frac{\sin 8^\circ}{\cos 8^\circ}} \quad \left[\text{Dividing numerator and denominator by } \cos 8^\circ \right] \\ &= \frac{1 - \tan 8^\circ}{1 + \tan 8^\circ} \quad \left[\because \tan \theta = \frac{\sin \theta}{\cos \theta} \right] \\ &= \frac{\tan 45^\circ - \tan 8^\circ}{1 + \tan 45^\circ \times \tan 8^\circ} \quad [\tan 45^\circ = 1] \\ &= \tan(45^\circ - 8^\circ) \quad \left[\because \tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B} \right] \\ &= \tan 37^\circ \\ &= \text{RHS} \end{aligned}$$

$$\therefore \text{LHS} = \text{RHS}$$

Hence proved.

Trigonometric Ratios of Compound Angles Ex 7.1 Q12

$$\begin{aligned} \text{LHS: } & \sin(60^\circ - \theta) \cos(30^\circ + \theta) + \cos(60^\circ - \theta) \times \sin(30^\circ + \theta) \\ &= \sin[(60^\circ - \theta) + (30^\circ + \theta)] \quad \left[\sin(A + B) = \sin A \cos B + \cos A \sin B \right] \\ &= \sin[60^\circ - \theta + 30^\circ + \theta] \\ &= \sin(90^\circ) \\ &= 1 \\ &= \text{RHS} \end{aligned}$$

$$\therefore \text{LHS} = \text{RHS}$$

Hence proved.

Trigonometric Ratios of Compound Angles Ex 7.1 Q13

$$\begin{aligned} \text{LHS: } & \frac{\tan 69^\circ + \tan 66^\circ}{1 - \tan 69^\circ \tan 66^\circ} \\ &= \tan(69^\circ + 66^\circ) \quad \left[\because \tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} \right] \\ &= \tan(135^\circ) \\ &= \tan(90^\circ + 45^\circ) \\ &= -\cot 45^\circ \quad [\because \tan \theta \text{ is negative in second quadrant}] \\ &= -1 \\ &= \text{RHS} \end{aligned}$$

$$\therefore \text{LHS} = \text{RHS}$$

Hence proved.

Trigonometric Ratios of Compound Angles Ex 7.1 Q14

We have,

$$\tan A = \frac{5}{6} \text{ and } \tan B = \frac{1}{11}$$

Now,

$$\begin{aligned} \tan(A + B) &= \frac{\tan A + \tan B}{1 - \tan A \tan B} \\ &= \frac{\frac{5}{6} + \frac{1}{11}}{1 - \frac{5}{6} \times \frac{1}{11}} \\ &= \frac{\frac{55 + 6}{66}}{1 - \frac{5}{66}} \\ &= \frac{\frac{61}{66}}{\frac{66 - 5}{66}} \\ &= \frac{61}{66} \times \frac{66}{61} \\ &= 1 \\ &= \tan \frac{\pi}{4} \end{aligned}$$

$$\left[\because \tan \frac{\pi}{4} = 1 \right]$$

$$\Rightarrow \tan(A + B) = \tan \frac{\pi}{4}$$

$$\Rightarrow A + B = \frac{\pi}{4}$$

Hence proved.

We have,

$$\tan A = \frac{m}{m-1} \text{ and } \tan B = \frac{1}{2m-1}$$

$$\begin{aligned} \text{Now, } \tan(A-B) &= \frac{\tan A - \tan B}{1 + \tan A \tan B} \\ &= \frac{\frac{m}{m-1} - \frac{1}{2m-1}}{1 + \frac{m}{m-1} \times \frac{1}{2m-1}} \\ &= \frac{\frac{m(2m-1) - (m-1)}{(m-1)(2m-1)}}{\frac{m(2m-1) - (m-1)}{(m-1)(2m-1)}} \\ &= \frac{1}{1} \\ &= 1 \end{aligned}$$

$$\therefore \tan(A-B) = 1 = \tan\left(\frac{\pi}{4}\right) \quad \left[\because \tan \frac{\pi}{4} = 1\right]$$

$$\Rightarrow \tan(A-B) = \tan\left(\frac{\pi}{4}\right)$$

$$\Rightarrow A-B = \left(\frac{\pi}{4}\right)$$

Trigonometric Ratios of Compound Angles Ex 7.1 Q15

$$\text{LHS: } \cos^2 45^\circ - \sin^2 15^\circ$$

$$\begin{aligned} &= \left(\frac{1}{\sqrt{2}}\right)^2 - \sin^2 15^\circ \quad \left[\because \cos 45 = \frac{1}{\sqrt{2}}\right] \\ &= \frac{1}{2} - \left(\frac{1 - \cos 2 \times 15^\circ}{2}\right) \quad \left[\because \cos 2\theta = 1 - \sin^2 \theta\right] \\ &= \frac{1}{2} - \left(\frac{1 - \cos 30^\circ}{2}\right) \\ &= \frac{1 - 1 + \cos 30^\circ}{2} \\ &= \frac{\cos 30^\circ}{2} \\ &= \frac{\sqrt{3}}{2} \times \frac{1}{2} \\ &= \frac{\sqrt{3}}{4} \\ &= \text{RHS} \end{aligned}$$

$$\therefore \text{LHS} = \text{RHS}$$

Hence proved.

We have,

$$\begin{aligned} \text{LHS } \sin^2(n+1)A - \sin^2 nA &= \sin[(n+1)A + nA] \sin[(n+1)A - nA] \\ &\quad \left[\because \sin^2 A - \sin^2 B = \sin(A+B) \sin(A-B)\right] \\ &= \sin[nA + A + nA] \sin[nA + A - nA] \\ &= \sin(2nA + A) \sin(A) \\ &= \sin(2n+1)A \sin A \\ &= \text{RHS} \end{aligned}$$

$$\therefore \text{LHS} = \text{RHS}$$

Hence proved.

Trigonometric Ratios of Compound Angles Ex 7.1 Q16

We have

$$\begin{aligned}
 \text{LHS} &= \frac{\sin(A+B) + \sin(A-B)}{\cos(A+B) + \cos(A-B)} \\
 &= \frac{2 \sin A \cos B}{2 \cos A \cos B} \quad \left[\begin{array}{l} \because 2 \sin A \cos B = \sin(A+B) + \sin(A-B) \\ \text{and, } 2 \cos A \cos B = \cos(A+B) + \cos(A-B) \end{array} \right] \\
 &= \frac{\sin A}{\cos A} \\
 &= \tan A \\
 &= \text{RHS}
 \end{aligned}$$

$\therefore$ LHS = RHS

Hence proved.

$$\begin{aligned}
 \text{LHS} &= \frac{\sin(A-B)}{\cos A \cos B} + \frac{\sin(B-C)}{\cos B \cos C} + \frac{\sin(C-A)}{\cos C \cos A} \\
 &= \frac{\sin A \cos B - \cos A \sin B}{\cos A \cos B} + \frac{\sin B \cos C - \cos B \sin C}{\cos B \cos C} + \frac{\sin C \cos A - \cos C \sin A}{\cos C \cos A} \\
 &= \frac{\sin A \cos B}{\cos A \cos B} - \frac{\cos A \sin B}{\cos A \cos B} + \frac{\sin B \cos C}{\cos B \cos C} - \frac{\cos B \sin C}{\cos B \cos C} + \frac{\sin C \cos A}{\cos C \cos A} - \frac{\cos C \sin A}{\cos C \cos A} \\
 &= \tan A - \tan B + \tan B - \tan C + \tan C - \tan A \\
 &= 0 \\
 &= \text{RHS}
 \end{aligned}$$

$\therefore$ LHS = RHS

Hence proved.

We have,

$$\begin{aligned}
 \text{LHS} &= \frac{\sin(A-B)}{\sin A \sin B} + \frac{\sin(B-C)}{\sin B \sin C} + \frac{\sin(C-A)}{\sin C \sin A} \\
 &= \frac{\sin A \cos B - \cos A \sin B}{\sin A \sin B} + \frac{\sin B \cos C - \cos B \sin C}{\sin B \sin C} + \frac{\sin C \cos A - \cos C \sin A}{\sin C \sin A} \\
 &= \frac{\sin A \cos B}{\sin A \sin B} - \frac{\cos A \sin B}{\sin A \sin B} + \frac{\sin B \cos C}{\sin B \sin C} - \frac{\cos B \sin C}{\sin B \sin C} + \frac{\sin C \cos A}{\sin C \sin A} - \frac{\cos C \sin A}{\sin C \sin A} \\
 &= \frac{\cos B}{\sin B} - \frac{\cos A}{\sin A} + \frac{\cos C}{\sin C} - \frac{\cos B}{\sin B} + \frac{\cos A}{\sin A} - \frac{\cos C}{\sin C} \\
 &= \cot B - \cot A + \cot C - \cot B + \cot A - \cot C \quad \left[\because \cot \theta = \frac{\cos \theta}{\sin \theta} \right] \\
 &= 0 \\
 &= \text{RHS}
 \end{aligned}$$

$\therefore$ LHS = RHS

Hence proved.

We have,

$$\begin{aligned}
 \text{RHS} &= \sin^2 A + \sin^2(A-B) - 2 \sin A \cos B \sin(A-B) \\
 &= \sin^2 A + \sin(A-B) [\sin(A-B) - 2 \sin A \cos B] \\
 &= \sin^2 A + \sin(A-B) [\sin A \cos B - \cos A \sin B - 2 \sin A \cos B] \\
 &= \sin^2 A + \sin(A-B) [-\sin A \cos B - \cos A \sin B] \\
 &= \sin^2 A - \sin(A-B) (\sin A \cos B + \cos A \sin B) \\
 &= \sin^2 A - \sin(A-B) \sin(A+B) \\
 &= \sin^2 A - (\sin^2 A - \sin^2 B) \quad \left[\because \sin(A-B) \sin(A+B) = \sin^2 A - \sin^2 B \right] \\
 &= \sin^2 A - \sin^2 A + \sin^2 B \\
 &= \sin^2 B \\
 &= \text{LHS}
 \end{aligned}$$

$\therefore$ LHS = RHS

Hence proved.

$$\begin{aligned}
 \text{RHS} &= \cos^2 A + \cos^2 B - 2 \cos A \cos B \cos(A+B) \\
 &= \cos^2 A + (1 - \sin^2 B) - 2 \cos A \cos B \cos(A+B)
 \end{aligned}$$

$$\begin{aligned}
&= [\cos^2 A - \sin^2 B] - 2 \cos A \cos B \cos(A+B) + 1 \\
&= [\cos(A+B) \cos(A-B)] - 2 \cos A \cos B \cos(A+B) + 1 \\
&= \cos(A+B) [\cos(A-B) - 2 \cos A \cos B] + 1 \\
&= \cos(A+B) [\cos A \cos B + \sin A \sin B - 2 \cos A \cos B] + 1 \\
&= \cos(A+B) [-\cos A \cos B + \sin A \sin B] + 1 \\
&= -\cos(A+B) [\cos A \cos B - \sin A \sin B] + 1 \\
&= -\cos(A+B) [\cos(A+B)] + 1 \\
&= -\cos^2(A+B) + 1 \\
&= 1 - \cos^2(A+B) \\
&= \sin^2(A+B) \quad \left[\sin^2 \theta = 1 - \cos^2 \theta \right] \\
&= \text{RHS}
\end{aligned}$$

$\therefore$ LHS = RHS

Hence proved.

We have,

$$\begin{aligned}
\text{LHS} &= \frac{\tan(A+B)}{\cot(A-B)} \\
&= \frac{\tan(A+B)}{\frac{1}{\tan(A-B)}} \quad \left[\because \cot \theta = \frac{1}{\tan \theta} \right] \\
&= \tan(A+B) \tan(A-B) \\
&= \left[\frac{\tan A + \tan B}{1 - \tan A \tan B} \right] \left[\frac{\tan A - \tan B}{1 + \tan A \tan B} \right] \\
&= \frac{(\tan A + \tan B)(\tan A - \tan B)}{(1 - \tan A \tan B)(1 + \tan A \tan B)} \\
&= \frac{\tan^2 A - \tan^2 B}{1 - (\tan A \tan B)^2} \quad \left[\because (a-b)(a+b) = a^2 - b^2 \right] \\
&= \frac{\tan^2 A - \tan^2 B}{1 - \tan^2 A \tan^2 B} \\
&= \text{RHS}
\end{aligned}$$

$\therefore$ LHS = RHS

Hence proved.

Trigonometric Ratios of Compound Angles Ex 7.1 Q17

We have,

$$8\theta = 6\theta + 2\theta$$

$$\begin{aligned}
\Rightarrow \tan 8\theta &= \tan(6\theta + 2\theta) \\
\Rightarrow \tan 8\theta &= \frac{\tan 6\theta + \tan 2\theta}{1 - \tan 6\theta \tan 2\theta} \quad \left[\because \tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} \right] \\
\Rightarrow \tan 8\theta (1 - \tan 6\theta \tan 2\theta) &= \tan 6\theta + \tan 2\theta \\
\Rightarrow \tan 8\theta - \tan 8\theta \tan 6\theta \tan 2\theta &= \tan 6\theta + \tan 2\theta \\
\Rightarrow \tan 8\theta - \tan 6\theta - \tan 2\theta &= \tan 8\theta \tan 6\theta \tan 2\theta
\end{aligned}$$

Hence proved.

We have,

$$45^\circ = 30^\circ + 15^\circ$$

$$\begin{aligned}
\Rightarrow \tan 45^\circ &= \tan(30^\circ + 15^\circ) \\
\Rightarrow 1 &= \frac{\tan 30^\circ + \tan 15^\circ}{1 - \tan 30^\circ \tan 15^\circ} \quad \left[\because \tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} \right] \\
\Rightarrow 1 - \tan 30^\circ \tan 15^\circ &= \tan 15^\circ + \tan 30^\circ \\
\Rightarrow 1 &= \tan 15^\circ + \tan 30^\circ + \tan 30^\circ \tan 15^\circ \\
\Rightarrow \tan 15^\circ + \tan 30^\circ + \tan 15^\circ \tan 30^\circ &= 1
\end{aligned}$$

Hence proved.

We have,

$$45^\circ = 9^\circ + 36^\circ$$

$$\Rightarrow \tan 45^\circ = \tan(9^\circ + 36^\circ)$$

$$\Rightarrow 1 = \frac{\tan 9^\circ + \tan 36^\circ}{1 - \tan 9^\circ \tan 36^\circ} \quad \left[\because \tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} \right]$$

$$\Rightarrow 1 - \tan 9^\circ \tan 36^\circ = \tan 9^\circ + \tan 36^\circ$$

$$\Rightarrow 1 = \tan 9^\circ + \tan 36^\circ + \tan 9^\circ \tan 36^\circ$$

$$\Rightarrow \tan 9^\circ + \tan 36^\circ + \tan 9^\circ \tan 36^\circ = 1$$

Hence proved.

We have,

$$13\theta = 9\theta + 4\theta$$

$$\Rightarrow \tan 13\theta = \tan(9\theta + 4\theta)$$

$$\Rightarrow \tan 13\theta = \frac{\tan 9\theta + \tan 4\theta}{1 - \tan 9\theta \tan 4\theta} \quad \left[\because \tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} \right]$$

$$\Rightarrow \tan 13\theta (1 - \tan 9\theta \tan 4\theta) = \tan 9\theta + \tan 4\theta$$

$$\Rightarrow \tan 13\theta - \tan 13\theta \tan 9\theta \tan 4\theta = \tan 9\theta + \tan 4\theta$$

$$\Rightarrow \tan 13\theta - \tan 9\theta - \tan 4\theta = \tan 13\theta \tan 9\theta \tan 4\theta$$

Hence proved.

Trigonometric Ratios of Compound Angles Ex 7.1 Q18

We have,

$$\text{RHS} = \tan 3\theta \tan \theta$$

$$= \tan(2\theta + \theta) \times \tan(2\theta - \theta)$$

$$= \left[\frac{\tan 2\theta + \tan \theta}{1 - \tan 2\theta \tan \theta} \right] \times \left[\frac{\tan 2\theta - \tan \theta}{1 + \tan 2\theta \tan \theta} \right]$$

$$= \frac{(\tan 2\theta + \tan \theta)(\tan 2\theta - \tan \theta)}{(1 - \tan 2\theta \tan \theta)(1 + \tan 2\theta \tan \theta)}$$

$$= \frac{\tan^2 2\theta - \tan^2 \theta}{1 - \tan^2 2\theta \tan^2 \theta} \quad \left[\because (a-b)(a+b) = a^2 - b^2 \right]$$

$$= \text{LHS}$$

$$\therefore \text{LHS} = \text{RHS}$$

Hence proved

Trigonometric Ratios of Compound Angles Ex 7.1 Q19

$$\frac{\sin x \cdot \cos y + \sin y \cdot \cos x}{\sin x \cdot \cos y - \sin y \cdot \cos x} = \frac{a+b}{a-b}$$

$$\Rightarrow \frac{\sin x \cdot \cos y + \sin y \cdot \cos x + \sin x \cdot \cos y - \sin y \cdot \cos x}{\sin x \cdot \cos y + \sin y \cdot \cos x - \sin x \cdot \cos y + \sin y \cdot \cos x} = \frac{a+b+a-b}{a+b-a+b} \quad [\text{Using Componendo and Dividendo}]$$

$$\Rightarrow \frac{2\sin x \cdot \cos y}{2\sin y \cdot \cos x} = \frac{2a}{2b}$$

$$\Rightarrow \frac{\tan x}{\tan y} = \frac{a}{b}$$

Hence Proved

Trigonometric Ratios of Compound Angles Ex 7.1 Q20

We have,

$$\tan A = x \tan B$$

$$\frac{\sin A}{\cos A} = x \frac{\sin B}{\cos B}$$

$$\left[\because \tan \theta = \frac{\sin \theta}{\cos \theta} \right]$$

$$\Rightarrow \sin A \cos B = x \cos A \sin B \quad \text{--- (i)}$$

$$\text{Now, } \frac{\sin(A-B)}{\sin(A+B)} = \frac{\sin A \cos B - \sin B \cos A}{\sin A \cos B + \cos A \sin B}$$

$$= \frac{x \cos A \sin B - \cos A \sin B}{x \cos A \sin B + \cos A \sin B}$$

[Using equation (i)]

$$= \frac{\cos A \sin B (x-1)}{\cos A \sin B (x+1)}$$

$$= \frac{x-1}{x+1}$$

$$x + 1$$

$$\therefore \frac{\sin(A - B)}{\sin(A + B)} = \frac{x - 1}{x + 1}$$

Hence proved.

Trigonometric Ratios of Compound Angles Ex 7.1 Q21

We have,

$$\tan(A + B) = x \text{ and } \tan(A - B) = y$$

$$\begin{aligned} \text{Now, } \tan 2A &= \tan[(A + B) + (A - B)] \\ &= \frac{\tan(A + B) + \tan(A - B)}{1 - \tan(A + B) \times \tan(A - B)} \\ &= \frac{x + y}{1 - xy} \end{aligned}$$

$$\therefore \tan 2A = \frac{x + y}{1 - xy}$$

$$\begin{aligned} \text{Now, } \tan 2B &= \tan[(A + B) - (A - B)] \\ &= \frac{\tan(A + B) - \tan(A - B)}{1 + \tan(A + B) \times \tan(A - B)} \\ &= \frac{x - y}{1 + xy} \end{aligned}$$

$$\therefore \tan 2B = \frac{x - y}{1 + xy}$$

Trigonometric Ratios of Compound Angles Ex 7.1 Q22

We have,

$$\cos A + \sin B = m \text{ and } \sin A + \cos B = n$$

$$\begin{aligned} \text{Now, } m^2 + n^2 - 2 &= (\cos A + \sin B)^2 + (\sin A + \cos B)^2 - 2 \\ &= \cos^2 A + \sin^2 B + 2 \cos A \sin B + \sin^2 A + \cos^2 B + 2 \sin A \cos B - 2 \\ &= (\sin^2 A + \cos^2 A) + (\sin^2 B + \cos^2 B) + 2 \cos A \sin B + 2 \sin A \cos B - 2 \\ &= 1 + 1 + 2 \cos A \sin B + 2 \sin A \cos B - 2 \\ &= 2 + 2(\sin A \cos B + \cos A \sin B) - 2 \\ &= 2(\sin A \cos B + \cos A \sin B) \quad [\because \sin(A + B) = \sin A \cos B + \cos A \sin B] \\ &= 2 \sin(A + B) \end{aligned}$$

$$\therefore 2 \sin(A + B) = m^2 + n^2 - 2$$

Hence proved

Trigonometric Ratios of Compound Angles Ex 7.1 Q23

We have,

$$\tan A + \tan B = a \text{ and } \cot A + \cot B = b$$

$$\text{Now, } \cot A + \cot B = b$$

$$\Rightarrow \frac{1}{\tan A} + \frac{1}{\tan B} = b \quad \left[\because \cot \theta = \frac{1}{\tan \theta} \right]$$

$$\Rightarrow \frac{\tan B + \tan A}{\tan A \tan B} = b$$

$$\Rightarrow \frac{a}{\tan A \tan B} = b \quad [\because \tan A + \tan B = a]$$

$$\Rightarrow \frac{a}{b} = \tan A \tan B$$

$$\begin{aligned} \therefore \cot(A + B) &= \frac{1}{\tan(A + B)} \\ &= \frac{1}{\frac{\tan A + \tan B}{1 - \tan A \tan B}} \end{aligned}$$

$$\begin{aligned}
 &= \frac{1 - \tan A \tan B}{\tan A + \tan B} \\
 &= \frac{1 - \frac{a}{b}}{\frac{a}{b}} \quad \left[\because \tan A \tan B = \frac{a}{b} \right] \\
 &= \frac{b - a}{ab} \\
 &= \frac{b}{ab} - \frac{a}{ab} \\
 &= \frac{1}{a} - \frac{1}{b}
 \end{aligned}$$

$$\therefore \cot(A + B) = \frac{1}{a} - \frac{1}{b}$$

Hence proved.

Trigonometric Ratios of Compound Angles Ex 7.1 Q24

We have,

$$\cos \theta = \frac{8}{17}$$

$$\begin{aligned}
 \therefore \sin \theta &= \sqrt{1 - \cos^2 \theta} = \sqrt{1 - \frac{64}{289}} \\
 &= \sqrt{\frac{225}{289}} \\
 &= \frac{15}{17}
 \end{aligned}$$

$$\begin{aligned}
 \text{Now, } \cos\left(\frac{\pi}{6} + \theta\right) + \cos\left(\frac{\pi}{4} - \theta\right) + \cos\left(\frac{2\pi}{3} - \theta\right) &= \left[\cos \frac{\pi}{6} \cos \theta - \sin \frac{\pi}{6} \sin \theta\right] + \left[\cos \frac{\pi}{4} \cos \theta + \sin \frac{\pi}{4} \sin \theta\right] \\
 &\quad + \left[\cos \frac{2\pi}{3} \cos \theta + \sin \frac{2\pi}{3} \sin \theta\right] \\
 &= \left[\cos \frac{\pi}{6} + \cos \frac{\pi}{4} + \cos \frac{2\pi}{3}\right] \cos \theta + \sin \theta \left[-\sin \frac{\pi}{6} + \sin \frac{\pi}{4} + \sin \frac{2\pi}{3}\right] \\
 &= \left[\frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} + \cos\left(\frac{\pi}{2} + \frac{\pi}{6}\right)\right] \times \frac{8}{17} + \frac{15}{17} \times \left[-\frac{1}{2} + \frac{1}{\sqrt{2}} + \sin\left(\frac{\pi}{2} + \frac{\pi}{6}\right)\right] \\
 &= \left[\frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} - \sin \frac{\pi}{6}\right] \times \frac{8}{17} + \frac{15}{17} \times \left[-\frac{1}{2} + \frac{1}{\sqrt{2}} + \cos \frac{\pi}{6}\right] \\
 &= \left[\frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} - \frac{1}{2}\right] \times \frac{8}{17} + \frac{15}{17} \times \left[-\frac{1}{2} + \frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2}\right] \\
 &\quad [\because \cos A \text{ is negative in second quadrant}] \\
 &= \left[\frac{\sqrt{3}-1}{2} + \frac{1}{\sqrt{2}}\right] \times \frac{8}{17} + \frac{15}{17} \times \left[\frac{\sqrt{3}-1}{2} + \frac{1}{\sqrt{2}}\right] \\
 &= \left(\frac{\sqrt{3}-1}{2} + \frac{1}{\sqrt{2}}\right) \left(\frac{8}{17} + \frac{15}{17}\right) \\
 &= \left(\frac{\sqrt{3}-1}{2} + \frac{1}{\sqrt{2}}\right) \left(\frac{8+15}{17}\right) \\
 &= \left(\frac{\sqrt{3}-1}{2} + \frac{1}{\sqrt{2}}\right) \times \frac{23}{17}
 \end{aligned}$$

$$\therefore \cos\left(\frac{\pi}{6} + \theta\right) + \cos\left(\frac{\pi}{4} - \theta\right) + \cos\left(\frac{2\pi}{3} - \theta\right) = \left(\frac{\sqrt{3}-1}{2} + \frac{1}{\sqrt{2}}\right) \times \frac{23}{17}$$

Hence proved.

Trigonometric Ratios of Compound Angles Ex 7.1 Q25

We have,

$$\tan x + \tan\left(x + \frac{\pi}{3}\right) + \tan\left(x + \frac{2\pi}{3}\right) = 3$$

$$\Rightarrow \tan x + \frac{\tan x + \tan \frac{\pi}{3}}{1 - \tan x \tan \frac{\pi}{3}} + \frac{\tan x + \tan \left(\frac{2\pi}{3}\right)}{1 - \tan x \tan \left(\frac{2\pi}{3}\right)} = 3$$

$$\left[\frac{1 - \tan x \tan \frac{\pi}{3}}{1 + \tan x \tan \frac{\pi}{3}} \right] \left[\frac{1 - \tan x \tan \frac{\pi}{3}}{1 + \tan x \tan \frac{\pi}{3}} \right]$$

$$\Rightarrow \tan x + \left[\frac{\tan x + \sqrt{3}}{1 - \sqrt{3} \tan x} \right] + \left[\frac{\tan x + \tan \left(\frac{\pi}{2} + \frac{\pi}{3} \right)}{1 - \tan x \tan \left(\frac{\pi}{2} + \frac{\pi}{3} \right)} \right] = 3$$

$$\Rightarrow \tan x + \frac{\tan x + \sqrt{3}}{1 - \sqrt{3} \tan x} + \frac{\tan x - \cot \frac{\pi}{3}}{1 + \tan x \cot \frac{\pi}{3}} = 3 \quad \left[\because \tan \theta \text{ is negative in second quadrant} \right]$$

$$\Rightarrow \tan x + \frac{\tan x + \sqrt{3}}{1 - \sqrt{3} \tan x} + \frac{\tan x - \sqrt{3}}{1 + \sqrt{3} \tan x} = 3$$

$$\Rightarrow \tan x + \frac{(\tan x + \sqrt{3})(1 + \sqrt{3} \tan x) + (\tan x - \sqrt{3})(1 - \sqrt{3} \tan x)}{(1 - \sqrt{3} \tan x)(1 + \sqrt{3} \tan x)} = 3$$

$$\Rightarrow \tan x + \frac{\tan x + \sqrt{3} \tan^2 x + \sqrt{3} + 3 \tan x + \tan x - \sqrt{3} \tan^2 x - \sqrt{3} + 3 \tan x}{1 - (\sqrt{3} \tan x)^2} = 3$$

$$\Rightarrow \tan x + \frac{8 \tan x}{1 - 3 \tan^2 x} = 3$$

$$\Rightarrow \frac{\tan x (1 - 3 \tan^2 x) + 8 \tan x}{1 - 3 \tan^2 x} = 3$$

$$\Rightarrow \frac{\tan x - 3 \tan^3 x + 8 \tan x}{1 - 3 \tan^2 x} = 3$$

$$\Rightarrow \frac{9 \tan x - 3 \tan^3 x}{1 - 3 \tan^2 x} = 3$$

$$\Rightarrow \frac{3 (3 \tan x - \tan^3 x)}{1 - 3 \tan^2 x} = 3$$

$$\Rightarrow \frac{3 \tan x - \tan^3 x}{1 - 3 \tan^2 x} = 1$$

Hence proved.

Trigonometric Ratios of Compound Angles Ex 7.1 Q26

We have,

$$\sin(\alpha + \beta) = 1$$

$$\Rightarrow \sin(\alpha + \beta) = \sin \frac{\pi}{2}$$

$$\Rightarrow \alpha + \beta = \frac{\pi}{2} \quad \text{--- (i)}$$

$$\text{and, } \sin(\alpha - \beta) = \frac{1}{2}$$

$$\Rightarrow \sin(\alpha - \beta) = \sin \frac{\pi}{6}$$

$$\Rightarrow \alpha - \beta = \frac{\pi}{6} \quad \text{--- (ii)}$$

Adding equations (i) and (ii), we get

$$2\alpha = \frac{\pi}{2} + \frac{\pi}{6} = \frac{4\pi}{6} = \frac{2\pi}{3}$$

$$\Rightarrow \alpha = \frac{\pi}{3}$$

Putting $\alpha = \frac{\pi}{3}$ in equation (i), we get

$$\frac{\pi}{3} + \beta = \frac{\pi}{2}$$

$$\Rightarrow \beta = \frac{\pi}{2} - \frac{\pi}{3}$$

$$\Rightarrow \beta = \frac{3\pi - 2\pi}{6}$$

$$= \frac{\pi}{6}$$

$$\Rightarrow \beta = \frac{\pi}{6}$$

$$\text{Now, } \tan(\alpha + 2\beta) = \tan\left(\frac{\pi}{3} + 2 \times \frac{\pi}{6}\right)$$

$$= \tan\left(\frac{\pi}{3} + \frac{\pi}{3}\right)$$

$$= \tan \frac{2\pi}{3}$$

$$\begin{aligned}
 &= -\tan \frac{\pi}{3} \\
 &= \tan \left(\frac{\pi}{2} + \frac{\pi}{6} \right) \\
 &= -\cot \frac{\pi}{6} \quad \left[\because \tan \theta \text{ is negative in second quadrant} \right] \\
 &= -\sqrt{3}
 \end{aligned}$$

$$\therefore t(\alpha + 2\beta) = -\sqrt{3}$$

$$\begin{aligned}
 \text{and, } \tan(2\alpha + \beta) &= \tan \left(2 \times \frac{\pi}{3} + \frac{\pi}{6} \right) \\
 &= \tan \left(\frac{2\pi}{3} + \frac{\pi}{6} \right) \\
 &= \tan \left(\frac{4\pi + \pi}{6} \right) \\
 &= \tan \left(\frac{5\pi}{6} \right) \\
 &= \tan \left(\frac{\pi}{2} + \frac{\pi}{6} \right) \\
 &= -\cot \frac{\pi}{6} \quad \left[\because \tan \theta \text{ is negative in second quadrant} \right] \\
 &= -\frac{1}{\sqrt{3}}
 \end{aligned}$$

$$\therefore \tan(2\alpha + \beta) = \frac{-1}{\sqrt{3}}$$

Trigonometric Ratios of Compound Angles Ex 7.1 Q27

We have,

$$6 \cos \theta + 8 \sin \theta = 9 \quad \text{--- (i)}$$

$$\begin{aligned}
 \Rightarrow 8 \sin \theta &= 9 - 6 \cos \theta \\
 \Rightarrow (8 \sin \theta)^2 &= (9 - 6 \cos \theta)^2 \quad \left[\because \text{Squaring both sides} \right] \\
 \Rightarrow 64 \sin^2 \theta &= 81 + 36 \cos^2 \theta - 108 \cos \theta \\
 \Rightarrow 64 \sin^2 \theta &= 81 + 36 \cos^2 \theta - 108 \cos \theta \\
 \Rightarrow 64(1 - \cos^2 \theta) &= 81 + 36 \cos^2 \theta - 108 \cos \theta \\
 \Rightarrow 64 - 64 \cos^2 \theta &= 81 + 36 \cos^2 \theta - 108 \cos \theta \\
 \Rightarrow 36 \cos^2 \theta + 64 \cos^2 \theta - 108 \cos \theta + 81 - 64 &= 0 \\
 \Rightarrow 100 \cos^2 \theta - 108 \cos \theta + 17 &= 0 \quad \text{--- (ii)}
 \end{aligned}$$

Since α, β are roots of equation (ii).

Therefore, $\cos \alpha$ and $\cos \beta$ are roots of equation (ii)

$$\therefore \cos \alpha + \cos \beta = \frac{17}{100} \quad \text{--- (iii)}$$

Again, $6 \cos \theta + 8 \sin \theta = 9$

$$\begin{aligned}
 \Rightarrow 6 \cos \theta &= 9 - 8 \sin \theta \\
 \Rightarrow (6 \cos \theta)^2 &= (9 - 8 \sin \theta)^2 \quad \left[\because \text{Squaring both sides} \right] \\
 \Rightarrow 36 \cos^2 \theta &= 81 + 64 \sin^2 \theta - 144 \sin \theta \\
 \Rightarrow 36(1 - \sin^2 \theta) &= 81 + 64 \sin^2 \theta - 144 \sin \theta \\
 \Rightarrow 36 - 36 \sin^2 \theta &= 81 + 64 \sin^2 \theta - 144 \sin \theta \\
 \Rightarrow 64 \sin^2 \theta + 36 \sin^2 \theta - 144 \sin \theta + 81 - 36 &= 0 \\
 \Rightarrow 100 \sin^2 \theta - 144 \sin \theta + 45 &= 0 \quad \text{--- (iv)}
 \end{aligned}$$

$$\therefore \sin \alpha \times \sin \beta = \frac{45}{100} \quad \text{--- (v)}$$

Now, $\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$

$$\begin{aligned}
 &= \frac{17}{100} - \frac{45}{100} \quad \left[\text{Using equation (iii) and (v)} \right] \\
 &= -\frac{28}{100}
 \end{aligned}$$

$$= -\frac{7}{25}$$

$$\begin{aligned}\text{Now, } \sin(\alpha + \beta) &= \sqrt{1 - (\cos \theta)^2} \\ &= \sqrt{1 - \left(-\frac{7}{25}\right)^2} \\ &= \sqrt{1 - \frac{49}{625}} \\ &= \sqrt{\frac{625 - 49}{625}} \\ &= \sqrt{\frac{576}{625}} \\ &= \frac{24}{25}\end{aligned}$$

$$\therefore \sin(\alpha + \beta) = \frac{24}{25}$$

Trigonometric Ratios of Compound Angles Ex 7.1 Q28

$$\sin(\alpha + \beta) = \sqrt{1 - \cos^2(\alpha + \beta)}$$

$$\begin{aligned}b^2 + a^2 &= (\cos \alpha + \cos \beta)^2 + (\sin \alpha + \sin \beta)^2 \\ \Rightarrow b^2 + a^2 &= (\cos^2 \alpha + \sin^2 \alpha) + (\cos^2 \beta + \sin^2 \beta) + 2(\cos \alpha \cos \beta + \sin \alpha \sin \beta) \\ \Rightarrow b^2 + a^2 &= 1 + 1 + 2 \cos(\alpha - \beta) = 2 + 2 \cos(\alpha - \beta)\end{aligned}$$

$$\begin{aligned}\text{and, } b^2 - a^2 &= (\cos \alpha + \cos \beta)^2 - (\sin \alpha + \sin \beta)^2 \\ b^2 - a^2 &= \cos^2 \alpha + \cos^2 \beta - \sin^2 \alpha - \sin^2 \beta + 2(\cos \alpha \cos \beta - \sin \alpha \sin \beta) \\ \Rightarrow b^2 - a^2 &= (\cos^2 \alpha - \sin^2 \beta) + (\cos^2 \beta - \sin^2 \alpha) + 2 \cos(\alpha + \beta) \\ \Rightarrow b^2 - a^2 &= \cos(\alpha + \beta) \cos(\alpha - \beta) + \cos(\beta + \alpha) \cos(\beta - \alpha) + 2 \cos(\alpha + \beta) \\ \Rightarrow b^2 - a^2 &= 2 \cos(\alpha + \beta) \cos(\alpha - \beta) + 2 \cos(\alpha + \beta) \\ &\quad [\because \cos(\beta - \alpha) = \cos\{-(\alpha - \beta)\} = \cos(\alpha - \beta)] \\ \Rightarrow b^2 - a^2 &= \cos(\alpha + \beta) \{2 \cos(\alpha - \beta) + 2\} \\ \Rightarrow b^2 - a^2 &= \cos(\alpha + \beta) (b^2 + a^2) \quad [\text{Using (i)}]\end{aligned}$$

$$\text{Thus, } b^2 - a^2 = (b^2 + a^2) \cos(\alpha + \beta)$$

$$\Rightarrow \cos(\alpha + \beta) = \frac{b^2 - a^2}{b^2 + a^2}$$

$$\Rightarrow \sin(\alpha + \beta) = \sqrt{1 - \left(\frac{b^2 - a^2}{b^2 + a^2}\right)^2} = \frac{\sqrt{4a^2b^2}}{(a^2 + b^2)^2} = \frac{2ab}{a^2 + b^2}$$

$$\begin{aligned}b^2 + a^2 &= (\cos \alpha + \cos \beta)^2 + (\sin \alpha + \sin \beta)^2 \\ \Rightarrow b^2 + a^2 &= (\cos^2 \alpha + \sin^2 \alpha) + (\cos^2 \beta + \sin^2 \beta) + 2(\cos \alpha \cos \beta + \sin \alpha \sin \beta) \\ \Rightarrow b^2 + a^2 &= 1 + 1 + 2 \cos(\alpha - \beta) = 2 + 2 \cos(\alpha - \beta)\end{aligned}$$

$$\begin{aligned}\text{and, } b^2 - a^2 &= (\cos \alpha + \cos \beta)^2 - (\sin \alpha + \sin \beta)^2 \\ b^2 - a^2 &= \cos^2 \alpha + \cos^2 \beta - \sin^2 \alpha - \sin^2 \beta + 2(\cos \alpha \cos \beta - \sin \alpha \sin \beta) \\ \Rightarrow b^2 - a^2 &= (\cos^2 \alpha - \sin^2 \beta) + (\cos^2 \beta - \sin^2 \alpha) + 2 \cos(\alpha + \beta) \\ \Rightarrow b^2 - a^2 &= \cos(\alpha + \beta) \cos(\alpha - \beta) + \cos(\beta + \alpha) \cos(\beta - \alpha) + 2 \cos(\alpha + \beta) \\ \Rightarrow b^2 - a^2 &= 2 \cos(\alpha + \beta) \cos(\alpha - \beta) + 2 \cos(\alpha + \beta) \\ &\quad [\because \cos(\beta - \alpha) = \cos\{-(\alpha - \beta)\} = \cos(\alpha - \beta)] \\ \Rightarrow b^2 - a^2 &= \cos(\alpha + \beta) \{2 \cos(\alpha - \beta) + 2\} \\ \Rightarrow b^2 - a^2 &= \cos(\alpha + \beta) (b^2 + a^2) \quad [\text{Using (i)}]\end{aligned}$$

$$\text{Thus, } b^2 - a^2 = (b^2 + a^2) \cos(\alpha + \beta)$$

$$\Rightarrow \cos(\alpha + \beta) = \frac{b^2 - a^2}{b^2 + a^2}$$

Trigonometric Ratios of Compound Angles Ex 7.1 Q29

$$\begin{aligned}
 \text{LHS} &= \frac{1}{\sin(x-a)\sin(x-b)} \\
 &= \frac{1}{\sin(a-b)} \left[\frac{\sin(a-b)}{\sin(x-a)\sin(x-b)} \right] \\
 &= \frac{1}{\sin(a-b)} \left[\frac{\sin\{(x-b)-(x-a)\}}{\sin(x-a)\sin(x-b)} \right] \\
 &= \frac{1}{\sin(a-b)} \left[\frac{\sin(x-b)\cos(x-a) - \cos(x-b)\sin(x-a)}{\sin(x-a)\sin(x-b)} \right] \\
 &= \frac{1}{\sin(a-b)} \left[\frac{\sin(x-b)\cos(x-a)}{\sin(x-a)\sin(x-b)} - \frac{\cos(x-b)\sin(x-a)}{\sin(x-a)\sin(x-b)} \right] \\
 &= \frac{1}{\sin(a-b)} [\cot(x-a) - \cot(x-b)] \\
 &= \frac{\cot(x-a) - \cot(x-b)}{\sin(a-b)} \\
 &= \text{RHS}
 \end{aligned}$$

∴ LHS=RHS

Hence proved

$$\begin{aligned}
 \text{LHS} &= \frac{1}{\sin(x-a)\cos(x-b)} \\
 &= \frac{1}{\cos(a-b)} \left[\frac{\cos(a-b)}{\sin(x-a)\cos(x-b)} \right] \\
 &= \frac{1}{\cos(a-b)} \left[\frac{\cos\{(x-b)-(x-a)\}}{\sin(x-a)\cos(x-b)} \right] \\
 &= \frac{1}{\cos(a-b)} \left[\frac{\cos(x-b)\cos(x-a) + \sin(x-b)\sin(x-a)}{\sin(x-a)\cos(x-b)} \right] \\
 &= \frac{1}{\cos(a-b)} \left[\frac{\cos(x-b)\cos(x-a)}{\sin(x-a)\cos(x-b)} + \frac{\sin(x-b)\sin(x-a)}{\sin(x-a)\cos(x-b)} \right] \\
 &= \frac{1}{\cos(a-b)} \left[\frac{\cos(x-a)}{\sin(x-a)} + \frac{\sin(x-b)}{\cos(x-b)} \right] \\
 &= \frac{1}{\cos(a-b)} [\cot(x-a) + \tan(x-b)] \\
 &= \frac{\cot(x-a) + \tan(x-b)}{\cos(a-b)} \\
 &= \text{RHS}
 \end{aligned}$$

∴ LHS=RHS

Hence proved

$$\begin{aligned}
 \text{LHS} &= \frac{1}{\cos(x-a)\cos(x-b)} \\
 &= \frac{1}{\sin(a-b)} \left[\frac{\sin(a-b)}{\cos(x-a)\cos(x-b)} \right] \\
 &= \frac{1}{\sin(a-b)} \left[\frac{\sin\{(x-b)-(x-a)\}}{\cos(x-a)\cos(x-b)} \right] \\
 &= \frac{1}{\sin(a-b)} \left[\frac{\sin(x-b)\cos(x-a) - \cos(x-b)\sin(x-a)}{\cos(x-a)\cos(x-b)} \right] \\
 &= \frac{1}{\sin(a-b)} \left[\frac{\sin(x-b)\cos(x-a)}{\cos(x-a)\cos(x-b)} - \frac{\cos(x-b)\sin(x-a)}{\cos(x-a)\cos(x-b)} \right] \\
 &= \frac{1}{\sin(a-b)} \left[\frac{\sin(x-b)}{\cos(x-b)} - \frac{\sin(x-a)}{\cos(x-a)} \right] \\
 &= \frac{1}{\sin(a-b)} [\tan(x-b) - \tan(x-a)] \\
 &= \frac{\tan(x-b) - \tan(x-a)}{\sin(a-b)} \\
 &= \text{RHS}
 \end{aligned}$$

$$\therefore \text{LHS} = \text{RHS}$$

Hence proved

Trigonometric Ratios of Compound Angles Ex 7.1 Q30

We have,

$$\sin \alpha \sin \beta - \cos \alpha \cos \beta + 1 = 0$$

$$\Rightarrow -(\cos \alpha \cos \beta - \sin \alpha \sin \beta) = -1$$

$$\Rightarrow \cos(\alpha + \beta) = 1 \quad \text{--- (i)}$$

$$\therefore \sin(\alpha + \beta) = \sqrt{1 - \cos^2(\alpha + \beta)}$$

$$= \sqrt{1 - 1^2}$$

$$= 0$$

$$\Rightarrow \sin(\alpha + \beta) = 0 \quad \text{--- (ii)}$$

Now,

$$\begin{aligned} 1 + \cot \alpha \tan \beta &= 1 + \frac{\cos \alpha}{\sin \alpha} \times \frac{\sin \beta}{\cos \beta} \\ &= \frac{\sin \alpha \times \cos \beta + \cos \alpha \times \sin \beta}{\sin \alpha \times \cos \beta} \\ &= \frac{\sin(\alpha + \beta)}{\sin \alpha \times \cos \beta} \\ &= \frac{0}{\sin \alpha \times \cos \beta} \quad \text{[Using equation (ii)]} \\ &= 0 \end{aligned}$$

$$\therefore 1 + \cot \alpha \tan \beta = 0$$

Hence proved

Trigonometric Ratios of Compound Angles Ex 7.1 Q31

We have,

$$\tan \alpha = x + 1 \text{ and } \tan \beta = x - 1$$

$$\begin{aligned} \text{Now, } 2 \cot(\alpha - \beta) &= \frac{2}{\tan(\alpha - \beta)} \\ &= \frac{2}{\frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta}} \\ &= \frac{2(1 + \tan \alpha \tan \beta)}{\tan \alpha - \tan \beta} \\ &= \frac{2[1 + (x + 1)(x - 1)]}{x + 1 - (x - 1)} \\ &= \frac{2[1 + x^2 - 1]}{x + 1 - x + 1} \\ &= \frac{2 \times x^2}{2} = x^2 \end{aligned}$$

$$\therefore 2 \cot(\alpha - \beta) = x^2$$

Hence proved

Trigonometric Ratios of Compound Angles Ex 7.1 Q32

Let the two parts of the angle be θ and $\theta - \emptyset$.

$$\tan(\theta - \emptyset) = \lambda \tan \emptyset \quad \text{[According to question]}$$

$$\Rightarrow \frac{\tan(\theta - \emptyset)}{\tan \emptyset} = \frac{\lambda}{1}$$

$$\Rightarrow \frac{\tan(\theta - \emptyset)}{\tan \emptyset} = \frac{\lambda}{1}$$

$$\Rightarrow \frac{\tan(\theta - \emptyset) + \tan \emptyset}{\tan(\theta - \emptyset) - \tan \emptyset} = \frac{\lambda + 1}{\lambda - 1} \quad \text{[Using Componendo and Dividendo]}$$

$$\begin{aligned}
 & \Rightarrow \frac{\frac{\tan \theta - \tan \phi}{1 + \tan \theta \cdot \tan \phi} + \tan \phi}{\frac{\tan \theta - \tan \phi}{1 + \tan \theta \cdot \tan \phi} - \tan \phi} = \frac{\lambda + 1}{\lambda - 1} \\
 & \Rightarrow \frac{\tan \theta - \tan \phi + \tan \phi(1 + \tan \theta \cdot \tan \phi)}{1 + \tan \theta \cdot \tan \phi - \tan \phi(1 + \tan \theta \cdot \tan \phi)} = \frac{\lambda + 1}{\lambda - 1} \\
 & \Rightarrow \frac{\tan \theta - \tan \phi + \tan \phi + \tan \theta \cdot \tan^2 \phi}{\tan \theta - \tan \phi - \tan \phi - \tan \theta \cdot \tan^2 \phi} = \frac{\lambda + 1}{\lambda - 1} \\
 & \Rightarrow \frac{\tan \theta + \tan \theta \cdot \tan^2 \phi}{\tan \theta - 2 \tan \phi - \tan \theta \cdot \tan^2 \phi} = \frac{\lambda + 1}{\lambda - 1} \\
 & \Rightarrow \tan \theta = \frac{\lambda + 1}{\lambda - 1} \cdot \frac{\tan \theta - 2 \tan \phi - \tan \theta \cdot \tan^2 \phi}{\tan \theta + \tan \theta \cdot \tan^2 \phi}
 \end{aligned}$$

Trigonometric Ratios of Compound Angles Ex 7.1 33

$$\begin{aligned}
 \tan \theta &= \frac{\sin \alpha - \cos \alpha}{\sin \alpha + \cos \alpha} \\
 \Rightarrow \tan \theta &= \frac{\tan \alpha - 1}{\tan \alpha + 1} \quad [\text{Dividing both Numerator and Denominator by } \cos \alpha] \\
 \Rightarrow \tan \theta &= \frac{\tan \alpha - \tan \frac{\pi}{4}}{1 + \tan \frac{\pi}{4} \cdot \tan \alpha} \\
 \Rightarrow \tan \theta &= \tan \left(\alpha - \frac{\pi}{4} \right) \\
 \Rightarrow \theta &= \alpha - \frac{\pi}{4} \quad [\text{Removing tan from both sides}] \\
 \Rightarrow \cos \theta &= \cos \left(\alpha - \frac{\pi}{4} \right) \quad [\text{Taking cos on both sides}] \\
 \Rightarrow \cos \theta &= \cos \alpha \cdot \cos \frac{\pi}{4} + \sin \alpha \cdot \sin \frac{\pi}{4} \\
 \Rightarrow \cos \theta &= \cos \alpha \cdot \frac{1}{\sqrt{2}} + \sin \alpha \cdot \frac{1}{\sqrt{2}} \\
 \Rightarrow \cos \theta &= \frac{\cos \alpha + \sin \alpha}{\sqrt{2}} \\
 \Rightarrow \sqrt{2} \cos \theta &= \sin \alpha + \cos \alpha \\
 \text{Hence Proved}
 \end{aligned}$$

Trigonometric Ratios of Compound Angles Ex 7.1 Q34

$$\begin{aligned}
 \text{RHS,} \\
 & \frac{p + q}{1 - pq} \\
 &= \frac{\tan(A + B) + \tan(A - B)}{1 - \tan(A + B) \cdot \tan(A - B)} \\
 &= \frac{\frac{\tan A + \tan B}{1 - \tan A \cdot \tan B} + \frac{\tan A - \tan B}{1 + \tan A \cdot \tan B}}{1 - \frac{\tan A + \tan B}{1 - \tan A \cdot \tan B} \cdot \frac{\tan A - \tan B}{1 + \tan A \cdot \tan B}} \\
 &= \frac{(\tan A + \tan B)(1 + \tan A \cdot \tan B) + (\tan A - \tan B)(1 - \tan A \cdot \tan B)}{(1 - \tan A \cdot \tan B)(1 + \tan A \cdot \tan B) - (\tan A + \tan B)(\tan A - \tan B)} \\
 &= \frac{\tan A + \tan B + \tan^2 A \cdot \tan B + \tan A \cdot \tan^2 B + \tan A - \tan B - \tan^2 A \cdot \tan B + \tan A \cdot \tan^2 B}{1 - \tan^2 A \cdot \tan^2 B - \tan^2 A + \tan^2 B} \\
 &= \frac{2 \tan A + 2 \tan A \cdot \tan^2 B}{(1 - \tan^2 A)(1 + \tan^2 B)} = \frac{2 \tan A(1 + \tan^2 B)}{(1 - \tan^2 A)(1 + \tan^2 B)} = \frac{2 \tan A}{1 - \tan^2 A} = \tan 2A = \text{LHS} \\
 \text{Hence Proved}
 \end{aligned}$$