RD Sharma Solutions Class 11 Maths Chapter 5 Ex 5.1

Trigonometric Functions Ex 5.1 Q1

$$\text{LHS} = \sec^4\theta - \sec^2\theta$$
$$= \sec^2\theta(\sec^2\theta - 1)$$
$$= (1+\tan^2\theta)\tan^2\theta \qquad [\because \sec^2\theta = 1+\tan^2\theta]$$
$$= \tan^2\theta + \tan^4\theta$$
$$= \tan^4\theta + \tan^2\theta$$
$$= \text{RHS}$$

LHS = RHS

Proved

Trigonometric Functions Ex 5.1 Q2

$$\text{LHS} = \sin^6\theta + \cos^6\theta$$
$$= (\sin^2\theta)^3 + (\cos^2\theta)^3$$
$$= (\sin^2\theta + \cos^2\theta)\left[(\sin^2\theta)^2 - \sin^2\theta\cos^2\theta + (\cos^2\theta)^2\right] \qquad \left(\because a^3+b^3 = (a+b)(a^2-ab+b^2)\right)$$
$$= (\sin^2\theta)^2 + (\cos^2\theta)^2 + 2\sin^2\theta\cos^2\theta - 2\sin^2\theta\cos^2\theta - \sin^2\theta\cos^2\theta)$$

[adding and subtracting $2\sin^2\theta\cos^2\theta$ and using identity $\sin^2\theta+\cos^2\theta = 1$]

$$= (\sin^2\theta + \cos^2\theta)^2 - 3\sin^2\theta\cos^2\theta$$
$$= (\sin^2\theta + \cos^2\theta)^2 - 3\sin^2\theta\cos^2\theta$$
$$= 1^2 - 3\sin^2\theta\cos^2\theta \qquad (\because \sin^2\theta+\cos^2\theta = 1)$$
$$= 1 - 3\sin^2\theta\cos^2\theta$$
$$= \text{RHS}$$

$\therefore$ LHS = RHS

Proved

Trigonometric Functions Ex 5.1 Q3

$$\text{LHS} = (\operatorname{cosec}\theta - \sin\theta)(\sec\theta - \cos\theta)(\tan\theta + \cot\theta)$$
$$= \left(\frac{1}{\sin\theta} - \sin\theta\right)\left(\frac{1}{\cos\theta} - \cos\theta\right)\left(\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right)$$

$\left[\because \operatorname{cosec}\theta = \frac{1}{\sin\theta}, \sec\theta = \frac{1}{\cos\theta}, \tan\theta = \frac{\sin\theta}{\cos\theta} \text{ and } \cot\theta = \frac{\cos\theta}{\sin\theta}\right]$

$$= \left(\frac{1-\sin^2\theta}{\sin\theta}\right)\left(\frac{1-\cos^2\theta}{\cos\theta}\right)\left(\frac{\sin^2\theta+\cos^2\theta}{\cos\theta\sin\theta}\right)$$
$$= \frac{\cos^2\theta.\sin^2\theta.1}{\sin^2\theta.\cos^2\theta} \qquad \left(\begin{array}{l}\because \sin^2\theta + \cos^2\theta = 1 \\ \Rightarrow 1-\sin^2\theta = \cos^2\theta, \text{ and} \\ \qquad 1-\cos^2\theta = \sin^2\theta\end{array}\right)$$
$$= 1$$
$$= \text{RHS}$$

Proved

Trigonometric Functions Ex 5.1 Q4

$$\text{LHS} = \operatorname{cosec}\theta(\sec\theta - 1) - \cot\theta(1-\cos\theta)$$
$$= \frac{1}{\sin\theta}\left(\frac{1}{\cos\theta} - 1\right) - \frac{\cos\theta}{\sin\theta}(1-\cos\theta) \qquad \left[\because \operatorname{cosec}\theta = \frac{1}{\sin\theta}, \sec\theta = \frac{1}{\cos\theta}\cot\theta = \frac{\cos\theta}{\sin\theta}\right]$$
$$= \frac{(1-\cos\theta)}{\sin\theta\cos\theta} - \frac{\cos\theta(1-\cos\theta)}{\sin\theta}$$
$$= \frac{(1-\cos\theta) - \cos^2\theta(1-\cos\theta)}{\sin\theta\cos\theta}$$
$$= \frac{(1-\cos\theta)(1-\cos^2\theta)}{\sin\theta\cos\theta}$$
$$= \frac{(1-\cos\theta)\sin^2\theta}{\sin\theta\cos\theta} \qquad (\because 1-\cos^2\theta = \sin^2\theta)$$

$$= (1 - \cos\theta)\frac{\sin\theta}{\cos\theta}$$

$$= \frac{\sin\theta}{\cos\theta} - \sin\theta$$

$$= \tan\theta - \sin\theta \quad (\because \tan\theta = \sin\theta - \cos\theta)$$

= RHS

Proved

Trigonometric Functions Ex 5.1 Q5

$$\text{LHS} = \frac{1 - \sin A\cos A}{\cos A(\sec A - \operatorname{cosec} A)} \cdot \frac{\sin^2 A - \cos^2 A}{\sin^3 A + \cos^3 A}$$

$$= \frac{1 - \sin A\cos A}{\cos A\left(\frac{1}{\cos A} - \frac{1}{\sin A}\right)} \cdot \frac{(\sin A + \cos A)(\sin A - \cos A)}{(\sin A + \cos A)(\sin^2 A + \cos^2 A - \sin A\cos A)}$$

$$\left[\begin{array}{l}\text{Using } a^2 - b^2 = (a - b)(a + b) \\ \text{and } a^3 + b^3 = (a + b)(a^2 + b^2 - ab)\end{array}\right]$$

$$= \frac{(1 - \sin A\cos A)}{\cos A\left(\frac{\sin A - \cos A}{\cos A\sin A}\right)} \cdot \frac{(\sin A - \cos A)}{(1 - \sin A\cos A)} \quad (\because \sin^2 A + \cos^2 A = 1)$$

$$= \frac{\cos A\sin A}{\cos A}$$

$$= \sin A$$

= RHS

Proved

Trigonometric Functions Ex 5.1 Q6

$$\text{LHS} = \frac{\tan A}{1 - \cot A} + \frac{\cot A}{1 - \tan A}$$

$$= \frac{(\sin A / \cos A)}{\left(1 - \frac{\cos A}{\sin A}\right)} + \frac{(\cos A / \sin A)}{1 - \frac{\sin A}{\cos A}}$$

$$= \frac{\sin A}{\cos A\frac{(\sin A - \cos A)}{\sin A}} + \frac{\cos A}{\sin A\frac{(\cos A - \sin A)}{\cos A}}$$

$$= \frac{\sin^2 A}{\cos A(\sin A - \cos A)} + \frac{\cos^2 A}{\sin A(\cos A - \sin A)}$$

$$= \frac{\sin^3 A - \cos^3 A}{\cos A\sin A(\sin A - \cos A)}$$

$$= \frac{(\sin A - \cos A)(\sin^2 A + \cos^2 A + \sin A\cos A)}{\cos A\sin A(\sin A - \cos A)} \quad \left[\text{Using } a^3 - b^3 = (a - b)(a^2 + b^2 + ab)\right]$$

$$= \frac{1 + \sin A\cos A}{\sin A\cos A} \quad \left[\because \sin^2 A + \cos^2 A = 1\right]$$

$$= \frac{1}{\sin A\cos A} + \frac{\sin A\cos A}{\sin A\cos A}$$

$$= \sec A\operatorname{cosec} A + 1 \quad \left[\because \frac{1}{\cos A} = \sec A, \frac{1}{\sin A} = \operatorname{cosec} A\right]$$

= RHS

Proved

Trigonometric Functions Ex 5.1 Q7

$$\text{LHS} = \frac{\sin^3 A + \cos^3 A}{\ldots} + \frac{\sin^3 A - \cos^3 A}{\ldots}$$

$$\frac{\quad}{\sin A+\cos A} \quad \frac{\quad}{\sin A-\cos A}$$

$$=\frac{(\sin A+\cos A)(\sin^2 A+\cos^2 A-\sin A\cos A)}{(\sin A+\cos A)}+\frac{(\sin A-\cos A)(\sin^2 A+\cos^2}{\sin A-\cos A}$$

$$\left(\text{Using } a^3+b^3=(a+b)(a^2+b^2-ab) \text{ and } a^3-b^3=(a-b)(a^2+b^2+ab)\right)$$

$$=(1-\sin A\cos A)+(1+\sin A\cos A)\left(\because \sin^2 A+\cos^2 A=1\right)$$

$= 2$

$= \text{RHS}$

Trigonometric Functions Ex 5.1 Q8

$$\text{LHS}=(\sec A\sec B+\tan A\tan B)^2-(\sec A\tan B+\tan A\sec B)^2$$

$$=(\sec A\sec B)^2+(\tan A\tan B)^2+2\sec A\sec B\tan A\tan B$$

$$-\left((\sec A\tan B)^2+(\tan A\sec B)^2+2\sec A\tan B\tan A\sec B\right) \quad \left[\text{Using}(a+b)^2=a^2+b^2+2ab\right]$$

$$=\sec^2 A\sec^2 B+\tan^2 A\tan^2 B+2\sec A\sec B\tan A\tan B$$

$$-\sec^2 A\tan^2 B-\tan^2 A\sec^2 B-2\sec A\sec B\tan A\tan B\left[\text{Using } (ab)^2=a^2b^2\right]$$

$$=\sec^2 A\sec^2 B-\sec^2 A\tan^2 B+\tan^2 A\tan^2 B-\tan^2 A\sec^2 B$$

$$=\sec^2 A\left(\sec^2 B-\tan^2 B\right)+\tan^2 A\left(\tan^2 B-\sec^2 B\right)$$

$$=\sec^2 A\,1-\tan^2 A.1 \qquad \left[\begin{array}{l}\because \sec^2\theta=1+\tan^2\theta\\ \Rightarrow \sec^2\theta-\tan^2\theta=1\end{array}\right]$$

$$=1+\tan^2 A-\tan^2 A$$

$= 1$

$= \text{RHS}$

Proved

Trigonometric Functions Ex 5.1 Q9

$$\text{RHS}=\frac{1+\cos\theta+\sin\theta}{1+\cos\theta-\sin\theta}$$

$$=\frac{((1+\cos\theta)+\sin\theta)}{(1+\cos\theta)-\sin\theta}\times\frac{((1+\cos\theta)+\sin\theta)}{(1+\cos\theta+\sin\theta)}$$

$$=\frac{((1+\cos\theta)+\sin\theta)^2}{(1+\cos\theta)^2-\sin^2\theta} \qquad \left(\begin{array}{l}\text{Using } (a+b)(a+b)=(a+b)^2\\ \& (a+b)(a-b)=a^2b^2\end{array}\right)$$

$$=\frac{(1+\cos\theta)^2+\sin^2\theta+2\sin\theta(1+\cos\theta)}{1+\cos^2\theta+2\cos\theta-\sin^2\theta} \qquad \left(\text{Using } (a+b)^2=a^2+b^2+2ab\right)$$

$$=\frac{1+\cos^2\theta+2.1\cos\theta+\sin^2\theta+2\sin\theta(1+\cos\theta)}{1+\cos^2+2\cos\theta-(1-\cos^2\theta)} \qquad \left(\text{Using } \sin^2\theta=1-\cos^2\theta\right)$$

$$=\frac{1+1+2\cos\theta+2\sin\theta(1+\cos\theta)}{1-1+\cos^2\theta+\cos^2\theta+2\cos\theta} \qquad \left(\text{Using } \sin^2\theta+\cos^2\theta=1\right)$$

$$=\frac{2+2\cos\theta+2\sin\theta(1+\cos\theta)}{2\cos^2\theta+2\cos\theta}$$

$$=\frac{2(1+\cos\theta)+2\sin\theta(1+\cos\theta)}{2\cos\theta(\cos\theta+1)}$$

$$=\frac{(1+\cos\theta)(2+2\sin\theta)}{2\cos\theta(1+\cos\theta)}$$

$$=\frac{1+\sin\theta}{\cos\theta}$$

$$=\frac{1+\sin\theta}{\cos\theta}\times\frac{1-\sin\theta}{1-\sin\theta}$$

$$=\frac{\cos^2\theta}{\cos\theta(1-\sin\theta)}$$

$$=\frac{\cos\theta}{\quad}$$

$= \frac{}{1-\sin\theta}$

Trigonometric Functions Ex 5.1 Q10

$$\text{LHS} = \frac{\tan^3\theta}{1+\tan^2\theta} + \frac{\cot^3\theta}{1+\cot^2\theta}$$

$$= \frac{\sin^3\theta}{\cos^3\theta\left(1+\frac{\sin^2\theta}{\cos^2\theta}\right)} + \frac{\cos^3\theta}{\sin^3\theta\left(1+\frac{\cos^2\theta}{\sin^2\theta}\right)} \quad \left(\because \tan\theta = \frac{\sin\theta}{\cos\theta} \text{ and } \cot\theta = \frac{\cos\theta}{\sin\theta}\right)$$

$$= \frac{\sin^3\theta\cos^2\theta}{\cos^3\theta\left(\cos^2\theta+\sin^2\theta\right)} + \frac{\cos^3\theta\sin^2\theta}{\sin^3\theta\left(\sin^2\theta+\cos^2\theta\right)}$$

$$= \frac{\sin^3\theta}{\cos\theta} + \frac{\cos^3\theta}{\sin\theta} \quad \left(\because \cos^2\theta+\sin^2\theta = 1\right)$$

$$= \frac{\sin^4\theta+\cos^4\theta}{\sin\theta\cos\theta}$$

$$= \frac{\left(\sin^2\theta\right)^2+\left(\cos^2\theta\right)^2+2\sin^2\theta\cos^2\theta-2\sin^2\theta\cos^2\theta}{\sin\theta\cos\theta} \quad \left(\text{adding and subtracting } 2\sin^2\theta\cos^2\theta\right)$$

$$= \frac{\left(\sin^2\theta+\cos^2\theta\right)^2-2\sin^2\theta\cos^2\theta}{\sin\theta\cos\theta}$$

$$= \frac{1^2-2\sin^2\theta\cos^2\theta}{\sin\theta\cos\theta} \quad \left(\because \sin^2\theta+\cos^2\theta = 1\right)$$

$$= \frac{1-2\sin^2\theta\cos^2\theta}{\sin\theta\cos\theta}$$

= RHS

Proved

Trigonometric Functions Ex 5.1 Q11

$$\text{LHS} = 1 - \frac{\sin^2\theta}{1+\cot\theta} - \frac{\cos^2\theta}{1+\tan\theta}$$

$$= 1 - \frac{\sin^2\theta}{1+\frac{\cos\theta}{\sin\theta}} - \frac{\cos^2\theta}{1+\frac{\sin\theta}{\cos\theta}} \quad \left(\because \cot\theta = \frac{\cos\theta}{\sin\theta}, \tan\theta = \frac{\sin\theta}{\cos\theta}\right)$$

$$= 1 - \frac{\sin^2\theta}{\frac{\sin\theta+\cos\theta}{\sin\theta}} - \frac{\cos^2\theta}{\frac{\cos\theta+\sin\theta}{\cos\theta}}$$

$$= 1 - \frac{\sin^3\theta}{\sin\theta+\cos\theta} - \frac{\cos^3\theta}{\cos\theta+\sin\theta}$$

$$= \frac{\sin\theta+\cos\theta-\left(\sin^3+\cos^3\theta\right)}{\sin\theta+\cos\theta}$$

$$= \frac{\sin\theta+\cos\theta-\left(\sin\theta+\cos\theta\right)\left(\sin^2\theta+\cos^2\theta-\sin\theta\cos\theta\right)}{\sin\theta+\cos\theta}$$

$$\left(\text{Using } a^3+b^3 = (a+b)\left(a^2+b^2-ab\right)\right)$$

$$\frac{\left(\sin\theta+\cos\theta\right)\left(1-\left(1-\sin\theta\cos\theta\right)\right)}{\sin\theta+\cos\theta} \quad \left(\text{Using} \sin^2\theta+\cos^2\theta = 1\right)$$

$= \sin\theta\cos\theta$

= RHS

Proved

Trigonometric Functions Ex 5.1 Q12

$$\text{LHS} = \left(\frac{1}{\sec^2\theta - \cos^2\theta} + \frac{1}{\operatorname{cosec}^2\theta - \sin^2\theta}\right)\sin^2\theta\cos^2\theta$$

$$= \left(\frac{1}{\frac{1}{\cos^2\theta} - \cos^2\theta} + \frac{1}{\left(\frac{1}{\sin^2\theta} - \sin^2\theta\right)}\right)\sin^2\theta\cos^2\theta$$

$$= \left(\frac{1}{\frac{1-\cos^4\theta}{\cos^2\theta}} + \frac{1}{\frac{1-\sin^4\theta}{\sin^2\theta}}\right)\sin^2\theta\cos^2\theta$$

$$= \left(\frac{\cos^2\theta}{\left(1-\cos^2\theta\right)\left(1+\cos^2\theta\right)} + \frac{\sin^2\theta}{\left(1-\sin^2\theta\right)\left(1+\sin^2\theta\right)}\right)\sin^2\theta\cos^2\theta$$

$$\left(\begin{aligned}\text{Using } 1-a^4 &= 1-\left(a^2\right)^2\\ &= \left(1-a^2\right)\left(1+a^2\right)\end{aligned}\right)$$

$$= \left(\frac{\cos^2\theta}{\sin^2\theta\left(1+\cos^2\theta\right)} + \frac{\sin^2\theta}{\cos^2\theta\left(1+\sin^2\right)}\right)\sin^2\theta\cos^2\theta \qquad \left(\begin{aligned}&\text{Using } 1-\cos^2\theta = \sin^2\theta\\ &\&\quad 1-\sin^2\theta = \cos^2\theta\end{aligned}\right)$$

$$= \left(\frac{\cos^4\theta\left(1+\sin^2\theta\right) + \sin^4\theta\left(1+\cos^2\theta\right)}{\sin^2\theta\cos^2\theta\left(1+\cos^2\theta\right)\left(1+\sin^2\theta\right)}\right).\sin^2\theta\cos^2\theta$$

$$= \frac{\cos^4\theta + \sin^2\theta\cos^4\theta + \sin^4\theta + \cos^2\theta\sin^4\theta}{\left(1+\cos^2\theta\right)\left(1+\sin^2\theta\right)}$$

$$= \frac{\left(\cos^2\theta\right)^2 + \left(\sin^2\theta\right)^2 + 2\cos^2\theta\sin^2\theta - 2\cos^2\theta\sin^2\theta + \sin^2\theta\cos^4\theta + \cos^2\theta\sin^4\theta}{\left(1+\cos^2\theta\right)\left(1+\sin^2\theta\right)}$$

$$\left(\text{adding and subtracting } 2\cos^2\theta\sin^2\theta\right)$$

$$= \frac{\left(\cos^2\theta + \sin^2\theta\right)^2 - 2\cos^2\theta\sin^2\theta + \sin^2\theta\cos^2\theta\left(\cos^2\theta + \sin^2\theta\right)}{1+\sin^2\theta + \cos^2\theta + \sin^2\theta\cos^2\theta}$$

$$= \frac{1^2 - 2\cos^2\theta\sin^2\theta + \sin^2\theta\cos^2\theta.1}{1+1+\sin^2\theta\cos^2\theta}$$

$$= \frac{1-\sin^2\theta\cos^2\theta}{2+\sin^2\theta\cos^2\theta}$$

= RHS

Proved

Trigonometric Functions Ex 5.1 Q13

$$\text{LHS} = \left(1+\tan\alpha\tan\beta\right)^2 + \left(\tan\alpha - \tan\beta\right)^2$$

$$= 1 + \left(\tan\alpha + \tan\beta\right)^2 + 2.1\tan\alpha\tan\beta + \left(\tan\alpha\right)^2 + \left(\tan\beta\right)^2 - 2\tan\alpha.\tan\beta$$

$$\left(\text{Using } \left(a+b\right)^2 = a^2+b^2+2ab \text{ and } \left(a-b\right)^2 = a^2+b^2-2ab\right)$$

$$= 1+\tan^2\alpha + \tan^2\beta + 2\tan\alpha\tan\beta + \tan^2\alpha + \tan^2\beta - 2\tan\alpha\tan\beta$$

$$= 1+\tan^2\alpha + \tan^2\alpha + \tan^2\beta + \tan^2\beta$$

$$= \sec^2\alpha + \tan^2\beta\left(1+\tan^2\alpha\right) \qquad \left(\because 1+\tan^2\alpha = \sec^2\alpha\right)$$

$$= \sec^2\alpha + \tan^2\beta.\sec^2\alpha$$

$$= \sec^2\alpha\left(1+\tan^2\beta\right)$$

$= \sec^2\alpha . \sec^2\beta \qquad \left(\because 1+\tan^2\beta = \sec^2\beta\right)$

$= \text{RHS}$

Proved

Trigonometric Functions Ex 5.1 Q14

$$\text{LHS} = \frac{(1+\cot\theta+\tan\theta)(\sin\theta-\cos\theta)}{\sec^3\theta-\operatorname{cosec}^3\theta}$$

$$= \frac{\left(1+\frac{\cos\theta}{\sin\theta}+\frac{\sin\theta}{\cos\theta}\right)}{\left(\frac{1}{\cos^3\theta}-\frac{1}{\sin^3\theta}\right)} \qquad \left(\begin{array}{l}\because \cot\theta = \frac{\cos\theta}{\sin\theta}, \tan\theta = \frac{\sin\theta}{\cos\theta} \\ \sec\theta = \frac{1}{\cos\theta}, \operatorname{cosec}\theta = \frac{1}{\sin\theta}\end{array}\right)$$

$$= \left(\frac{1+\frac{\cos^2\theta+\sin^2\theta}{\sin\theta\cos\theta}}{\frac{\sin^3\theta-\cos^3\theta}{\cos^3\theta\sin^3\theta}}\right)(\sin\theta-\cos\theta)$$

$$= \frac{(\sin\theta\cos\theta+1)\sin^3\theta\cos^3\theta}{\sin\theta\cos\theta.\left(\sin^3\theta-\cos^3\theta\right)}.(\sin\theta-\cos\theta) \qquad \left(\because \sin^2\theta+\cos^2\theta = 1\right)$$

$$= \frac{(1+\sin\theta\cos\theta)\sin^2\theta\cos^2\theta.(\sin\theta-\cos\theta)}{(\sin\theta-\cos\theta)\left(\sin^2\theta+\cos^2\theta+\sin\theta\cos\theta\right)} \qquad \left(\because a^3-b^3 = (a-b)\left(a^2+b^2+ab\right)\right)$$

$$= \frac{(1+\sin\theta\cos\theta).\sin^2\theta\cos^2\theta}{(1+\sin\theta\cos\theta)}$$

$= \sin^2\theta\cos^2\theta$

$= \text{RHS}$

Proved

Trigonometric Functions Ex 5.1 Q15

$$\text{LHS} = \frac{2\sin\theta\cos\theta-\cos\theta}{1-\sin\theta+\sin^2\theta-\cos^2\theta}$$

$$= \frac{\cos\theta(2\sin\theta-1)}{1-\cos^2\theta+\sin^2\theta-\sin\theta}$$

$$= \frac{\cos\theta(2\sin\theta-1)}{\sin^2\theta+\sin^2\theta-\sin\theta} \qquad \left(\because 1-\cos^2\theta = \sin^2\theta\right)$$

$$= \frac{\cos\theta(2\sin\theta-1)}{2\sin^2\theta-\sin\theta}$$

$$= \frac{\cos\theta(2\sin\theta-1)}{\sin\theta(2\sin\theta-1)}$$

$$= \frac{\cos\theta}{\sin\theta}$$

$= \cot\theta$

$= \text{RHS}$

Proved

Trigonometric Functions Ex 5.1 Q16

$\text{LHS} = \cos\theta\,(\tan\theta+2)(2\tan\theta+1)$

$$= \cos\theta\left(\frac{\sin\theta}{\cos\theta}+2\right)\left(\frac{2\sin\theta}{\cos\theta}+1\right)\left(\because \tan\theta = \frac{\sin\theta}{\cos\theta}\right)$$

$$= \cos\frac{(\sin\theta+2\cos\theta)(2\sin\theta+\cos\theta)}{\cos\theta.\cos\theta}$$

$$\left(2\sin^2\theta+\sin\theta\cos\theta+4\sin\theta\cos\theta+2\cos^2\theta\right)$$

$= \frac{[illegible]}{\cos\theta}$

$= \frac{2\left(\sin^2\theta + \cos^2\theta\right) + 5\sin\theta\cos\theta}{\cos\theta}$

$= \frac{2 + 5\sin\theta\cos\theta}{\cos\theta} \quad \left(\because \sin^2\theta + \cos^2\theta\right) = 1$

$= \frac{2}{\cos\theta} + \frac{5\sin\theta\cos\theta}{\cos\theta}$

$= 2\sec\theta + 5\sin\theta$

= RHS

Proved

Trigonometric Functions Ex 5.1 Q17

$$\frac{2\sin\theta}{1+\cos\theta+\sin\theta} = x$$

$$\Rightarrow \frac{2\sin\theta(1-\cos\theta+\sin\theta)}{(1+\cos\theta+\sin\theta)(1-\cos\theta+\sin\theta)} = x \quad [\textit{Rationalizing the denominator}]$$

$$\Rightarrow \frac{2\sin\theta(1-\cos\theta+\sin\theta)}{(1+\sin\theta)^2-\cos^2\theta} = x$$

$$\Rightarrow \frac{2\sin\theta - 2\sin\theta\cos\theta + 2\sin^2\theta}{1+\sin^2\theta+2\sin\theta-\cos^2\theta} = x$$

$$\Rightarrow \frac{2\sin\theta(1+\cos\theta-\sin\theta)}{2\sin^2\theta+2\sin\theta} = x$$

$$\Rightarrow \frac{2\sin\theta(1+\cos\theta-\sin\theta)}{2\sin\theta(1+\sin\theta)} = x$$

$$\Rightarrow \frac{1+\cos\theta-\sin\theta}{1+\sin\theta} = x \ [\textit{Cancelling the } 2\sin\theta \textit{ in both Numerator and Denominator}]$$

Hence Proved

Trigonometric Functions Ex 5.1 Q18

Now, $\cos\theta = \sqrt{1-\sin^2\theta}$

$= \sqrt{\frac{1-\left(a^2-b^2\right)^2}{\left(a^2+b^2\right)^2}} \qquad \left[\because \sin\theta = \frac{a^2-b^2}{a^2+b^2}\right]$

$= \sqrt{\frac{\left(a^2+b^2\right)^2-\left(a^2-b^2\right)^2}{\left(a^2+b^2\right)^2}}$

$= \sqrt{\frac{\left(a^2+b^2+a^2-b^2\right)\left(a^2+b^2-a^2+b^2\right)}{a^2+b^2}} \quad \left(\text{Using } x^2-y^2 = (x-y)(x+y)\right)$

$= \sqrt{\frac{2a^2\times 2b^2}{a^2+b^2}}$

$= \frac{2ab}{a^2+b^2}$ ....(ii)

Now $\tan\theta = \frac{\sin\theta}{\cos\theta}$

$= \frac{\frac{a^2-b^2}{a^2+b^2}}{\frac{2ab}{a^2+b^2}}$

$= \frac{a^2-b^2}{2ab}$

$\ldots\ 1 \quad a^2+b^2 \quad \ldots$

$\sec\theta = \frac{}{\cos\theta} = \frac{}{2ab}$ (from (ii))

and $\operatorname{cosec}\theta = \frac{1}{\sin\theta} = \frac{a^2+b^2}{a^2-b^2}$ (from (i))

Trigonometric Functions Ex 5.1 Q19

$$\sqrt{\frac{a+b}{a-b}} + \sqrt{\frac{a-b}{a+b}}$$

$$= \sqrt{\frac{\frac{a}{b}+1}{\frac{a}{b}-1}} + \sqrt{\frac{\frac{a}{b}-1}{\frac{a}{b}+1}} \quad [\textit{Dividing both Numerator and denominator by } b]$$

$$= \sqrt{\frac{\tan\theta+1}{\tan\theta-1}} + \sqrt{\frac{\tan\theta-1}{\tan\theta+1}}$$

$$= \sqrt{\frac{\frac{\sin\theta}{\cos\theta}+1}{\frac{\sin\theta}{\cos\theta}-1}} + \sqrt{\frac{\frac{\sin\theta}{\cos\theta}-1}{\frac{\sin\theta}{\cos\theta}+1}}$$

$$= \sqrt{\frac{\frac{\sin\theta+\cos\theta}{\cos\theta}}{\frac{\sin\theta-\cos\theta}{\cos\theta}}} + \sqrt{\frac{\frac{\sin\theta-\cos\theta}{\cos\theta}}{\frac{\sin\theta+\cos\theta}{\cos\theta}}}$$

$$= \sqrt{\frac{\sin\theta+\cos\theta}{\sin\theta-\cos\theta}} + \sqrt{\frac{\sin\theta-\cos\theta}{\sin\theta+\cos\theta}}$$

$$= \frac{\sin\theta+\cos\theta+\sin\theta-\cos\theta}{\sqrt{\sin^2\theta-\cos^2\theta}}$$

$$= \frac{2\sin\theta}{\sqrt{\sin^2\theta - \cos^2\theta}}$$

Trigonometric Functions Ex 5.1 Q20

Given $= \tan\theta = \frac{a}{b}$

To show: $\frac{a\sin\theta - b\cos\theta}{a\sin\theta + b\cos\theta} = \frac{a^2-b^2}{a^2+b^2}$

Since, $\tan\theta = \frac{a}{b}$

$\Rightarrow \quad \frac{\sin\theta}{\cos\theta} = \frac{a}{b}$

$\Rightarrow \quad b\sin\theta = a\cos\theta = \lambda$ (say)

$\Rightarrow \quad \sin\theta = \frac{\lambda}{b}$ and $\cos\theta = \frac{\lambda}{a}$

how $\quad \frac{a\sin\theta - b\cos\theta}{a\sin\theta + b\cos\theta} = \frac{\frac{a.\lambda}{b} - \frac{b.\lambda}{a}}{\frac{a.\lambda}{b} + \frac{b.\lambda}{a}}$

$$= \frac{\lambda\left(\frac{a}{b}-\frac{b}{a}\right)}{\lambda\left(\frac{a}{b}+\frac{b}{a}\right)}$$

$$= \frac{\frac{a}{b}-\frac{b}{a}}{\frac{a}{b}+\frac{b}{a}}$$

$$= \frac{\frac{a^2-b^2}{ab}}{\frac{a^2+b^2}{ab}}$$

$$= \frac{a^2-b^2}{}$$

$= a^2 + b^2$

Proved

Trigonometric Functions Ex 5.1 Q21

Given, $\cos ec\theta - \sin\theta = a^3, \sec\theta - \cos\theta = b^3$

To show: $a^2b^2\left(a^2+b^2\right) = 1$

Since, $\cos ec\theta - \sin\theta = a^3$

$$\Rightarrow \quad \frac{1}{\sin\theta} - \sin\theta = a^3 \qquad \left(\because \cos ec\theta = \frac{1}{\sin\theta}\right)$$

$$\Rightarrow \quad \frac{1-\sin^2\theta}{\sin\theta} = a^3$$

$$\Rightarrow \quad \frac{\cos^2\theta}{\sin\theta} = a^3 \qquad \left(\because 1-\sin^2\theta = \cos^2\theta\right)$$

$$\Rightarrow \quad a = \frac{\cos^{2/3}\theta}{\sin^{1/3}\theta}$$

Since, $\dfrac{1}{\cos\theta} - \cos\theta = b^3 \qquad \left(\because \sec\theta = \dfrac{1}{\cos\theta}\right)$

$$\Rightarrow \quad \frac{1-\cos^2\theta}{\cos\theta} = b^3$$

$$\Rightarrow \quad \frac{\sin^2\theta}{\cos\theta} = b^3 \qquad \left(\because 1-\cos^2\theta = \sin^2\theta\right)$$

$$\Rightarrow \quad b = \frac{\sin^{2/3}\theta}{\cos^{1/3}\theta}$$

Now, $a^2b^2\left(a^2+b^2\right) = \dfrac{\cos^{4/3}\theta}{\sin^{2/3}\theta} \times \dfrac{\sin^{4/3}\theta}{\cos^{2/3}\theta}\left(\dfrac{\cos^{4/3}\theta}{\sin^{2/3}\theta} + \dfrac{\sin^{4/3}\theta}{\cos^{2/3}\theta}\right)$

$$= \cos^{2/3}\theta \times \sin^{2/3}\theta \frac{\left(\cos^{6/3}\theta + \sin^{6/3}\theta\right)}{\sin^{2/3}\theta.\cos^{2/3}\theta}$$

$$= \cos^2\theta + \sin^2\theta$$

$$= 1$$

Proved

Trigonometric Functions Ex 5.1 Q22

Let,

$\cot\theta\left(1+\sin\theta\right) = 4m \qquad$ ---(i)

and, $\cot\theta\left(1-\sin\theta\right) = 4n \qquad$ ---(ii)

To show: $\left(m^2-n^2\right)^2 = mn$

From (i) and(ii), we get

$$m = \frac{\cot\theta\left(1+\sin\theta\right)}{4} \quad \& \quad n = \frac{\cot\theta\left(1-\sin\theta\right)}{4}$$

LHS $= \left(m^2-n^2\right)^2$

$= \left(\left(m+n\right)\left(m-n\right)\right)^2$

$= \left(m+n\right)^2\left(m-n\right)^2$

$$= \left(\frac{\cot\theta\left(1+\sin\theta\right)+\cot\theta\left(1-\sin\theta\right)}{4}\right)^2\left(\frac{\cot\theta\left(1+\sin\theta\right)-\cot\theta\left(1-\sin\theta\right)}{4}\right)^2$$

$$= \left(\frac{\cot\theta\left(1+\sin\theta+1-\sin\theta\right)}{4}\right)^2 \times \left(\frac{\cot\theta\left(1+\sin\theta-1+\sin\theta\right)}{4}\right)^2$$

$\cot^2\theta \quad \cot^2\theta$

$$= \frac{\cot^2\theta}{16} \times 4 \times \frac{\cot^2\theta}{16} \times 4\sin^2\theta$$

$$= \frac{\cot^2\theta}{16} \times \frac{\cos^2\theta}{\sin^2\theta}\sin^2\theta \qquad \left[\because \cot\theta = \frac{\cos\theta}{\sin\theta}\right]$$

$$= \frac{\cot\theta}{4} \times \frac{\cot\theta}{4} \times \left(1-\sin^2\theta\right) \qquad \left[\because \cos^2\theta = 1-\sin^2\theta\right]$$

$$= \frac{\cot\theta\left(1+\sin\theta\right)}{4} \times \frac{\cot\theta\left(1-\sin\theta\right)}{4}$$

$$= mn$$

Trigonometric Functions Ex 5.1 Q23

To show: $sin^6\theta + cos^6\theta = \frac{4-3\left(m^2-1\right)^2}{4}$, where $m^2 \le 2$

$Since,\ sin\theta + cos\theta = m$ ...(i)

$\Rightarrow \quad \left(sin\theta + cos\theta\right)^2 = m^2$

$\Rightarrow \quad sin^2\theta + cos^2\theta + 2\,sin\theta cos\theta = m^2$

$\Rightarrow \quad 1 + 2\,sin\theta cos\theta = m^2 \quad \left(\because sin^2\theta + cos^2\theta = 1\right)$

$\Rightarrow \quad 2\,sin\theta cos\theta = m^2 - 1$

$\Rightarrow \quad sin\theta cos\theta = \frac{m^2-1}{2}$ ...(ii)

$\therefore$ LHS $= sin^6\theta + cos^2\theta$

$= \left(sin^2\theta\right)^3 + \left(cos^2\theta\right)^3$

$= \left(sin^2\theta + cos^2\theta\right)\left(\left(sin^2\theta\right)^2 + \left(cos^2\theta\right)^2 - sin^2\theta cos^2\theta\right)$

$= 1.\left(\left(sin^2\theta\right)^2 + \left(cos^2\theta\right)^2 + 2\,sin^2\theta cos^2\theta - 2\,sin^2\theta cos^2\theta - sin^2\theta cos^2\theta\right)$

(adding and subtracting $2\,sin^2\theta cos^2\theta$)

$= \left(sin^2\theta + cos^2\theta\right)^2 - 3\,sin^2\theta cos^2\theta$

$= 1 - 3\,sin^2\theta cos^2\theta$

$= 1 - 3\left(sin\theta cos\theta\right)^2$

$= 1 - 3\frac{\left(m^2-1\right)^2}{4}$ (from (ii))

$= \frac{4-3\left(m^2-1\right)^2}{4}$, where $m^2 \le 2$

= RHS

Proved

Trigonometric Functions Ex 5.1 Q24

$$LHS = ab + a - b + 1$$

$$= (sec\ \theta - \tan\ \theta)(cosec\ + cot\ ) + sec\ \theta - \tan\ \theta - cosec\ \theta - cot\ \theta + 1$$

$$= \left(\frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}\right)\left(\frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta}\right) + \frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta} - \frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta} + 1$$

$$= \frac{1}{\sin\theta\cos\theta} + \frac{1}{\cos\theta} \times \frac{\cos\theta}{\sin\theta} - \frac{\sin\theta}{\cos\theta} \times \frac{1}{\sin\theta} - \tan\theta \times cot\ \theta + \frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta} - \frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta} + 1$$

$$= \frac{1}{\sin\theta\cos\theta} + \frac{1}{\sin\theta} - \frac{1}{\cos\theta} - 1 + \frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta} - \frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta} + 1$$

$$= \frac{1}{\sin\theta\cos\theta} - \frac{\sin\theta}{\cos\theta} - \frac{\cos\theta}{\sin\theta}$$

$$= \frac{1}{\sin\theta\cos\theta} - \frac{\sin\theta}{\cos\theta} - \frac{\cos\theta}{\sin\theta}$$

$$= \frac{1-\sin^2\theta-\cos^2\theta}{\sin\theta.\cos\theta}$$

$$= \frac{1-(\cos^2\theta+\sin^2\theta)}{\sin\theta.\cos\theta}$$

$$= \frac{1-1}{\sin\theta.\cos\theta} = 0 = RHS.\ Hence\ Proved$$

Trigonometric Functions Ex 5.1 Q25

$LHS = \left|\sqrt{1-sin\,\theta} \quad \sqrt{1+sin\,\theta}\right|$

LHS = $\left|\sqrt{\frac{1-\sin\theta}{1+\sin\theta}}+\sqrt{\frac{1+\sin\theta}{1-\sin\theta}}\right|$

$= \left|\frac{\left(\sqrt{1-\sin\theta}\right)^2+\left(\sqrt{1+\sin\theta}\right)^2}{\sqrt{(1+\sin\theta)(1-\sin\theta)}}\right|$

$= \left|\frac{1-\sin\theta+1+\sin\theta}{\sqrt{1-\sin^2\theta}}\right|$

$= \left|\frac{2}{\cos\theta}\right| \qquad \left(\because\ 1-\sin^2\theta=\cos^2\theta \ \Rightarrow \sqrt{1-\sin^2\theta}=\cos\theta\right)$

$= \frac{-2}{\cos\theta} \qquad \left(\because \frac{\pi}{2}<\theta<\pi \Rightarrow \cos\theta<0\right)$

= RHS

RD Sharma Solutions Class 11 Maths Chapter 5 Ex 5.2

Trigonometric Functions Ex 5.2 Q 1

We have,

$$\operatorname{cosec}^2\theta - \cot^2\theta = 1$$

$$\Rightarrow \quad \operatorname{cosec}^2\theta = 1 + \cot^2\theta$$

$$\Rightarrow \quad \operatorname{cosec}\theta = \pm\sqrt{1+\cot^2\theta}$$

In the third quadrant $\operatorname{cosec}\theta$ is negative

$$\therefore \quad \operatorname{cosec}\theta = -\sqrt{1+\cot^2\theta}$$

$$= -\sqrt{1+\left(\frac{12}{5}\right)^2} \qquad \left[\because \quad \cot\theta = \frac{12}{5}\right]$$

$$= -\sqrt{1+\frac{144}{25}}$$

$$= -\sqrt{\frac{169}{25}}$$

$$= -\frac{13}{5}$$

$$\therefore \quad \operatorname{cosec}\theta = -\frac{13}{5}$$

Now, $\tan\theta = \dfrac{1}{\cot\theta}$

$$= \frac{1}{\frac{12}{5}}$$

$$= \frac{5}{12}$$

We have,

$$\sin^2\theta + \cos^2\theta = 1$$

$$\Rightarrow \quad \sin^2\theta = 1 - \cos^2\theta$$

$$\Rightarrow \quad \sin\theta = \pm\sqrt{1-\cos^2\theta}$$

In the 2^{nd} quadrant $\sin\theta$ is positive and $\tan\theta$ is negative

$$\therefore \quad \sin\theta = \sqrt{1-\cos^2\theta}$$

$$= \sqrt{1-\left(-\frac{1}{2}\right)^2} \qquad \left[\because \quad \cos\theta = -\frac{1}{2}\right]$$

$$= \sqrt{1-\frac{1}{4}}$$

$$= \sqrt{\frac{3}{4}}$$

$$= \frac{\sqrt{3}}{2}$$

and, $\tan\theta = \dfrac{\sin\theta}{\cos\theta} = \dfrac{\frac{\sqrt{3}}{2}}{-\frac{1}{2}} = -\sqrt{3}$

Now, $\operatorname{cosec}\theta = \dfrac{1}{\sin\theta} = \dfrac{1}{\frac{\sqrt{3}}{2}} = \dfrac{2}{\sqrt{3}}$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{-\frac{1}{2}} = -2$$

and $$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{-\sqrt{3}} = \frac{-1}{\sqrt{3}}$$

Hence, $\sin\theta = \frac{\sqrt{3}}{2}$, $\tan\theta = -\sqrt{3}$,

$\operatorname{cosec}\theta = \frac{2}{\sqrt{3}}$, $\sec\theta = -2$ and $\cot\theta = \frac{-1}{\sqrt{3}}$

In the third quadrant $\operatorname{cosec}\theta$ is negative

$$\therefore \quad \operatorname{cosec}\theta = -\sqrt{1+\cot^2\theta}$$
$$= -\sqrt{1+\left(\frac{4}{3}\right)^2}$$
$$= -\sqrt{1+\frac{16}{9}}$$
$$= -\sqrt{\frac{25}{9}}$$
$$= -\frac{5}{3}$$

Now, $$\sin\theta = \frac{1}{\operatorname{cosec}\theta} = \frac{1}{\frac{-5}{3}} = \frac{-3}{5}$$

and, $$\cos\theta = \frac{1}{\sec\theta} = \frac{1}{\frac{-5}{4}} = \frac{-4}{5}$$

Hence, $\sin\theta = \frac{-3}{5}$, $\cos\theta = \frac{-4}{5}$,

$\operatorname{cosec}\theta = -\frac{5}{3}$, $\sec\theta = \frac{-5}{4}$ and $\cot\theta = \frac{4}{3}$

We have,

$$\sin^2\theta + \cos^2\theta = 1$$

$$\Rightarrow \quad \cos^2\theta = 1 - \sin^2\theta$$
$$\Rightarrow \quad \cos\theta = \pm\sqrt{1-\sin^2\theta}$$

In the 1st quadrant $\cos\theta$ is positive and $\tan\theta$ is also positive

$$\therefore \quad \cos\theta = \sqrt{1-\sin^2\theta}$$
$$= \sqrt{1-\left(\frac{3}{5}\right)^2} \qquad \left[\because \quad \sin\theta = \frac{3}{5}\right]$$
$$= \sqrt{1-\frac{9}{25}}$$
$$= \sqrt{\frac{16}{25}}$$
$$= \frac{4}{5}$$

and, $$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{3}{5}}{\frac{4}{5}} = \frac{3}{4}$$

Now, $$\operatorname{cosec}\theta = \frac{1}{\sin\theta} = \frac{1}{\frac{3}{5}} = \frac{5}{3}$$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{\frac{4}{5}} = \frac{5}{4}$$

and, $$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{\frac{3}{4}} = \frac{4}{3}$$

Hence, $\cos\theta = \frac{4}{5}$, $\operatorname{cosec}\theta = \frac{5}{3}$, $\tan\theta = \frac{3}{4}$,

$\sec\theta = \frac{5}{4}$, and $\cot\theta = \frac{4}{3}$

Trigonometric Functions Ex 5.2 Q 2

We have,

$$\sin^2\theta + \cos^2\theta = 1$$

$$\Rightarrow \quad \cos^2\theta = 1 - \sin^2\theta$$
$$\Rightarrow \quad \cos\theta = \pm\sqrt{1-\sin^2\theta}$$

In the 2^{st} quadrant $\cos\theta$ is negative and $\tan\theta$ is also negative

$\therefore \quad \cos\theta = -\sqrt{1-\sin^2\theta}$

$= -\sqrt{1-\left(\frac{12}{13}\right)^2} \quad \left[\because \quad \sin\theta = \frac{12}{13}\right]$

$= -\sqrt{1-\frac{144}{169}}$

$= -\sqrt{\frac{25}{169}}$

$= -\frac{5}{13}$

and, $\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{12}{13}}{\frac{-5}{13}} = -\frac{13}{5}$

Now, $\sec\theta = \frac{1}{\cos\theta} = \frac{1}{\frac{-5}{13}} = -\frac{13}{5}$

$\therefore \quad \sec\theta + \tan\theta = -\frac{13}{5} - \frac{12}{5}$

$= \frac{-13-12}{5}$

$= -\frac{25}{5}$

$= -5$

$\Rightarrow \quad \sec\theta + \tan\theta = -5$

Trigonometric Functions Ex 5.2 Q 3

We have,

$\sin\theta = \frac{3}{5}, \quad \tan\phi = \frac{1}{2} \quad \text{and} \frac{\pi}{2} < \theta < \pi < \frac{3\pi}{2}$

$\Rightarrow$ θ lies in the second quadrant and ϕ lies in the third quadrant.

Now, $\sin^2\theta + \cos^2\theta = 1$

$\Rightarrow \quad \cos^2\theta = 1 - \sin^2\theta$

$\Rightarrow \quad \cos\theta = \pm\sqrt{1-\sin^2\theta}$

In the 2^{st} quadrant $\cos\theta$ is negative and $\tan\theta$ is also negative

$\therefore \quad \cos\theta = -\sqrt{1-\sin^2\theta}$

$= -\sqrt{1-\left(\frac{3}{5}\right)^2}$

$= -\sqrt{1-\frac{9}{25}}$

$= -\sqrt{\frac{16}{25}}$

$= -\frac{4}{5}$

$\Rightarrow \quad \cos\theta = -\frac{4}{5}$

and, $\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{3}{5}}{\frac{-4}{5}} = -\frac{3}{4}$ - - - - - (i)

Now, $\sec^2\phi - \tan^2\phi = 1$

$\Rightarrow \quad \sec^2\phi = 1 + \tan^2\phi$

$\Rightarrow \quad \sec\phi = \pm\sqrt{1+\tan^2\phi}$

In the third quadrant $\sec\phi$ is negative

$\therefore \quad \sec\phi = -\sqrt{1+\left(\frac{1}{2}\right)^2}$

$= -\sqrt{1+\frac{1}{4}}$

$= -\sqrt{\frac{5}{4}}$

$\Rightarrow \quad \sec\phi = -\frac{\sqrt{5}}{2}$ - - - - - - (ii)

$\therefore \quad 8\tan\theta - \sqrt{5}\sec\phi$

$= 8 \times \left(\frac{-3}{4}\right) - \sqrt{5} \times \left(-\frac{\sqrt{5}}{2}\right)$ [by equations (i) and (ii)]

$= -2 \times 3 + \frac{5}{2}$

$= -6 + \frac{5}{2}$

$= \frac{-12+5}{2}$

$= \frac{-7}{2}$

$\therefore \quad 8\tan\theta - \sqrt{5}\sec\phi = -\frac{7}{2}$

Trigonometric Functions Ex 5.2 Q 4

We have,

$\sin\theta + \cos\theta = 0$

$\Rightarrow \quad \sin\theta = -\cos\theta$ - - - - - - (i)

$\Rightarrow \quad \frac{\sin\theta}{\cos\theta} = -1$

$\Rightarrow \quad \tan\theta = -1$

We know that,

$\sec^2\theta - \tan^2\theta = 1$

$\Rightarrow \quad \sec^2\theta = 1 + \tan^2\theta$

$\Rightarrow \quad \sec\theta = \pm\sqrt{1+\tan^2\theta}$

In the 4th quadrant $\sec\theta$ is positive.

$\therefore \quad \sec\theta = \sqrt{1+\tan^2\theta}$

$= \sqrt{1+(-1)^2}$

$= \sqrt{1+1}$

$= \sqrt{2}$

$\therefore \quad \cos\theta = \frac{1}{\sec\theta} = \frac{1}{\sqrt{2}}$

putting $\cos\theta = \frac{1}{\sqrt{2}}$ in equation (i), we get,

$\sin\theta = -\left(\frac{1}{\sqrt{2}}\right) = -\frac{1}{\sqrt{2}}$

Hence, $\sin\theta = -\frac{1}{\sqrt{2}}$ and $\cos\theta = \frac{1}{\sqrt{2}}$.

Chapter 5 Trigonometric Functions Ex 5.2 Q 5.

We have,

$\cos\theta = -\frac{3}{5}$, and $\pi < \theta < \frac{3\pi}{2}$

$\Rightarrow \quad \theta$ lies in the 3rd quadrant

We know that,

$\Rightarrow \quad \sin\theta = \pm\sqrt{1-\cos^2\theta}$

In the 3rd quadrant $\sin\theta$ is negative and $\tan\theta$ is positive.

$\therefore \quad \sin\theta = -\sqrt{1-\cos^2\theta}$

$= -\sqrt{1-\left(-\frac{3}{5}\right)^2}$ $\left[\because \quad \cos\theta = -\frac{3}{5}\right]$

$= -\sqrt{1-\frac{9}{25}}$

$$= -\sqrt{\frac{16}{25}}$$

$$= -\frac{4}{5}$$

$\Rightarrow \quad \sin\theta = -\frac{4}{5}$

and, $\quad \tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{-4}{5}}{\frac{-3}{5}} = \frac{4}{5}$

Now, $\quad \operatorname{cosec}\theta = \frac{1}{\sin\theta} = \frac{1}{\frac{-4}{5}} = \frac{-5}{4}$

$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{\frac{-3}{5}} = \frac{-5}{3}$

and, $\quad \cot\theta = \frac{1}{\tan\theta} = \frac{1}{\frac{4}{3}} = \frac{3}{4}$

$\therefore \quad \frac{\operatorname{cosec}\theta + \cot\theta}{\sec\theta - \tan\theta} = \frac{\frac{-5}{4} + \frac{3}{4}}{\frac{-5}{3} - \frac{4}{3}}$

$$= \frac{\frac{-5+3}{4}}{\frac{-5-4}{3}}$$

$$= \frac{-\frac{2}{4}}{-\frac{9}{3}}$$

$$= \frac{2}{4} \times \frac{3}{9}$$

$$= \frac{1}{6}$$

$\therefore \quad \frac{\operatorname{cosec}\theta + \cot\theta}{\sec\theta - \tan\theta} = \frac{1}{6}$

RD Sharma Solutions Class 11 Maths Chapter 5 Ex 5.3

Chapter 5 Trigonometric Functions Ex 5.3 Q 1 i

$$\sin\frac{5\pi}{3} = \sin\left(2\pi - \frac{\pi}{3}\right)$$

$$= -\sin\frac{\pi}{3} \qquad (\because \sin(2\pi - \theta) = -\sin\theta)$$

$$= \frac{-\sqrt{3}}{2}$$

Chapter 5 Trigonometric Functions Ex 5.3 Q 1 ii

$$3060^\circ = 17\pi \qquad (\because \pi = 180^\circ)$$

$$\therefore \sin 3060^\circ = \sin 17\pi$$

$$= 0 \qquad (\because \sin n\pi = 0 \text{ for all } n \in z)$$

Chapter 5 Trigonometric Functions Ex 5.3 Q 1 iii

$$\tan\frac{11\pi}{6} = \tan\left(2\pi - \frac{\pi}{6}\right)$$

$$= -\tan\frac{\pi}{6} \qquad (\because \tan(2\pi - \theta) = -\tan\theta)$$

$$= \frac{-1}{\sqrt{3}}$$

Chapter 5 Trigonometric Functions Ex 5.3 Q 1 iv

$$1125^\circ = 6\pi + \frac{\pi}{4} (\pi = 180^\circ)$$

$$\cos(-1125^\circ) = \cos\left(-\left(6\pi + \frac{\pi}{4}\right)\right)$$

$$= \cos\left(6\pi + \frac{\pi}{4}\right) \qquad (\because \cos(-\theta) = \cos\theta)$$

$$= \cos\left(2 \times 3\pi + \frac{\pi}{4}\right)$$

$$= \cos\frac{\pi}{4} \qquad (\because \cos(2k\pi + \theta) = \cos\theta, k \in n)$$

$$= \frac{1}{\sqrt{2}}$$

Chapter 5 Trigonometric Functions Ex 5.3 Q 1.v

$$\tan 315^\circ = \tan\left(2\pi - \frac{\pi}{4}\right)$$

$$= -\tan\frac{\pi}{4} \qquad \left(\because \tan\left(2\pi - \theta\right) = -\tan\theta\right)$$

$$= -1$$

Chapter 5 Trigonometric Functions Ex 5.3 Q 1.vi

$$\sin 510^\circ = \sin\left(3\pi - \frac{\pi}{6}\right)$$

$$= \sin\frac{\left(\pi\right)}{6} \qquad \left(\because 3\pi - \frac{\pi}{6} \text{ lies in second quadrant}\right)$$

$$= \frac{1}{2}$$

Alternative solution

$$\sin 510^\circ = \sin\left(3\pi - \frac{\pi}{6}\right)$$

$$= \sin\left(2\pi + \left(\pi - \frac{\pi}{6}\right)\right)$$

$$= \sin\left(\pi - \frac{\pi}{6}\right) \qquad \left(\because \sin\left(2\pi + \theta\right) = \sin\theta, \text{ as sine is periodic with period } 2\pi\right)$$

$$= \sin\frac{\pi}{6} \qquad \left(\because \sin\left(\pi - \theta\right) = \sin\theta\right)$$

$$= \frac{1}{2}$$

Chapter 5 Trigonometric Functions Ex 5.3 Q 1.vii

$$\cos 570^\circ = \cos\left(3\pi + \frac{\pi}{6}\right)$$

$$= \cos\left(2\pi + \left(\pi + \frac{\pi}{6}\right)\right)$$

$$= \cos\left(\pi + \frac{\pi}{6}\right) \qquad \left(\because \cos\left(2\pi + \theta\right) = \cos\theta, \text{ as cosine is periodoc with period } 2\pi\right)$$

$$= -\cos\frac{\pi}{6} \qquad \left(\because \cos\left(\pi + \theta\right) = -\cos\theta\right)$$

$$= \frac{-\sqrt{3}}{2}$$

Chapter 5 Trigonometric Functions Ex 5.3 Q 1.viii

$$\sin\left(-330^\circ\right) = \sin\left(-\left(2\pi - \frac{\pi}{6}\right)\right)$$

$$= \sin\left(2\pi - \frac{\pi}{6}\right) \qquad \left(\because \sin\left(-\theta\right) = -\sin\theta\right)$$

$$= -\left(-\sin\frac{\pi}{6}\right) \qquad \left(\because \sin\left(2\pi - \theta\right) = -\sin\theta\right)$$

$$= \sin\frac{\pi}{6}$$

$$= \frac{1}{2}$$

Chapter 5 Trigonometric Functions Ex 5.3 Q 1.ix

$$\operatorname{cosec}\left(-1200^\circ\right) = \operatorname{cosec}\left(-\left(7\pi - \frac{\pi}{3}\right)\right)$$

$$= \operatorname{cosec}\left(7\pi - \frac{\pi}{3}\right) \qquad \left(\because \operatorname{cosec}\left(-\theta\right) = -\operatorname{cosec}\theta\right)$$

$$= -\operatorname{cosec}\left(2 \times 3\pi + \left(\pi - \frac{\pi}{3}\right)\right)$$

$$= -\operatorname{cosec}\left(\pi - \frac{\pi}{3}\right) \qquad \left(\because \operatorname{cosec} \text{ is periodic of period } 2\pi, \therefore \operatorname{cosec}\left(2\pi + \theta\right) = \operatorname{cosec}\left(2n\pi + \theta\right) = \operatorname{cosec}\theta \text{ for all } n \in N\right)$$

$$= -\operatorname{cosec}\frac{\pi}{3} \qquad \left(\therefore \operatorname{cosec}\left(\pi - \theta\right) = \operatorname{cosec}\theta\right)$$

$$= \frac{-2}{\sqrt{3}}$$

Chapter 5 Trigonometric Functions Ex 5.3 Q 1.x

$$\tan\left(-585^\circ\right) = -\tan\left(585\right) \qquad \left(\because \tan\left(-\theta\right) = -\tan\theta\right)$$

$$= -\tan\left(3\pi + \frac{\pi}{4}\right)$$

$$= -\tan\left(2\pi + \left(\pi + \frac{\pi}{4}\right)\right) \qquad \left(\because \tan\left(2\pi + \theta\right) = \tan\theta\right)$$

$$= -\tan\frac{\pi}{4} \qquad \left(\because \tan\left(\pi + \theta\right) = \tan\theta\right)$$

$$= -1$$

Chapter 5 Trigonometric Functions Ex 5.3 Q 1.xi

$$\cos\left(855^\circ\right) = \cos\left(5\pi - \frac{\pi}{4}\right)$$
$$= \cos\left(2\times 2\pi + \left(\pi - \frac{\pi}{4}\right)\right)$$
$$= \cos\left(\pi - \frac{\pi}{4}\right) \qquad \left(\because \cos\left(2k\pi + \theta\right) = \cos\theta \text{ for all } k \in N\right)$$
$$= -\cos\frac{\pi}{4} \qquad \left(\because \cos\left(\pi - \theta\right) = -\cos\theta\right)$$
$$= \frac{-1}{\sqrt{2}}$$

Chapter 5 Trigonometric Functions Ex 5.3 Q 1.xii

$$\sin 1845^\circ = \sin\left(10\pi + \frac{\pi}{4}\right)$$
$$= \left(2\times 5\pi + \frac{\pi}{4}\right)$$
$$= \sin\pi \qquad \left(\because \sin\left(2k\pi + \theta\right) = \sin\theta,\ \text{for all } k \in N\right)$$
$$= \frac{1}{\sqrt{2}}$$

Chapter 5 Trigonometric Functions Ex 5.3 Q 1.xiii

$$\cos 1755^\circ = \cos\left(10\pi - \frac{\pi}{4}\right)$$
$$= \cos\left(2\times 5\pi - \frac{\pi}{4}\right)$$
$$= \cos\frac{\pi}{4} \qquad \left(\because \cos\left(2k\pi - \theta\right) = \cos\theta, k \in N\right)$$
$$= \frac{1}{\sqrt{2}}$$

Chapter 5 Trigonometric Functions Ex 5.3 Q 1.xiv

$$4530^\circ = \left(25\pi + \frac{\pi}{6}\right)$$
$$\therefore\ \sin 4530 = \sin\left(25\pi + \frac{\pi}{6}\right)$$
$$= \sin\left(2\times 12\pi + \left(\pi + \frac{\pi}{6}\right)\right)$$
$$= \sin\left(\pi \frac{\pi}{6}\right) \qquad \left(\because \sin\left(2k\pi + \theta\right) = \sin\theta, k \in N\right)$$
$$= -\sin\frac{\pi}{6} \qquad \left(\because \sin\left(\pi + \theta\right) = -\sin\theta\right)$$
$$= \frac{-1}{2}$$

Chapter 5 Trigonometric Functions Ex 5.3 Q 2.i

$$\text{LHS} = \tan 225^\circ \cot 405^\circ + \tan 765^\circ \cot 675^\circ$$
$$= \tan\left(\pi + \frac{\pi}{4}\right)\cot\left(2\pi + \frac{\pi}{4}\right) + \tan\left(4\pi + \frac{\pi}{4}\right)\cot\left(4\pi - \frac{\pi}{4}\right)$$
$$= \tan\frac{\pi}{4}.\ \cot\frac{\pi}{4} + \tan\frac{\pi}{4}\times\left(-\cot\frac{\pi}{4}\right) \qquad \left(\because \cot\left(4\pi - \frac{\pi}{4}\right) = -\cot\frac{\pi}{4}\right)$$
$$= 1.1 + 1.\left(-1\right)$$
$$= 0$$
$$= \text{RHS}$$

Proved

Chapter 5 Trigonometric Functions Ex 5.3 Q 2.ii

$$\text{LHS} = \sin\frac{8\pi}{3}\cos\frac{23\pi}{6} + \cos\frac{13\pi}{3}\sin\frac{35\pi}{6}$$
$$= \sin\left(3\pi - \frac{\pi}{3}\right)\cos\left(4\pi - \frac{\pi}{6}\right) + \cos\left(4\pi + \frac{\pi}{3}\right)\sin\left(6\pi - \frac{\pi}{6}\right)$$
$$= \sin\frac{\pi}{3}\cos\frac{\pi}{6} + \cos\frac{\pi}{3}\left(-\sin\frac{\pi}{6}\right) \qquad \left(\because \sin\left(6\pi - \theta\right) = -\sin\theta\right)$$
$$= \frac{\sqrt{3}}{2}\times\frac{\sqrt{3}}{22} + \frac{1}{2}\times\left(\frac{-1}{2}\right)$$
$$= \frac{3}{4} - \frac{1}{4}$$
$$= \frac{2}{4}$$
$$= \frac{1}{2}$$
$$= \text{RHS}$$

Proved

Chapter 5 Trigonometric Functions Ex 5.3 Q 2.iii

$$\text{LHS} = \cos 24^\circ + \cos 55^\circ + \cos 125^\circ + \cos 204^\circ + \cos 300^\circ$$
$$= \cos 24^\circ + \cos 204^\circ + \cos 55^\circ + \cos 125^\circ + \cos 300^\circ$$

$= \cos 24^\circ + \cos 204^\circ + \cos 55^\circ + \cos 125^\circ + \cos 300^\circ$

$= \cos 24^\circ + \cos\left(\pi + 24^\circ\right) + \cos 55^\circ + \cos\left(\pi - 55^\circ\right) + \cos\left(2\pi - \frac{\pi}{3}\right)$

$= \cos 24^\circ - \cos 24^\circ + \cos 55^\circ - \cos 55^\circ + \cos\frac{\pi}{3}$

$= \cos\frac{\pi}{3}$

$= \frac{1}{2}$

= RHS

Proved

Chapter 5 Trigonometric Functions Ex 5.3 Q 1 2.iv

LHS $= \tan\left(-225^\circ\right)\cot\left(-405^\circ\right) - \tan\left(-765^\circ\right)\cot\left(675^\circ\right)$

$= -\tan 225^\circ\left(-\cot 405^\circ\right) + \tan 765^\circ \cot 765^\circ$ $\left(\because \tan(-\theta) = -\tan\theta \text{ \& } \cot(-\theta) = -\cot\theta\right)$

$= \tan\left(\pi + \frac{\pi}{4}\right)\cot\left(2\pi \frac{\pi}{4}\right) + \tan\left(4\pi + \frac{\pi}{4}\right)\cot\left(4\pi - \frac{\pi}{4}\right)$

$= \tan\frac{\pi}{4}\cot\frac{\pi}{4} + \tan\frac{\pi}{4} \times \left(-\cot\frac{\pi}{4}\right)$ $\left(\because \cot(4\pi - \theta) = -\cot\theta\right)$

$= 1.1 + 1(-1)$

$= 1 - 1$

$= 0$

= RHS

Proved

Chapter 5 Trigonometric Functions Ex 5.3 Q 2.v

LHS $= \cos 570^\circ \sin 510^\circ + \sin\left(-330^\circ\right)\cos\left(-390^\circ\right)$

$= \cos\left(3\pi + \frac{\pi}{6}\right)\sin\left(3\pi - \frac{\pi}{6}\right) - \sin 330^\circ \cos 390^\circ$ $\left(\because \sin(-\theta) = -\sin\theta \text{ and } \cos(-\theta) = \cos\theta\right)$

$= -\cos\frac{\pi}{6}\sin\frac{\pi}{6} - \sin\left(2\pi - \frac{\pi}{6}\right)\cos\left(2\pi + \frac{\pi}{6}\right)$

$= -\sin\frac{\pi}{6}\cos\frac{\pi}{6} + \sin\frac{\pi}{6}.\cos\frac{\pi}{6}$ $\left(\because \sin(2\pi - \theta) = -\sin\theta\right)$

$= 0$

= RHS

Proved

Chapter 5 Trigonometric Functions Ex 5.3 Q 2.vi

LHS $= \tan\frac{11\pi}{3} - 2\sin\frac{4\pi}{6} - \frac{3}{4}\operatorname{cosec}^2\frac{\pi}{4} + 4\cos^2\frac{17\pi}{6}$

$= \tan\left(4\pi - \frac{\pi}{3}\right) - 2\sin\frac{2\pi}{3} - \frac{3}{4} \times \left(\sqrt{2}\right)^2 + 4\cos^2\left(3\pi - \frac{\pi}{6}\right)$

$= -\tan\frac{\pi}{3} - 2\sin\left(\pi - \frac{\pi}{3}\right) - \frac{3}{4} \times 2 + 4\cos^2\frac{\pi}{6}$

$\left(\because \tan\left(4\pi - \frac{\pi}{3}\right) = -\tan\frac{\pi}{3}, \cos\left(3\pi - \frac{\pi}{6}\right) = -\cos\frac{\pi}{6}\right)$

$= -\sqrt{3} - 2\sin\frac{\pi}{3} - \frac{3}{2} + 4 \times \left(\frac{\sqrt{3}}{2}\right)^2$

$= -\sqrt{3} - 2 \times \frac{\sqrt{3}}{2} - \frac{3}{2} + 4 \times \frac{3}{4}$

$= -\sqrt{3} - \sqrt{3} - \frac{3}{2} + 3$

$= -2\sqrt{3}\frac{-3+6}{2}$

$= -2\sqrt{3} + \frac{3}{2}$

$= \frac{3 - 4\sqrt{3}}{2}$

= RHS

Proved

Chapter 5 Trigonometric Functions Ex 5.3 Q 2.vii

LHS $= 3\sin\frac{\pi}{6}\sec\frac{\pi}{3} - 4\sin\frac{5\pi}{6}\cot\frac{\pi}{4}$

$= 3 \times \frac{1}{2} \times 2 - 4\sin\left(\pi - \frac{\pi}{6}\right) \times 1$

$= 3 - 4\sin\frac{\pi}{6}$ $\left(\because \sin(\pi - \theta) = \sin\theta\right)$

$= 3 - 4 \times \frac{1}{\ }$

$$= 3 - 2^2$$

$= 1$

$= RHS$

Proved

Chapter 5 Trigonometric Functions Ex 5.3 Q 3.i

$$LHS = \frac{\cos(2\pi+\theta)\operatorname{cosec}(2\pi+\theta)\tan\left(\frac{\pi}{2}+\theta\right)}{\sec\left(\frac{\pi}{2}+\theta\right)\cos\theta\cot(\pi+\theta)}$$

$$= \frac{\cos\theta \times \operatorname{cosec}\theta(-\cot\theta)}{-\operatorname{cosec}\theta.\cos\theta\cot\theta} \quad \left(\because \tan\left(\frac{\pi}{2}+\theta\right) = -\cot\theta \ \& \ \sec\left(\frac{\pi}{2}+\theta\right) = -\operatorname{cosec}\theta\right)$$

$= 1$

$= RHS$

Proved

Chapter 5 Trigonometric Functions Ex 5.3 Q 3.ii

$$LHS = \frac{\operatorname{cosec}(90^\circ+\theta)+\cot(450^\circ+\theta)}{\operatorname{cosec}(90^\circ-\theta)+\tan(180^\circ-\theta)} + \frac{\tan(180^\circ+\theta)+\sec(180^\circ-\theta)}{\tan(360^\circ+\theta)-\sec(-\theta)}$$

$$= \frac{\sec\theta+\cot\left(2\pi+\frac{\pi}{2}+\theta\right)}{\sec\theta-\tan\theta} + \frac{\tan\theta-\sec\theta}{\tan\theta-\sec\theta}$$

$$\left(\because \operatorname{cosec}(90^\circ+\theta) = \sec\theta, \operatorname{cosec}(90^\circ+\theta) = \sec\theta, \tan(180^\circ-\theta) = -\tan\theta \sec(-\theta) = \sec\theta\right)$$

$$= \frac{\sec\theta+\cot\left(\frac{\pi}{2}+\theta\right)}{\sec\theta-\tan\theta}+1 \quad \left(\because \cot(2\pi+\theta) = \cot\theta\right)$$

$$= \frac{\sec\theta-\tan\theta}{\sec\theta-\tan\theta}+1 \quad \left(\because \cot\left(\frac{\pi}{2}+\theta\right) = -\tan\theta\right)$$

$= 1 + 1$

$= 2$

$= RHS$

Proved

Chapter 5 Trigonometric Functions Ex 5.3 Q 3.iii

$$LHS = \frac{\sin(180^\circ+\theta)\cos(90^\circ+\theta)\tan(270^\circ-\theta)\cot(360^\circ-\theta)}{\sin(360^\circ-\theta)\cos(360^\circ+\theta)\operatorname{cosec}(-\theta)\sin(270^\circ+\theta)}$$

$$= \frac{\sin\theta(-\sin\theta)\cot\theta(-\cot\theta)}{-\sin\theta\cos\theta(-\operatorname{cosec}\theta)(-\cos\theta)} \quad \left(\because \tan(270^\circ-\theta) = \cot\theta \ \& \ \sin(270^\circ+\theta) = -\cos\theta\right)$$

$$= \frac{-\sin\theta\times\sin\theta\times\cos\theta\times\cos\theta\times\sin\theta}{-\sin\theta\times\cos\theta\times\sin\theta\times\sin\theta\times\cos\theta} \quad \left(\because \cot\theta = \frac{\cos\theta}{\sin\theta} \ \& \ \operatorname{cosec}\theta = \frac{1}{\sin\theta}\right)$$

$= 1$

$= RHS$

Proved

Chapter 5 Trigonometric Functions Ex 5.3 Q 3.iv

$$LHS = \left\{1+\cot\theta-\sec\left(\frac{\pi}{2}+\theta\right)\right\}\left\{1+\cot\theta+\sec\left(\frac{\pi}{2}+\theta\right)\right\}$$

$$= \{1+\cot\theta-(-\operatorname{cosec}\theta)\}\{1+\cot\theta-\operatorname{cosec}\theta\}$$

$$\left(\because \sec\left(\frac{\pi}{2}+\theta\right) = -\operatorname{cosec}\theta\right)$$

$$= \{(1+\cot\theta)+\operatorname{cosec}\theta\}\{(1+\cot\theta)-\operatorname{cosec}\theta\}$$

$$= (1+\cot\theta)^2 - \operatorname{cosec}^2\theta$$

$$= 1+\cot^2\theta+2\cot\theta-\operatorname{cosec}^2\theta$$

$$= \operatorname{cosec}^2+2\cot\theta-\operatorname{cosec}^2 \quad \left(\because 1+\cot^2\theta = \operatorname{cosec}^2\theta\right)$$

$= 2\cot\theta$

$= RHS$

Proved

Chapter 5 Trigonometric Functions Ex 5.3 Q 3 v

$$\text{LHS} = \frac{\tan\left(90^\circ - \theta\right)\sec\left(180^\circ - \theta\right)\sin\left(-\theta\right)}{\sin\left(180^\circ + \theta\right)\cot\left(360^\circ - \theta\right)\operatorname{cosec}\left(90^\circ - \theta\right)}$$

$$= \frac{\cot\theta \times \left(-\sec\theta\right) \times \left(-\sin\theta\right)}{-\sin\theta \times \left(-\cot\theta\right) \times \sec\theta}$$

$= 1$

$= \text{RHS}$

Proved

Chapter 5 Trigonometric Functions Ex 5.3 Q 4

$$\text{LHS} = \sin^2\frac{\pi}{18} + \sin^2\frac{\pi}{9} + \sin^2\frac{7\pi}{18} + \sin^2\frac{4\pi}{9}$$

$$= \sin^2\frac{\pi}{18} + \sin^2\frac{4\pi}{9} + \sin^2\frac{\pi}{9} + \sin^2\frac{7\pi}{18}$$

$$= \sin^2\left(\frac{\pi}{2} - \frac{4\pi}{9}\right) + \sin^2\frac{4\pi}{9} + \sin^2\frac{\pi}{9} + \sin^2\left(\frac{\pi}{2} - \frac{\pi}{9}\right) \qquad \left(\because \frac{\pi}{18} = \frac{\pi}{2} - \frac{4\pi}{9} \text{ and } \frac{7\pi}{18} = \frac{\pi}{2} - \frac{\pi}{9}\right)$$

$$= \cos^2\frac{4\pi}{9} + \sin^2\frac{4\pi}{9} + \sin^2\frac{\pi}{9} + \cos^2\frac{\pi}{9} \qquad \left(\because \sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta\right)$$

$= 1 + 1 \quad \left(\because \sin^2\theta + \cos^2\theta = 1\right)$

$= 2$

$= \text{RHS}$

Proved

Chapter 5 Trigonometric Functions Ex 5.3 Q 5

$$\text{LHS} = \sec\left(\frac{3\pi}{2} - \theta\right)\sec\left(\theta - \frac{5\pi}{2}\right) + \tan\left(\frac{5\pi}{2} + \theta\right)\tan\left(\theta - \frac{3\pi}{2}\right)$$

$$= \sec\left(\frac{3\pi}{2} - \theta\right)\sec\left(-\left(\frac{5\pi}{2} - \theta\right)\right) + \tan\left(\frac{5\pi}{2} + \theta\right)\tan\left(-\left(\frac{3\pi}{2} - \theta\right)\right)$$

$$= -\operatorname{cosec}\theta . \sec\left(\frac{5\pi}{2} - \theta\right) - \cot\theta \times (-)\tan\left(\frac{3\pi}{2} - \theta\right)$$

$$\left[\because \left(\sec\left(\frac{3\pi}{2} - \theta\right)\right) = -\operatorname{cosec}\theta,\ \sec\left(-\theta\right) = \sec\theta, \tan\left(\frac{5\pi}{2} + \theta\right) = -\cot\theta \ \& \tan\left(-\theta\right) = -\tan\theta\right)\right]$$

$$= -\operatorname{cosec}\theta \times \operatorname{cosec}\theta - \cot\theta \times (-1) \times \cot\theta \qquad \left[\left(\because \sec\left(\frac{5\pi}{2} - \theta\right)\right) = \operatorname{cosec}\theta \ \& \tan\left(\frac{3\pi}{2} - \theta\right) = \cot\theta\right]$$

$= -\operatorname{cosec}^2\theta + \cot^2\theta$

$= -\operatorname{cosec}^2\theta + \operatorname{cosec}^2\theta - 1 \qquad \left(\because \operatorname{cosec}^2\theta = 1 + \cot^2\theta\right)$

$= -1$

$= \text{RHS}$

Proved

Chapter 5 Trigonometric Functions Ex 5.3 Q 6

We have $A + B + C = \pi$ $\quad \left(\because \text{sum of 3 angles of a triangle is } \pi = 180^\circ\right)$

$\Rightarrow \quad A + B = \pi - C$

$\Rightarrow \quad \cos\left(A + B\right) = \cos\left(\pi - C\right)$

$\Rightarrow \quad = -\cos C \qquad \left(\because \cos\left(\pi - \theta\right) = -\cos\theta\right)$

$\Rightarrow \quad \cos\left(A + B\right) + \cos C = 0$

Proved

Chapter 5 Trigonometric Functions Ex 5.3 Q 6 ii

We have $A + B + C = \pi$ $\quad \left(\because \text{sum of 3 angles of a triangle is } \pi = 180^\circ\right)$

$\Rightarrow \quad A + B = \pi - C$

$\Rightarrow \quad \frac{A + B}{2} = \frac{\pi - C}{2}$

$\Rightarrow \quad \frac{A + B}{2} = \frac{\pi}{2} - \frac{C}{2}$

$\Rightarrow \quad \cos = \left(\frac{A + B}{2}\right) = \cos\left(\frac{\pi}{2} - \frac{C}{2}\right)$

$\Rightarrow \quad = \sin\frac{C}{2} \qquad \left(\because \cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta\right)$

Hence $\cos\left(\frac{A + B}{2}\right) = \sin\frac{C}{2}$

Proved

Chapter 5 Trigonometric Functions Ex 5.3 Q 6 iii

We have $A + B + C = \pi$ $\quad \left(\because \text{sum of 3 angles of a triangle is } \pi = 180^\circ\right)$

We have $A + B + C = \pi$ $\left(\because \text{ sum of 3 angles of a triangle is } \pi = 180^\circ\right)$

$\Rightarrow \quad A + B = \pi - C$

$\Rightarrow \quad \dfrac{A+B}{2} = \dfrac{\pi - C}{2}$

$\Rightarrow \quad \dfrac{A+B}{2} = \dfrac{\pi}{2} - \dfrac{C}{2}$

$\Rightarrow \quad \tan\left(\dfrac{A+B}{2}\right) = \tan\left(\dfrac{\pi}{2} - \dfrac{C}{2}\right)$

$= \cot\dfrac{C}{2} \qquad \left(\because \tan\left(\dfrac{\pi}{2} - \theta\right) = \cot\theta\right)$

Hence $\tan\left(\dfrac{A+B}{2}\right) = \cot\dfrac{C}{2}$

Proved

Chapter 5 Trigonometric Functions Ex 5.3 Q 7

$\because$ A, B, C, D are the angles of a cyclic quadrilateral in order,

$\therefore$ $A + C = \pi$ & $B + D = \pi$

$\Rightarrow \quad \pi - A = C$ & $\pi - D = B$

$\Rightarrow \quad \cos(\pi - A) = \cos C$ ....(i)

& $\cos(\pi - D) = \cos B$ ....(ii)

Now, $\cos(180^\circ - A) + \cos(180^\circ + B) + (180^\circ + C) - \sin(90^\circ + D)$

$= \cos C + (-\cos B) - \cos C - \cos D$

$\left(\because \cos(180^\circ + B) = -\cos B, \cos(180^\circ + C) = -\cos C \text{ \& using (i)}\right)$

$= -\cos B - \cos D$

$= -\cos B - (-\cos B)$ (using (ii))

$= -\cos B + \cos B$

$= 0$

Proved

Chapter 5 Trigonometric Functions Ex 5.3 Q 8i.

$\operatorname{cosec}(90^\circ + \theta) + x\cos\theta\cot(90^\circ + \theta) = \sin(90^\circ + \theta)$

$\Rightarrow \quad \sec\theta + x\cos\theta \times (-\tan\theta) = \cos\theta$

$\Rightarrow \quad \dfrac{1}{\cos\theta} + x\cos\theta \times \dfrac{(-\sin\theta)}{\cos\theta} = \cos\theta$

$\Rightarrow \quad \dfrac{1}{\cos\theta} - x\sin\theta = \cos\theta$

$\Rightarrow \quad \dfrac{1 - x\sin\theta\cos\theta}{\cos\theta} = \cos\theta$

$\Rightarrow \quad 1 - x\sin\theta\cos\theta = \cos^2\theta$

$\Rightarrow \quad 1 - \cos^2\theta = x\sin\theta\cos\theta$

$\Rightarrow \quad \sin^2\theta = x\sin\theta\cos\theta$

$\Rightarrow \quad \sin\theta = x\cos\theta$

$\Rightarrow \quad x = \dfrac{\sin\theta}{\cos\theta}$

$= \tan\theta$

Hence $x = \tan\theta$

Chapter 5 Trigonometric Functions Ex 5.3 Q 8. ii.

We have $x\cot(90^\circ + \theta) + \tan(90^\circ + \theta)\sin\theta + \operatorname{cosec}(90^\circ + \theta) = 0$

$\Rightarrow \quad x(-\tan\theta) - \cot\theta \times \sin\theta + \sec\theta = 0$

$\Rightarrow \quad -x\tan\theta - \dfrac{\cos\theta}{\sin\theta} \times \sin\theta + \dfrac{1}{\cos\theta} = 0$

$\Rightarrow \quad -x\dfrac{\sin\theta}{\cos\theta} - \cos\theta + \dfrac{1}{\cos\theta} = 0$

$\Rightarrow \quad \dfrac{-x\sin\theta - \cos^2\theta + 1}{\cos\theta} = 0$

$\Rightarrow \quad -x\sin\theta + 1 - \cos^2\theta = 0$

$\Rightarrow \quad -x\sin\theta + \sin^2\theta = 0$

$\Rightarrow \quad x\sin\theta = \sin^2\theta$

$\Rightarrow \quad x = \dfrac{\sin^2\theta}{\sin\theta}$

$\Rightarrow \quad x = \sin\theta$

Chapter 5 Trigonometric Functions Ex 5.3 Q 9. i.

LHS = $\tan 720^\circ - \cos 270^\circ - \sin 150^\circ \cos 120^\circ$

$= \tan 4\pi - \cos\left(\frac{3\pi}{2}\right) - \sin\left(\pi \frac{\pi}{6}\right)\cos\left(\frac{\pi}{2} + \frac{\pi}{6}\right) \left(\because \pi = 180^\circ\right)$

$= 0 - 0 - \sin\frac{\pi}{6}\left(-\sin\frac{\pi}{6}\right) \qquad \left(\because \tan n\pi = 0 \text{ for all } n \in Z \ \& \ \cos\frac{3\pi}{2} = 0\right)$

$= \sin^2\frac{\pi}{6}$

$= \left(\frac{1}{2}\right)^2$

$= \frac{1}{4}$

= RHS

Proved

Chapter 5 Trigonometric Functions Ex 5.3 Q 9. ii.

LHS = $\sin 780^\circ \sin 480^\circ + \cos 120^\circ \sin 150^\circ$

$= \sin\left(4\pi + \frac{\pi}{3}\right)\sin\left(3\pi - \frac{\pi}{3}\right) + \cos\left(\frac{\pi}{2} + \frac{\pi}{6}\right)\sin\left(\pi - \frac{\pi}{6}\right) \qquad \left(\because \pi = 180^\circ\right)$

$= \sin\frac{\pi}{3} \times \sin\frac{\pi}{3} + \left(-\sin\frac{\pi}{6}\right)\sin\frac{\pi}{6} \qquad \left(\because \sin\left(4\pi + \frac{\pi}{3}\right) = \sin\frac{\pi}{3} \ \& \ \sin\left(3\pi - \frac{\pi}{3}\right) = \sin\frac{\pi}{3}\right)$

$= \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} - \frac{1}{2} \times \frac{1}{2}$

$= \frac{3}{4} - \frac{1}{4}$

$= \frac{2}{4}$

$= \frac{1}{2}$

= RHS

Proved

Chapter 5 Trigonometric Functions Ex 5.3 Q 9. iii.

LHS = $\sin 780^\circ \sin 120^\circ + \cos 240^\circ \sin 390^\circ$

$= \sin\left(4\pi + \frac{\pi}{3}\right)\sin\left(\frac{\pi}{2} + \frac{\pi}{6}\right) + \cos\left(\pi + \frac{\pi}{6}\right)\sin\left(2\pi + \frac{\pi}{6}\right)$

$= \sin\frac{\pi}{3} \times \cos\frac{\pi}{6} - \cos\frac{\pi}{3} \times \left(+\sin\frac{\pi}{6}\right)$

$= \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} - \frac{1}{2} \times \frac{1}{2}$

$= \frac{3}{4} - \frac{1}{4}$

$= \frac{1}{2}$

$= \frac{1}{2}$

= RHS

Proved

Chapter 5 Trigonometric Functions Ex 5.3 Q 9.iv.

LHS = $\sin 600^\circ \cos 390^\circ + \cos 480^\circ \sin 150^\circ$

$= \sin\left(3\pi + \frac{\pi}{3}\right)\cos\left(2\pi + \frac{\pi}{6}\right) + \cos\left(3\pi - \frac{\pi}{3}\right)\sin\left(\pi - \frac{\pi}{6}\right)$

$= -\sin\frac{\pi}{3}\cos\frac{\pi}{6} - \cos\frac{\pi}{3} - \sin\frac{\pi}{6} \qquad \left(\because \sin\left(3\pi + \frac{\pi}{3}\right) = -\sin\frac{\pi}{3} \ \& \ \cos\left(3\pi - \frac{\pi}{3}\right) = -\cos\frac{\pi}{3}\right)$

$= \frac{-\sqrt{3}}{2} \times \frac{-\sqrt{3}}{2} - \frac{1}{2} \times \frac{1}{2}$

$= \frac{-3}{4} - \frac{1}{4}$

$= \frac{-4}{4}$

$= -1$

$= RHS$

Proved

Chapter 5 Trigonometric Functions Ex 5.3 Q 9.v.

$LHS = \tan 225^\circ \cot 405^\circ + \tan 765^\circ \cot 675^\circ$

$= \tan\left(\pi + \frac{\pi}{4}\right)\cot\left(2\pi + \frac{\pi}{4}\right) + \tan\left(4\pi + \frac{\pi}{4}\right)\cot\left(4\pi - \frac{\pi}{4}\right)$

$= \tan\frac{\pi}{4}\cot\frac{\pi}{4} + \tan\frac{\pi}{4}\left(-\cot\frac{\pi}{4}\right)$

$= 1.1 + 1.(-1)$

$= 1 - 1$

$= 0$

$= RHS$

Proved