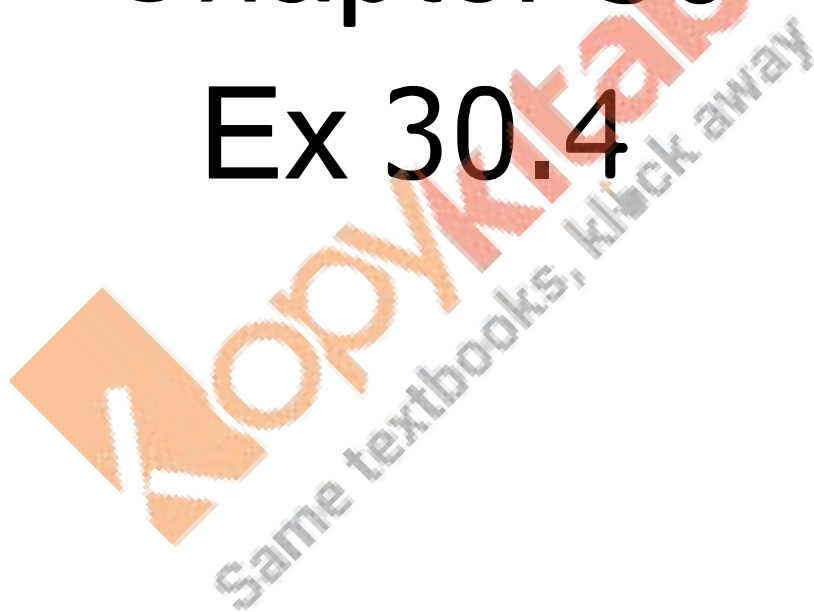


RD Sharma
Solutions
Class 11 Maths
Chapter 30
Ex 30.4



Derivatives Ex 30.4 Q1

We have,

$$\begin{aligned} & \frac{d}{dx}(x^3 \sin x) \\ &= \sin x \frac{d(x^3)}{dx} + x^3 \frac{d(\sin x)}{dx} \quad [\text{Using product rule}] \\ &= \sin x \cdot 3x^2 + x^3 \cdot \cos x \\ &= x^2(3 \sin x + x \cos x) \end{aligned}$$

Derivatives Ex 30.4 Q2

We have,

$$\begin{aligned} & \frac{d}{dx}(x^3 e^x) \\ &= e^x \frac{d}{dx}(x^3) + x^3 \frac{d}{dx}(e^x) \quad [\text{Using product rule}] \\ &= e^x 3x^2 + x^3 e^x \\ &= x^2 e^x (3 + x) \end{aligned}$$

Derivatives Ex 30.4 Q3

We have,

$$\begin{aligned} & \frac{d}{dx}(x^2 e^x \log x) \\ &= e^x \log x \frac{d}{dx}(x^2) + x^2 \log x \frac{d}{dx}(e^x) + x^2 e^x \frac{d}{dx}(\log x) \quad [\text{Using product rule}] \\ &= e^x \log x \cdot 2x + x^2 \log x \cdot e^x + x^2 e^x \cdot \frac{1}{x} \\ &= x e^x (2 \log x + x \log x + 1) \end{aligned}$$

Derivatives Ex 30.4 Q4

We have,

$$\frac{d}{dx} (x^n \tan x)$$

$$= \tan x \frac{d}{dx} (x^n) + x^n \frac{d}{dx} (\tan x) \quad [\text{Using product rule}]$$

$$= \tan x \cdot nx^{n-1} + x^n \sec^2 x$$

$$= x^{n-1} (n \tan x + x \sec^2 x) \quad [x^n = x^{n-1} \cdot x^1 = x^{n-1+1}]$$

Derivatives Ex 30.4 Q5

We have,

$$\frac{d}{dx} (x^n \log_a x)$$

$$= \log_a x \frac{d}{dx} (x^n) + x^n \frac{d}{dx} (\log_a x) \quad [\text{Using product rule}]$$

$$= nx^{n-1} \log_a x + \frac{x^n}{\log a} \cdot \frac{1}{x} \quad \left[\because \log_a x = \frac{\log x}{\log a} \right]$$

$$= x^{n-1} \left[n \log_a x + \frac{1}{\log a} \right]$$

Derivatives Ex 30.4 Q6

We have,

$$\frac{d}{dx} (x^3 + x^2 + 1) \sin x$$

$$= \sin x \frac{d}{dx} (x^3 + x^2 + 1) + (x^3 + x^2 + 1) \frac{d}{dx} (\sin x) \quad [\text{Using product rule}]$$

$$= \sin x (3x^2 + 2x) + (x^3 + x^2 + 1) \cos x$$

$$\therefore (x^3 + x^2 + 1) \cos x + (3x^2 + 2x) \sin x$$

Derivatives Ex 30.4 Q7

We have,

$$\frac{d}{dx}(\sin x \times \cos x)$$

$$\cos x \times \frac{d}{dx}(\sin x) + \sin x \times \frac{d}{dx}(\cos x) \quad [\text{using product rule}]$$

$$= \cos x (\cos x) + \sin x (-\sin x)$$

$$= \cos^2 x - \sin^2 x \quad [\because \cos 2x = \cos^2 x - \sin^2 x]$$

$$= \cos 2x$$

Derivatives Ex 30.4 Q8

We have,

$$\frac{d}{dx}(2^x \times \cot x \times x^{-\frac{1}{2}})$$

$$= \cot x \times \frac{1}{\sqrt{x}} \times \frac{d}{dx}(2^x) + 2^x \times \frac{1}{\sqrt{x}} \times \frac{d}{dx}(\cot x) + 2^x \times \cot x \times \frac{d}{dx}(x^{-\frac{1}{2}}) \quad [\text{Using product rule}]$$

$$= \frac{\cot x}{\sqrt{x}} \times 2^x \times \log 2 + \frac{2^x}{\sqrt{x}} (-\operatorname{cosec}^2 x) + 2^x \times \cot x \left(-\frac{1}{2}\right) \frac{1}{2\sqrt{x}}$$

$$= \frac{2^x}{\sqrt{x}} \left(\cot x \times \log 2 - \operatorname{cosec}^2 x - \frac{\cot x}{2x} \right)$$

Derivatives Ex 30.4 Q9

$$\frac{d}{dx}(x^2 \sin x \log x)$$

$$= \sin x \log x \times \frac{d}{dx}(x^2) + x^2 \log x \times \frac{d}{dx}(\sin x) + x^2 \sin x \times \frac{d}{dx}(\log x) \quad [\text{Using product rule}]$$

$$= \sin x \log x \times 2x + x^2 \log x \times \cos x + x^2 \sin x \times \frac{1}{x}$$

$$= 2x \times \sin x \times \log x + x^2 \times \cos x \times \log x + x \sin x$$

Derivatives Ex 30.4 Q10

We have,

$$\begin{aligned}
 & \frac{d}{dx} (x^5 e^x + x^6 \log x) \\
 &= \frac{d}{dx} (x^5 e^x) + \frac{d}{dx} (x^6 \log x) \\
 &= e^x \frac{dx^5}{dx} + x^5 \frac{de^x}{dx} + \log x \frac{d}{dx} (x^6) + x^6 \frac{d}{dx} (\log x) \quad [\text{Using product rule}] \\
 &= e^x \times 5x^4 + x^5 \times e^x + \log x \times 6x^5 + x^6 \times \frac{1}{x} \\
 &= 5x^4 \times e^x + x^5 \times e^x + 6x^5 \times \log x + x^5 \\
 &= x^4 (5e^x + ex^x + 6x \log x + x)
 \end{aligned}$$

Derivatives Ex 30.4 Q11

We have,

$$\frac{d}{dx} \{(x \sin x + \cos x)(x \cos x - \sin x)\}$$

We will apply product rule,

$$\begin{aligned}
 &= (x \cos x - \sin x) \frac{d}{dx} (x \sin x + \cos x) + (x \sin x + \cos x) \frac{d}{dx} (x \cos x - \sin x) \\
 &= (x \cos x - \sin x) \left\{ \frac{d}{dx} (x \sin x) + \frac{d}{dx} (\cos x) \right\} + (x \sin x + \cos x) \left\{ \frac{d}{dx} (x \cos x) - \frac{d}{dx} (\sin x) \right\}
 \end{aligned}$$

Again apply product rule,

$$\begin{aligned}
 &= (x \cos x - \sin x) \left\{ \left(\sin x \frac{dx}{dx} + x \frac{d \sin x}{dx} \right) \right\} + (-\sin x) + (x \cos x + \sin x) \left\{ \left(\sin x \frac{dx}{dx} + x \frac{d \cos x}{dx} - \cos x \right) \right\} \\
 &= (x \cos x - \sin x) \{ (\sin x + x \cos x) - \sin x \} + (-\sin x) + (x \cos x + \sin x) \{ (\cos x - x \sin x - \cos x) \} \\
 &= (x \cos x - \sin x) \times x \cos x + (x \sin x + \cos x) (-x \sin x) \\
 &= (x^2 \cos^2 x - x \sin x \times \cos x) + (-x^2 \sin^2 x - x \sin x \times \cos x) \\
 &= x^2 (\cos^2 x - \sin^2 x) - x (\sin x \cos x + \sin x \cos x) \\
 &= x^2 - \cos 2x - x \times 2 \sin x \cos x
 \end{aligned}$$

$$\begin{aligned}
 &= x^2 \cos x - x \sin x \\
 &= x \{x \cos x - \sin x\}
 \end{aligned}$$

Derivatives Ex 30.4 Q12

We have,

$$\frac{d}{dx} \{ (x \sin x + \cos x) (e^x + x^2 \log x) \}$$

We will apply product rule,

$$\begin{aligned}
 &= (e^x + x^2 \log x) \frac{d}{dx} (x \sin x + \cos x) + (x \sin x + \cos x) \frac{d}{dx} (e^x + x^2 \log x) \\
 &= (e^x + x^2 \log x) \left(\frac{d}{dx} (x \sin x) + \frac{d}{dx} \cos x \right) + (x \sin x + \cos x) \times \left\{ \frac{d}{dx} (e^x) + \frac{d}{dx} (x^2 \log x) \right\}
 \end{aligned}$$

Again apply product rule,

$$\begin{aligned}
 &= (e^x + x^2 \log x) \left(\sin x \frac{d}{dx} (x) + x \frac{d}{dx} (\sin x) \right) - \sin x + (x \sin x + \cos x) \left\{ e^x + \left(\log x \frac{d}{dx} (x^2) + x^2 \frac{d}{dx} (\log x) \right) \right\} \\
 &= (e^x + x^2 \log x) (\sin x + x \cos x - \sin x) + (x \sin x + \cos x) \left(e^x + \log x \times 2x + x^2 \frac{1}{x} \right) \\
 &= (e^x + x^2 \log x) x \cos x + (x \sin x + \cos x) (e^x + 2x \log x + x) \\
 &= x \cos x e^x + e^3 \cos x \log x + x e^x \sin x + e^x \cos x + 2x^2 \sin x \log x + 2x \cos x \log x + x^2 \sin x + x \cos x \\
 &= x \cos x (e^x + x^2 \log x) + (x \sin x + \cos x) (e^x + x + 2x \log x)
 \end{aligned}$$

Derivatives Ex 30.4 Q13

We have,

$$\begin{aligned}
 &\frac{d}{dx} \{ (1 - 2 \tan x) (5 + 4 \sin x) \} \\
 &= (5 + 4 \sin x) \frac{d}{dx} (1 - 2 \tan x) + (1 - 2 \tan x) \frac{d}{dx} (5 + 4 \sin x) \quad [\text{Using product rule,}] \\
 &= (5 + 4 \sin x) (0 - 2 \sec^2 x) + (1 - 2 \tan x) (0 + 4 \cos x) \\
 &= -10 \sec^2 x - 8 \sin x \times \sec^2 x + 4 \cos x - 8 \cos x \times \tan x \\
 &= 4 \left(-\frac{5}{2} \sec^2 x - 2 \sin x \times \frac{1}{\cos^2 x} + \cos x - 2 \cos x \times \frac{\sin x}{\cos x} \right) \\
 &= 4 \left(-\frac{5}{2} \sec^2 x - 2 \tan x \sec x + \cos x - 2 \sin x \right)
 \end{aligned}$$

$$= 4 \left(\cos x - 2 \sin x - 2 \tan x \sec x - \frac{5}{2} \sec^2 x \right)$$

Derivatives Ex 30.4 Q14

We have,

$$\frac{d}{dx} \{(1+x^2) \cos x\}$$

$$= \cos x \frac{d}{dx} (1+x^2) + (1+x^2) \frac{d}{dx} (\cos x) \quad (\text{using product rule})$$

$$= \cos x \times 2x + (1+x^2)(-\sin x)$$

$$= 2x \cos x - (1+x^2) \sin x$$

Derivatives Ex 30.4 Q15

We have,

$$\frac{d}{dx} (\sin^2 x)$$

$$= \frac{d}{dx} (\sin x)(\sin x)$$

$$= \sin x \frac{d}{dx} (\sin x) + \sin x \frac{d}{dx} (\sin x) \quad [\text{Using product rule}]$$

$$= \sin x \times \cos x + \sin x \times \cos x$$

$$= 2 \sin x \cos x$$

$$= \sin 2x$$

$$[\because \sin 2A = 2 \sin A \cos A]$$

Derivatives Ex 30.4 Q16

We have,

$$\frac{d}{dx} (\log_x x)$$

$$\log_x x = \frac{\log x}{\log x^2}$$

$$= \frac{\log x}{2 \log x}$$

$$= \frac{1}{2}$$

$$\frac{d}{dx} \left(\frac{1}{2} \right) = 0$$

$$\therefore \frac{d}{dx} (\log_x x) = 0$$

Derivatives Ex 30.4 Q17

$$\frac{d}{dx} (e^x \log \sqrt{x} \tan x)$$

Apply product rule,

$$= \log \sqrt{x} \times \tan x \frac{d}{dx} (e^x) + e^x \times \tan x \frac{d}{dx} (\log \sqrt{x}) + e^x \log \sqrt{x} \frac{d}{dx} (\tan x)$$

$$= \log \sqrt{x} \times \tan e^x + e^x \tan x \frac{1}{2x} + e^x \log \sqrt{x} \times \sec^2 x$$

$$= \frac{1}{2} \log x \times \tan x \times e^x + \frac{1 \tan x}{2x} e^x + e^x \frac{1}{2} \log x \sec^2 x \quad \left[\because \log \sqrt{x} = \frac{1}{2} \log x \right]$$

$$= \frac{1}{2} e^x \left(\log x \times \tan x + \frac{\tan x}{x} + \log x \sec^2 x \right)$$