

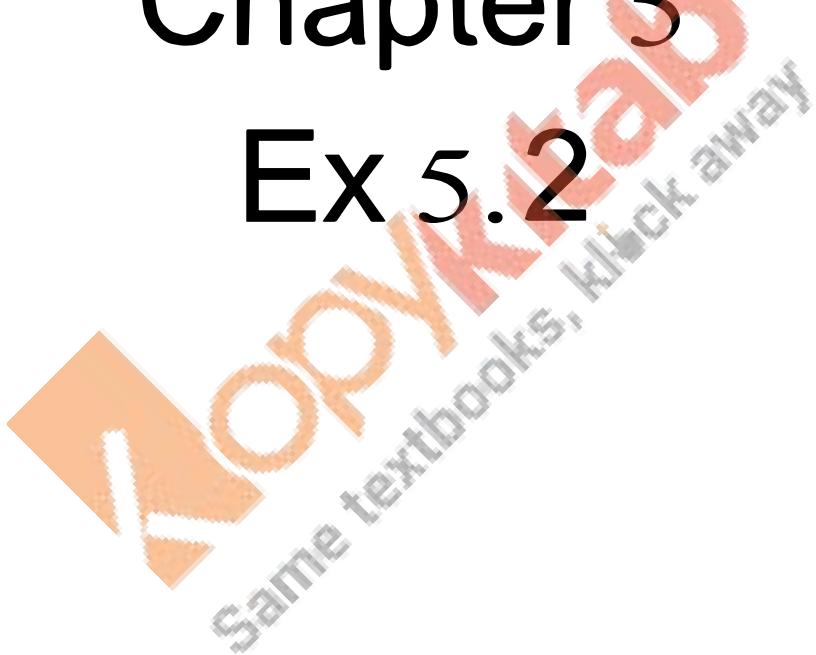
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Solutions

Class 11 Maths

Chapter 5

Ex 5.2



Trigonometric Functions Ex 5.2 Q 1

We have,

$$\operatorname{cosec}^2 \theta - \cot^2 \theta = 1$$

$$\Rightarrow \operatorname{cosec}^2 \theta = 1 + \cot^2 \theta$$

$$\Rightarrow \operatorname{cosec} \theta = \pm \sqrt{1 + \cot^2 \theta}$$

In the third quadrant $\operatorname{cosec} \theta$ is negative

$$\begin{aligned}\therefore \operatorname{cosec} \theta &= -\sqrt{1 + \cot^2 \theta} \\&= -\sqrt{1 + \left(\frac{12}{5}\right)^2} \quad \left[\because \cot \theta = \frac{12}{5} \right] \\&= -\sqrt{1 + \frac{144}{25}} \\&= -\sqrt{\frac{169}{25}} \\&= -\frac{13}{5} \\&\therefore \operatorname{cosec} \theta = -\frac{13}{5}\end{aligned}$$

$$\text{Now, } \tan \theta = \frac{1}{\cot \theta}$$

$$\begin{aligned}&= \frac{1}{\frac{12}{5}} \\&= \frac{5}{12}\end{aligned}$$

We have,

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\Rightarrow \sin^2 \theta = 1 - \cos^2 \theta$$

$$\Rightarrow \sin \theta = \pm \sqrt{1 - \cos^2 \theta}$$

In the 2nd quadrant $\sin \theta$ is positive and $\tan \theta$ is negative

$$\begin{aligned}\therefore \sin \theta &= \sqrt{1 - \cos^2 \theta} \\&= \sqrt{1 - \left(-\frac{1}{2}\right)^2} \quad \left[\because \cos \theta = -\frac{1}{2} \right] \\&= \sqrt{1 - \frac{1}{4}} \\&= \sqrt{\frac{3}{4}} \\&= \frac{\sqrt{3}}{2}\end{aligned}$$

$$\text{and, } \tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{\frac{\sqrt{3}}{2}}{-\frac{1}{2}} = -\sqrt{3}$$

$$\text{Now, } \operatorname{cosec} \theta = \frac{1}{\sin \theta} = \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}}$$

$$\sec \theta = \frac{1}{\cos \theta} = \frac{1}{-\frac{1}{2}} = -2$$

$$\text{and } \cot \theta = \frac{1}{\tan \theta} = \frac{1}{-\sqrt{3}} = \frac{-1}{\sqrt{3}}$$

$$\text{Hence, } \sin \theta = \frac{\sqrt{3}}{2}, \quad \tan \theta = -\sqrt{3},$$

$$\cosec \theta = \frac{2}{\sqrt{3}}, \quad \sec \theta = -2 \text{ and } \cot \theta = \frac{-1}{\sqrt{3}}$$

In the third quadrant $\cosec \theta$ is negative

$$\begin{aligned} \therefore \cosec \theta &= -\sqrt{1 + \cot^2 \theta} \\ &= -\sqrt{1 + \left(\frac{4}{3}\right)^2} \\ &= -\sqrt{1 + \frac{16}{9}} \\ &= -\sqrt{\frac{25}{9}} \\ &= -\frac{5}{3} \end{aligned}$$

$$\text{Now, } \sin \theta = \frac{1}{\cosec \theta} = \frac{1}{-\frac{5}{3}} = \frac{-3}{5}$$

$$\text{and, } \cos \theta = \frac{1}{\sec \theta} = \frac{1}{-\frac{5}{3}} = \frac{-4}{5}$$

$$\text{Hence, } \sin \theta = \frac{-3}{5}, \quad \cos \theta = \frac{-4}{5},$$

$$\cosec \theta = -\frac{5}{3}, \quad \sec \theta = \frac{-5}{4} \text{ and } \cot \theta = \frac{4}{3}$$

We have,

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\Rightarrow \cos^2 \theta = 1 - \sin^2 \theta$$

$$\Rightarrow \cos \theta = \pm \sqrt{1 - \sin^2 \theta}$$

In the 1st quadrant $\cos \theta$ is positive and $\tan \theta$ is also positive

$$\begin{aligned} \therefore \cos \theta &= \sqrt{1 - \sin^2 \theta} \\ &= \sqrt{1 - \left(\frac{3}{5}\right)^2} \\ &= \sqrt{1 - \frac{9}{25}} \\ &= \sqrt{\frac{16}{25}} \\ &= \frac{4}{5} \end{aligned}$$

$$\text{and, } \tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{\frac{3}{5}}{\frac{4}{5}} = \frac{3}{4}$$

$$\text{Now, } \cosec \theta = \frac{1}{\sin \theta} = \frac{1}{\frac{3}{5}} = \frac{5}{3}$$

$$\sec \theta = \frac{1}{\cos \theta} = \frac{1}{\frac{4}{5}} = \frac{5}{4}$$

$$\text{and, } \cot \theta = \frac{1}{\tan \theta} = \frac{1}{\frac{3}{4}} = \frac{4}{3}$$

$$\text{Hence, } \cos \theta = \frac{4}{5}, \quad \cosec \theta = \frac{5}{3}, \quad \tan \theta = \frac{3}{4},$$

$$\sec \theta = \frac{5}{4}, \quad \text{and } \cot \theta = \frac{4}{3}$$

Trigonometric Functions Ex 5.2 Q 2

We have,

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\Rightarrow \cos^2 \theta = 1 - \sin^2 \theta$$

$$\Rightarrow \cos \theta = \pm \sqrt{1 - \sin^2 \theta}$$

In the 2nd quadrant $\cos\theta$ is negative and $\tan\theta$ is also negative

$$\begin{aligned}\therefore \cos\theta &= -\sqrt{1 - \sin^2\theta} \\ &= -\sqrt{1 - \left(\frac{12}{13}\right)^2} \quad \left[\because \sin\theta = \frac{12}{13}\right] \\ &= -\sqrt{1 - \frac{144}{169}} \\ &= -\sqrt{\frac{25}{169}} \\ &= -\frac{5}{13}\end{aligned}$$

$$\text{and, } \tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{12}{13}}{-\frac{5}{13}} = -\frac{12}{5}$$

$$\text{Now, } \sec\theta = \frac{1}{\cos\theta} = \frac{1}{-\frac{5}{13}} = -\frac{13}{5}$$

$$\begin{aligned}\therefore \sec\theta + \tan\theta &= -\frac{13}{5} - \frac{12}{5} \\ &= \frac{-13 - 12}{5} \\ &= -\frac{25}{5} \\ &= -5\end{aligned}$$

$$\Rightarrow \sec\theta + \tan\theta = -5$$

Trigonometric Functions Ex 5.2 Q 3

We have,

$$\sin\theta = \frac{3}{5}, \tan\phi = \frac{1}{2} \text{ and } \frac{\pi}{2} < \theta < \pi < \frac{3\pi}{2}$$

$\Rightarrow \theta$ lies in the second quadrant and ϕ lies in the third quadrant.

$$\text{Now, } \sin^2\theta + \cos^2\theta = 1$$

$$\Rightarrow \cos^2\theta = 1 - \sin^2\theta$$

$$\Rightarrow \cos\theta = \pm\sqrt{1 - \sin^2\theta}$$

In the 2nd quadrant $\cos\theta$ is negative and $\tan\theta$ is also negative

$$\begin{aligned}\therefore \cos\theta &= -\sqrt{1 - \sin^2\theta} \\ &= -\sqrt{1 - \left(\frac{3}{5}\right)^2} \\ &= -\sqrt{1 - \frac{9}{25}} \\ &= -\sqrt{\frac{16}{25}} \\ &= -\frac{4}{5}\end{aligned}$$

$$\Rightarrow \cos\theta = -\frac{4}{5}$$

$$\text{and, } \tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{3}{5}}{-\frac{4}{5}} = -\frac{3}{4} \quad \text{--- (i)}$$

$$\text{Now, } \sec^2\phi - \tan^2\phi = 1$$

$$\Rightarrow \sec^2\phi = 1 + \tan^2\phi$$

$$\Rightarrow \sec\phi = \pm\sqrt{1 + \tan^2\phi}$$

In the third quadrant $\sec\phi$ is negative

$$\begin{aligned}\therefore \sec\phi &= -\sqrt{1 + \left(\frac{1}{2}\right)^2} \\ &= -\sqrt{1 + \frac{1}{4}}\end{aligned}$$

$$= -\sqrt{\frac{5}{4}}$$

$$\Rightarrow \sec \phi = -\frac{\sqrt{5}}{2} \quad \text{--- (ii)}$$

$$\begin{aligned}\therefore 8 \tan \theta - \sqrt{5} \sec \phi \\&= 8 \times \left(\frac{-3}{4}\right) - \sqrt{5} \times \left(-\frac{\sqrt{5}}{2}\right) \quad [\text{by equations (i) and (ii)}] \\&= -2 \times 3 + \frac{5}{2} \\&= -6 + \frac{5}{2} \\&= \frac{-12+5}{2} \\&= \frac{-7}{2}\end{aligned}$$

$$\therefore 8 \tan \theta - \sqrt{5} \sec \phi = -\frac{7}{2}$$

Trigonometric Functions Ex 5.2 Q 4

We have,

$$\sin \theta + \cos \theta = 0$$

$$\Rightarrow \sin \theta = -\cos \theta \quad \text{--- (i)}$$

$$\Rightarrow \frac{\sin \theta}{\cos \theta} = -1$$

$$\Rightarrow \tan \theta = -1$$

We know that,

$$\sec^2 \theta - \tan^2 \theta = 1$$

$$\Rightarrow \sec^2 \theta = 1 + \tan^2 \theta$$

$$\Rightarrow \sec \theta = \pm \sqrt{1 + \tan^2 \theta}$$

In the 4th quadrant $\sec \theta$ is positive.

$$\begin{aligned}\therefore \sec \theta &= \sqrt{1 + \tan^2 \theta} \\&= \sqrt{1 + (-1)^2} \\&= \sqrt{1+1} \\&= \sqrt{2}\end{aligned}$$

$$\therefore \cos \theta = \frac{1}{\sec \theta} = \frac{1}{\sqrt{2}}$$

putting $\cos \theta = \frac{1}{\sqrt{2}}$ in equation (i), we get,

$$\sin \theta = -\left(\frac{1}{\sqrt{2}}\right) = -\frac{1}{\sqrt{2}}$$

$$\text{Hence, } \sin \theta = -\frac{1}{\sqrt{2}} \text{ and } \cos \theta = \frac{1}{\sqrt{2}}.$$

Chapter 5 Trigonometric Functions Ex 5.2 Q 5.

We have,

$$\cos \theta = -\frac{3}{5}, \quad \text{and } \pi < \theta < \frac{3\pi}{2}$$

$\Rightarrow \theta$ lies in the 3rd quadrant

We know that,

$$\Rightarrow \sin \theta = \pm \sqrt{1 - \cos^2 \theta}$$

In the 3rd quadrant $\sin \theta$ is negative and $\tan \theta$ is positive.

$$\begin{aligned}\therefore \sin \theta &= -\sqrt{1 - \cos^2 \theta} \\&= -\sqrt{1 - \left(-\frac{3}{5}\right)^2} \quad \left[\because \cos \theta = -\frac{3}{5}\right] \\&= -\sqrt{1 - \frac{9}{25}}\end{aligned}$$

$$= -\sqrt{\frac{16}{25}} \\ = -\frac{4}{5}$$

$$\Rightarrow \sin \theta = -\frac{4}{5}$$

$$\text{and, } \tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{\frac{-4}{5}}{\frac{-3}{5}} = \frac{4}{3}$$

$$\text{Now, cosec} \theta = \frac{1}{\sin \theta} = \frac{1}{\frac{-4}{5}} = \frac{-5}{4}$$

$$\sec \theta = \frac{1}{\cos \theta} = \frac{1}{\frac{-3}{5}} = \frac{-5}{3}$$

$$\text{and, } \cot \theta = \frac{1}{\tan \theta} = \frac{1}{\frac{4}{3}} = \frac{3}{4}$$

$$\therefore \frac{\cosec \theta + \cot \theta}{\sec \theta - \tan \theta} = \frac{\frac{-5}{4} + \frac{3}{4}}{\frac{-5}{3} - \frac{4}{3}} \\ = \frac{\frac{-5+3}{4}}{\frac{-5-4}{3}} \\ = \frac{\frac{-2}{4}}{\frac{-9}{3}} \\ = \frac{\frac{2}{4} \times \frac{3}{9}}{}$$

$$= \frac{1}{6}$$

$$\therefore \frac{\cosec \theta + \cot \theta}{\sec \theta - \tan \theta} = \frac{1}{6}$$

