RD Sharma Solutions Class 11 Maths Chapter 30 Ex 30.2

Derivatives Ex 30.2 Q1(i)

We have,

$$\because f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0} \frac{f\frac{2}{x+h} - \frac{2}{x}}{h}$$

$$= \lim_{h\to 0} \frac{(2x-2x-2h)}{hx(x+h)}$$

$$= \lim_{h\to 0} \frac{2(x-x-h)}{h.x(x+h)}$$

$$= \lim_{h\to 0} \frac{-2h}{hx(x+h)}$$

$$= \lim_{h\to 0} \frac{-2}{x(x+h)}$$

$$= \frac{-2}{x^2}$$

Derivatives Ex 30.2 Q1(ii)

We have,

$$\because f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\frac{1}{\sqrt{x+h}} - \frac{1}{\sqrt{x}}}{h}$$

$$= \lim_{h\to 0} \frac{\sqrt{x} - \sqrt{x+h}}{\sqrt{x}\sqrt{x+h}.h}$$

$$= \lim_{h\to 0} \frac{\sqrt{x} - \sqrt{x+h}}{\sqrt{x}\sqrt{x+h}.h} \times \frac{\sqrt{x} + \sqrt{x+h}}{\sqrt{x} + \sqrt{x+h}\,h}$$

$$= \lim_{h\to 0} \frac{x-(x+h)}{\sqrt{x}\sqrt{x+h}.h\left(\sqrt{x} + \sqrt{x+h}\right)}$$

$$= \lim_{h\to 0} \frac{-h}{\sqrt{x}\sqrt{x+h}.h\left(\sqrt{x} + \sqrt{x+h}\right)}$$

$$= \lim_{h\to 0} \frac{-1}{x\sqrt{x+h} + \sqrt{x}\left(\sqrt{x+h}\right)}$$

$$= \lim_{h\to 0} \frac{-1}{2x\sqrt{x}}$$

$$= \frac{-1}{2}x^{-\frac{3}{2}}$$

Derivatives Ex 30.2 Q1(iii)

We have,

$$f(x) = \frac{1}{x^3}$$

$$\because f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\frac{1}{(x+h)^3} - \frac{1}{x^3}}{h}$$

$$= \lim_{h\to 0} \frac{x^3 - (x+h)^3}{x^3 h (x+h)^3}$$

$$= \lim_{h\to 0} \frac{x^3 - (x^3 + 3x^2h + 3xh^2 + h^3)}{x^3 h (x+h)^3}$$

$$= \lim_{h\to 0} \frac{x^3 - x^3 - 3x^2 - 3xh - h^2}{x^3 (x+h)^3}$$

$$= \lim_{h\to 0} \frac{-3x^2 - 3xh - h^2}{x^3 (x+h)^3}$$

$$= \frac{-3x^2}{x^6}$$

$$= \frac{-3}{x^4}$$

Derivatives Ex 30.2 Q1(iv)

We have,

$$f(x) = \frac{x^2+1}{x}$$

$$\because f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\frac{(x+h)^2+1}{(x+h)} - \frac{x^2+1}{x}}{h}$$

$$= \lim_{h\to 0} \frac{x\left[x^2 + h^2 + 2xh + 1\right] - (x^2+1)(x+h)}{hx(x+h)}$$

$$= \lim_{h\to 0} \frac{x^3 + xh^2 + 2x^2h + x - x^3 - x - x^2h - h}{h.x(x+h)}$$

$$= \lim_{h\to 0} \frac{xh + 2x^2 - x^2 - 1}{x(x+h)}$$

$$= \frac{x^2 - 1}{x^2}$$

$$= 1 - \frac{1}{x^2}$$

Derivatives Ex 30.2 Q1(v)

We have,

$$f(x) = \frac{x^2 - 1}{x}$$

$$\therefore\ f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\frac{(x+h)^2 - 1}{(x+h)} - \frac{x^2 - 1}{x}}{h}$$

$$= \lim_{h\to 0} \frac{x\left(x^2 + h^2 + 2xh - 1\right) - (x+h)\left(x^2 - 1\right)}{x(x+h)h}$$

$$= \lim_{h\to 0} \frac{xh + 2x^2 - x^2 + 1}{x(x+h)}$$

$$= \frac{x^2 + 1}{x^2}$$

$$= 1 + \frac{1}{x^2}$$

Derivatives Ex 30.2 Q1(vi)

We have,

$$f(x) = \frac{x+1}{x+2}$$

$$\because\ f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\frac{(x+h)+1}{(x+h)+2} - \frac{x+1}{x+2}}{h}$$

$$= \lim_{h\to 0} \frac{(x+2)(x+h+1) - (x+1)(x+h+2)}{(x+h+2)(x+2)h}$$

$$= \lim_{h\to 0} \frac{\left(x^2 + 2x + xh + 2h + 2 + x\right) - \left(x^2 + xh + 2x + x + h + 2\right)}{(x+h+2)(x+2)h}$$

$$= \lim_{h\to 0} \frac{h}{(x+h+2)(x+2)h}$$

$$= \frac{1}{(x+2)^2}$$

Derivatives Ex 30.2 Q1(vii)

We have,

$$f(x) = \frac{x+2}{3x+5}$$

$$\because f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\frac{(x+h+2)}{3(x+h)+5} - \frac{x+2}{3x+5}}{h}$$

$$= \lim_{h\to 0} \frac{(3x+5)(x+h+2) - (x+2)(3x+3h+5)}{(3x+5)(3x+3h+5)h}$$

$$= \lim_{h\to 0} \frac{(3x^2+5x+3xh+5h+6x+10) - (3x^2+3xh+5x+6x+6h+10)}{(3x+5)(3x+3h+5)h}$$

$$= \lim_{h\to 0} \frac{-h}{(3x+5)(3x+3h+5)h}$$

$$= \lim_{h\to 0} \frac{-1}{(3x+5)(3x+3h+5)}$$

$$= \frac{-1}{(3x+5)^2}$$

Derivatives Ex 30.2 Q1(viii)

We have,

$$f(x) = kx^n$$

$$\because f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{k(x+h)^n - kx^n}{h}$$

$$= k \lim_{h\to 0} \frac{\left(x^n + nx^{n-1}h + \frac{n(n-1)}{2}x^{n-2}h^2 + \dots\right) - x^n}{h} \qquad \left[\because (x+y)^n = x^n + nx^{n-1}y\dots\right]$$

$$= k \lim_{h\to 0} nx^{n-1} + \frac{n(n-1)}{2!}x^{n-2}h + \frac{n(n-1)(n-2)}{3!}x^{n-3}h^2\dots$$

$$= k\, nx^{n-1} + 0 + 0\dots$$

$$= k\, nx^{n-1}$$

Derivatives Ex 30.2 Q1(ix)

We have,

$$f(x) = \frac{1}{\sqrt{3-x}}$$

$$\because f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{1}{\sqrt{3-(x+h)}} - \frac{1}{\sqrt{3-x}}}{h}$$

$$= \lim_{h \to 0} \frac{\sqrt{3-x} - \sqrt{3-(x+h)}}{\sqrt{3-x}\sqrt{3-(x+h)} \times h}$$

$$= \lim_{h \to 0} \frac{\sqrt{3-x} - \sqrt{3-(x+h)}}{\sqrt{3-x}\sqrt{3-(x+h)}h} \times \frac{\sqrt{3-x} + \sqrt{3-(x+h)}}{\sqrt{3-x} + \sqrt{3-(x+h)}}$$ [Rationalising the numerator by $\sqrt{3-x} + \sqrt{3-(x+h)}$]

$$= \lim_{h \to 0} \frac{(3-x) - (3-(x+h))}{\sqrt{3-x}\sqrt{3-(x+h)} \times h\left(\sqrt{3-x} + \sqrt{3-(x+h)}\right)}$$

$$= \lim_{h \to 0} \frac{h}{\sqrt{3-x}\sqrt{3-(x+h)} \times h\left(\sqrt{3-x} + \sqrt{3-(x+h)}\right)}$$

$$= \frac{1}{(3-x) \times 2\sqrt{3-x}}$$

$$= \frac{1}{2(3-x)^{\frac{3}{2}}}$$

Derivatives Ex 30.2 Q1(x)

We have,

$$f(x) = x^2 + x + 3$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\left\{(x+h)^2 + (x+h) + 3\right\} - x^2 + x + 3}{h}$$

$$= \lim_{h \to 0} \frac{x^2 + h^2 + 2xh + x + h + 3 - x^2 - x - 3}{h}$$

$$= \lim_{h \to 0} \frac{2xh + h^2 + h}{h}$$

$$= \lim_{h \to 0} \frac{h(2x + h + 1)}{h}$$

$$= 2x + 0 + 1$$

$$= 2x + 1$$

Derivatives Ex 30.2 Q1(xi)

We have,

$$f(x) = (x+2)^3$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{(x+h+2)^3 - (x+2)^3}{h}$$

$$= \lim_{h \to 0} \frac{\left\{(x+2) + h\right\}^3 - (x+2)^3}{h}$$

$$= \lim_{h \to 0} \frac{(x+2)^3 + h^3 + 3h(x+2)^2 + 3(x+2)h^2 - (x+2)^3}{h}$$

$$= \lim_{h \to 0} 3(x+2)^2 + 3(x+2)h + h^2$$

$$= 3(x+2)^2$$

Derivatives Ex 30.2 Q1(xii)

We have,

$$f(x) = x^3 + 4x^2 + 3x + 2$$

$$\because f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)^3 + 4(x+h)^2 + 3(x+h) + 2 - (x^3 + 4x^2 + 3x + 2)}{h}$$

On solving we get,

$$= \lim_{h \to 0} \frac{x^3 + h^3 + 3x^2h + 3h^2x + 4x^2 + 4h^2 + 8hx + 3x + 3h + 2 - x^3 - 4x^2 - 3x - 2}{h}$$

$$= \lim_{h \to 0} \frac{3x^2h + 3xh^2 + h^3 + 4h^2 + 8hx + 3h}{h}$$

$$= \lim_{h \to 0} 3x^2 + 3xh + h^2 + 4h + 8x + 3$$

$$= 3x^2 + 8x + 3$$

Derivatives Ex 30.2 Q1(xiii)

We have,

$$f(x) = x^3 - 5x^2 + x - 5$$

$$\because f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\{(x+h)^3 + (x+h) - 5(x+h)^2 - 5\} - (x^3 - 5x^2 + x - 5)}{h}$$

$$= \lim_{h \to 0} \frac{(x^3 + h^3 + 3x^2h + 3h^2x + x + h - 5x^2 - 5h^2 - 10xh - 5) - (x^3 - 5x^2 + x - 5)}{h}$$

$$= \lim_{h \to 0} \frac{(3x^2h + 3h^2x + h^3 + h - 5h^2 - 10xh)}{h}$$

$$= \lim_{h \to 0} 3x^2 + 3xh + h^2 + 1 - 5h - 10x$$

$$= 3x^2 - 10x + 1$$

Derivatives Ex 30.2 Q1(xiv)

We have,

$$f(x) = \sqrt{2x^2+1}$$

$$\because f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\sqrt{2(x+h)^2+1} - \sqrt{2x^2+1}}{h}$$

Multiplying Numerator and Denominator by $\sqrt{2(x+h)^2+1} + \sqrt{2x^2+1}$

$$= \lim_{h\to 0} \frac{\left\{2(x+h)^2+1-\left(2x^2+1\right)\right\}}{h\left(\sqrt{2(x+h)^2+1}+\sqrt{2x^2+1}\right)}$$

$$= \lim_{h\to 0} \frac{2x^2+2h^2+4xh+1-2x^2-1}{h\left(\sqrt{2(x+h)^2+1}+\sqrt{2x^2+1}\right)}$$

$$= \lim_{h\to 0} \frac{4xh+2h^2}{h\sqrt{2(x+h)^2+1}+\sqrt{2x^2+1}}$$

$$= \frac{4x}{2\sqrt{2x^2+1}}$$

$$= \frac{2x}{\sqrt{2x^2+1}}$$

Derivatives Ex 30.2 Q1(xv)

We have, $f(x) = \dfrac{2x+3}{x-2}$

Therefore,

$$f'(x) = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$$

$$= \lim_{h\to 0} \frac{\left(\frac{2x+2h+3}{x+h-2}\right)-\left(\frac{2x+3}{x-2}\right)}{h}$$

$$= \lim_{h\to 0} \frac{2x^2+2hx+3x-4x-4h-6-2x^2-2hx+4x-3x-3h+6}{h(x+h-2)(x-2)}$$

$$= \lim_{h\to 0} \frac{-7}{(x+h-2)(x-2)}$$

$$= \boxed{\frac{-7}{(x-2)^2}}$$

Derivatives Ex 30.2 Q2(i)

We have,

$$f(x) = e^{-x}$$

$$\because f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0} \frac{e^{-(x+h)}-e^{-x}}{h}$$

$$= \lim_{h\to 0} \frac{e^{-x}\left(e^{-h}-1\right)}{h}$$

$$= \lim_{h\to 0} \frac{-e^{-x}\left(e^{-h}-1\right)}{-h}$$

$$= -e^{-x} \qquad \left[\because \lim_{\theta\to 0} \frac{e^{\theta}-1}{\theta} = 1\right]$$

Derivatives Ex 30.2 Q2(ii)

We have,

$f(x) = e^{3x}$

$$\because f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{e^{3(x+h)} - e^{3x}}{h}$$

$$= \lim_{h \to 0} \frac{e^{3(x)}.e^{3h} - e^{3x}}{h}$$

$$= \lim_{h \to 0} \frac{e^{3(x)}\left(e^{3h} - 1\right)}{h}$$

Multiplying Numerator and Denominator by 3

$$= \lim_{h \to 0} e^{3(x)} \frac{\left(e^{3h} - 1\right)}{3h} \qquad \left[\lim_{h \to 0} \frac{e^{3h} - 1}{3} = 1\right]$$

$$= 3e^{3x}$$

Derivatives Ex 30.2 Q2(iii)

We have,

$f(x) = e^{ax+b}$

$$\because f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{e^{a(x+h)+b} - e^{ax+b}}{h}$$

$$= \lim_{h \to 0} \frac{e^{ax} \times e^{ah} \times e^{b} - e^{ax} \times e^{b}}{h}$$

$$= \lim_{h \to 0} \frac{e^{b} \times e^{ax}\left(e^{ah} - 1\right)}{h}$$

$$= \lim_{h \to 0} e^{ax+b} \times \frac{a\left(e^{ah} - 1\right)}{a.h}$$

Multiplying Numerator and denominator by a $\qquad \left[\lim_{h \to 0} \frac{\left(e^{ah} - 1\right)}{ah} = 1\right]$

$$= ae^{ax+b}$$

Derivatives Ex 30.2 Q2(iv)

We have,

$f(x) = xe^x$

$$\because f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$
$$= \lim_{h\to 0}\frac{(x+h)e^{(x+h)} - xe^x}{h}$$
$$= \lim_{h\to 0}\frac{xe^x.e^h + he^x.e^h - xe^x}{h}$$
$$= \lim_{h\to 0}\ xe^x\left(\frac{e^h-1}{h}\right) + \frac{he^{x+h}}{h}$$
$$= xe^x + e^x$$
$$= e^x(x+1)$$

Derivatives Ex 30.2 Q2(v)

Let $f(x) = -x$. Then, $f(x+h) = -(x+h)$

$$\therefore \frac{d}{dx}(f(x)) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$
$$\Rightarrow \frac{d}{dx}(f(x)) = \lim_{h\to 0}\frac{-(x+h)+(x)}{h}$$
$$\Rightarrow \frac{d}{dx}(f(x)) = \lim_{h\to 0}\frac{-h}{h}$$
$$\Rightarrow \frac{d}{dx}(f(x)) = \lim_{h\to 0} -1$$
$$\Rightarrow \frac{d}{dx}(f(x)) = -1$$

Derivatives Ex 30.2 Q2(vi)

Let $f(x) = (-x)^{-1}$. Then, $f(x+h) = (-(x+h))^{-1}$

$$\therefore \frac{d}{dx}(f(x)) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$
$$\Rightarrow \frac{d}{dx}(f(x)) = \lim_{h\to 0}\frac{(-(x+h))^{-1}-(-x)^{-1}}{h}$$
$$\Rightarrow \frac{d}{dx}(f(x)) = \lim_{h\to 0}\frac{\frac{-1}{x+h}+\frac{1}{x}}{h}$$
$$\Rightarrow \frac{d}{dx}(f(x)) = \lim_{h\to 0}\frac{\frac{-x+x+h}{x(x+h)}}{h}$$
$$\Rightarrow \frac{d}{dx}(f(x)) = \lim_{h\to 0}\frac{h}{hx(x+h)}$$
$$\Rightarrow \frac{d}{dx}(f(x)) = \lim_{h\to 0}\frac{1}{x(x+h)}$$
$$\Rightarrow \frac{d}{dx}(f(x)) = \frac{1}{x(x+0)}$$
$$\Rightarrow \frac{d}{dx}(f(x)) = \frac{1}{x^2}$$

Derivatives Ex 30.2 Q2(vii)

Let $f(x) = \sin(x+1)$. Then, $f(x+h) = \sin((x+h)+1)$

$$\therefore \frac{d}{dx}(f(x)) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$\Rightarrow \frac{d}{dx}(f(x)) = \lim_{h\to 0}\frac{\sin((x+h)+1)-\sin(x+1)}{h}$$

$$\Rightarrow \frac{d}{dx}(f(x)) = \lim_{h\to 0}\frac{2\sin\left[\frac{((x+h)+1)-(x+1)}{2}\right]\cos\left[\frac{((x+h)+1)+(x+1)}{2}\right]}{h}$$

$$\Rightarrow \frac{d}{dx}(f(x)) = \lim_{h\to 0}\frac{2\sin\left[\frac{h}{2}\right]\cos\left[\frac{2x+2+h}{2}\right]}{h}$$

$$\Rightarrow \frac{d}{dx}(f(x)) = \lim_{h\to 0}\frac{\sin\left[\frac{h}{2}\right]}{\frac{h}{2}} \times \lim_{h\to 0}\cos\left[\frac{2x+2+h}{2}\right]$$

$$\Rightarrow \frac{d}{dx}(f(x)) = 1 \times \cos\left[\frac{2x+2+0}{2}\right]$$

$$\Rightarrow \frac{d}{dx}(f(x)) = \cos(x+1)$$

Derivatives Ex 30.2 Q2(viii)

Let $f(x) = \cos\left(x-\frac{\pi}{8}\right)$. Then, $f(x+h) = \cos\left(x+h-\frac{\pi}{8}\right)$

$$\therefore \frac{d}{dx}(f(x)) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$\Rightarrow \frac{d}{dx}(f(x)) = \lim_{h\to 0}\frac{\cos\left(x+h-\frac{\pi}{8}\right)-\cos\left(x-\frac{\pi}{8}\right)}{h}$$

$$\Rightarrow \frac{d}{dx}(f(x)) = \lim_{h\to 0}\frac{-2\sin\left[\frac{\left(x+h-\frac{\pi}{8}\right)+\left(x-\frac{\pi}{8}\right)}{2}\right]\sin\left[\frac{\left(x+h-\frac{\pi}{8}\right)-\left(x-\frac{\pi}{8}\right)}{2}\right]}{h}$$

$$\Rightarrow \frac{d}{dx}(f(x)) = \lim_{h\to 0}\frac{-2\sin\left[\frac{2x+h-\frac{2\pi}{8}}{2}\right]\sin\left[\frac{h}{2}\right]}{h}$$

$$\Rightarrow \frac{d}{dx}(f(x)) = \lim_{h\to 0}-\sin\left[\frac{2x+h-\frac{2\pi}{8}}{2}\right]\times\lim_{h\to 0}\frac{\sin\left[\frac{h}{2}\right]}{\frac{h}{2}}$$

$$\Rightarrow \frac{d}{dx}(f(x)) = -\sin\left[\frac{2x+0-\frac{2\pi}{8}}{2}\right]\times 1$$

$$\Rightarrow \frac{d}{dx}(f(x)) = -\sin\left(x-\frac{\pi}{8}\right)$$

Derivatives Ex 30.2 Q2(ix)

We have,

$f(x) = x \sin x$

$$\because f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{(x+h)\sin(x+h) - x\sin x}{h}$$

$$= \lim_{h\to 0} \frac{x\left(\sin(x+h) - \sin x\right)}{h} + \sin(x+h) \qquad \left[\sin c - \sin d = 2\cos\frac{c+d}{2}\sin\frac{c-d}{2}\right]$$

$$= \lim_{h\to 0} \frac{x \times 2\cos\left(x+\frac{h}{2}\right)\sin\frac{h}{2}}{h} + \sin(x+h) \qquad \left[\because \lim_{\theta\to 0}\frac{\sin\theta}{\theta} = 1\right]$$

$$= 2x \times \cos x \times \frac{1}{2} + \sin x$$

$$= x \times \cos x + \sin x$$

$$= \sin x + x\cos x$$

Derivatives Ex 30.2 Q2(x)

We have,

$f(x) = x\cos x$

$$\because f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{(x+h)\cos(x+h) - x\cos x}{h}$$

$$= \lim_{h\to 0} \frac{x\cos(x+h)h\cos(x+h) - x\cos x}{h}$$

$$= \lim_{h\to 0} \frac{x\left\{\cos(x+h) - \cos x\right\}}{h} + \cos(x+h)$$

$$= \lim_{h\to 0} x.2\sin\left(x - x - \frac{h}{2}\right)\sin\left(x + \frac{h}{2}\right) + \cos(x+h) \qquad \left[\because \cos A - \cos B = 2\sin\frac{B-A}{2}\sin\frac{B+A}{2}\right]$$

$$= \lim_{h\to 0} 2x.\sin\left(\frac{-h}{2}\right)\sin\left(x + \frac{h}{2}\right) + \cos(x+h)$$

$$= -x\sin x + \cos x$$

Derivatives Ex 30.2 Q2(xi)

We have,

$f(x) = \sin(2x-3)$

$$\because f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\sin\{2(x+h) - 3\} - \sin(2x-3)}{h}$$

$$= \lim_{h\to 0} \frac{2\cos\frac{(2x+2h-3+2x-3)}{2} \times \frac{\sin(2x+2h-3-2-x+3)}{2}}{h} \qquad \left[\because \sin C - \sin D = 2\cos\frac{C+D}{2}\sin\frac{C-D}{2}\right]$$

$$= \lim_{h\to 0} 2\cos(2x-3+h).\frac{\sinh}{2} \qquad \left[\because \lim_{\theta\to 0}\frac{\sin\theta}{\theta} = 1\right]$$

$$= 2\cos(2x-3)$$

Derivatives Ex 30.2 Q3(i)

We have,

$f(x) = \sqrt{\sin 2x}$

$$\because f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\sqrt{\sin 2(x+h)} - \sqrt{\sin 2x}}{h}$$

Multiplying Numerator and Denominator by $\left(\sqrt{\sin 2(x+h)} + \sqrt{\sin 2x}\right)$

$$= \lim_{h\to 0} \frac{\sqrt{\sin 2(x+h)} - \sqrt{\sin 2x}}{h} \times \frac{\sqrt{\sin 2(x+h)} + \sqrt{\sin 2x}}{\sqrt{\sin 2(x+h)} + \sqrt{\sin 2x}}$$

$$= \lim_{h\to 0} \frac{\sin(2x+2h) - \sin 2x}{h\left(\sqrt{\sin 2(x+h)} + \sqrt{\sin 2x}\right)}$$

$$= \lim_{h\to 0} \frac{\sin(2x+2h) - \sin 2x}{h\left(\sqrt{\sin(2x+2h)} + \sqrt{\sin 2x}\right)} \qquad \left[\sin C - \sin D = 2\cos\frac{C+D}{2}\sin\frac{C-D}{2}\right]$$

$$= \lim_{h\to 0} \frac{2\cos(2x+h) \times \sin h}{h} \times \frac{1}{\sqrt{\sin(2x+2h)} + \sqrt{\sin 2x}}$$

$$= \frac{2\cos 2x}{2\sqrt{\sin 2x}}$$

$$= \frac{\cos 2x}{\sqrt{\sin 2x}}$$

Derivatives Ex 30.2 Q3(ii)

We have,

$$f(x)=\frac{\sin x}{x}$$

$$\because f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$=\lim_{h\to 0}\frac{\frac{\sin(x+h)}{x+h}-\frac{\sin x}{x}}{h}$$

$$=\lim_{h\to 0}\frac{x\sin(x+h)-(x+h)\sin x}{xh(x+h)}$$

$$=\lim_{h\to 0}\frac{x(\sin x.\cos h+\cos x.\sin h)-x.\sin x-h.\sin x}{xh(x+h)}\qquad [\because \sin(A+B=\sin A.\cos B+\cos A.\sin B)]$$

$$=\lim_{h\to 0}\frac{x.\sin x(\cos h-1)}{xh(x+h)}+\frac{x.\cos x.\sin h}{(x+h)xh}-\frac{h\sin x}{(x+h)xh}\qquad \left[\because 1-\cos h=2\sin^2\frac{h}{2}\right]$$

$$=\frac{-x\sin x}{x(x+h)}\times\frac{2\sin^2\frac{h}{2}}{\frac{h^2}{4}}\times\frac{h}{4}+\frac{x\cos x}{x^2}-\frac{\sin x}{x^2}$$

$$\because h\to 0\Rightarrow\frac{h}{2}\to 0\quad and\quad \lim_{\theta\to 0}\frac{\sin\theta}{\theta}=1$$

$$=0+\frac{x\cos x-\sin x}{x^2}$$

$$=\frac{x\cos x-\sin x}{x^2}$$

Derivatives Ex 30.2 Q3(iii)

We have,

$$f(x)-\frac{\cos x}{x}$$

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$=\lim_{h\to 0}\frac{\frac{\cos(x+h)}{x+h}-\frac{\cos x}{x}}{h}$$

$$=\lim_{h\to 0}\frac{x.\cos(x+h)-(x+h)\cos x}{(x+h)xh}\qquad [\because \cos(A+B)=\cos A.\cos B-\sin A.\sin B]$$

$$=\lim_{h\to 0}\frac{x[\cos x.\cos h-\sin x.\sin h]-x.\cos x-h.\cos x}{(x+h)x.h}$$

$$=\lim_{h\to 0}\frac{x\cos x(\cos h-1)}{(x+h)x.h}-\frac{x.\cos x.\sin h}{(x+h)xh}-\frac{h.\cos x}{(x+h)xh}$$

$$=\lim_{h\to 0}\frac{-x\cos x.2\sin^2\frac{h}{2}}{(x+h)x\frac{h^2}{4}}\times\frac{h^2}{4}-\frac{x.\sin x}{x(x+h)}-\frac{\cos x}{x(x+h)}$$

$$=0-\frac{x\sin x}{x^2}-\frac{\cos x}{x^2}$$

$$=-\frac{x\sin x-\cos x}{x^2}$$

Derivatives Ex 30.2 Q3(iv)

We have,

$f(x) = x^2 \sin x$

$\therefore f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$

$= \lim_{h\to 0} \frac{(x+h)^2 \sin(x+h) - x^2 \sin x}{h}$

$= \lim_{h\to 0} \frac{(x^2+h^2+2hx)(\sin x.\cos h + \cos x.\sin h) - x^2 \sin x}{h}$ $\quad [\because \sin(A+B) = \sin A.\cos B + \cos B.\sin A]$

$= \lim_{h\to 0} \frac{x^2 \sin x(\cos h - 1)}{h} + \frac{h(h+2x)\sin x.\cos h}{h} + (x+h)^2 \cos x \frac{\sin h}{h}$

$= \lim_{h\to 0} -x^2 \sin x \times \frac{2\sin^2 \frac{h}{2}}{\left(\frac{h}{2}\right)^2} \times \frac{h^2}{4} + (h+2x)\sin x.\cos h + (x+h)^2 \cos x$

$= 0 + (2x \sin x + x^2 \cos x)$

$= 2x \sin x + x^2 \cos x$

Derivatives Ex 30.2 Q3(v)

We have,

$f(x) = \sqrt{\sin(3x+1)}$

$\therefore f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$

$= \lim_{h\to 0} \frac{\sqrt{\sin 3(x+h)+1} - \sqrt{\sin(3x+1)}}{h}$

$= \lim_{h\to 0} \frac{\sqrt{\sin(3x+3h)+1} - \sqrt{\sin(3x+1)}}{h} \times \frac{\sqrt{\sin(3x+3h)+1} + \sqrt{\sin(3x+1)}}{\sqrt{\sin(3x+3h)+1} + \sqrt{\sin(3x+1)}}$

$= \lim_{h\to 0} \frac{\sin(3x+3h+1) - \sin(3x+1)}{h\left(\sqrt{\sin(3x+3h)+1} + \sqrt{\sin(3x+1)}\right)}$

$= \lim_{h\to 0} 2\cos\left(3x+1+\frac{3h}{2}\right) \times \frac{\sin\frac{3h}{2}}{\frac{3h}{2}} \times \frac{3}{2} \times \frac{1}{\sqrt{\sin(3x+3h)+1} + \sqrt{\sin(3x+1)}}$

$= \frac{3\cos(3x+1)}{2\sqrt{\sin(3x+1)}}$ $\quad \left[\lim_{h\to 0} \frac{\sin\frac{3h}{2}}{\frac{3h}{2}} = 1\right]$

Derivatives Ex 30.2 Q3(vi)

We have,

$f(x) = \sin x + \cos x$

$$\therefore f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\{\sin(x+h) + \cos(x+h)\} - \sin x + \cos x}{h}$$

$$= \lim_{h\to 0} \frac{\{\sin(x+h) + \cos(x+h) - \sin x - \cos x\}}{h}$$

$$= \lim_{h\to 0} \frac{\{\sin(x+h) - \sin x\} + \{\cos(x+h) - \cos x\}}{h}$$

$$= \lim_{h\to 0} \frac{\left\{2\sin\frac{(x+h-x)}{2}\cos\frac{(x+h+x)}{2}\right\} + \left\{-2\sin\frac{x+h+x}{2}\sin\frac{x+h-x}{2}\right\}}{h}$$

$$\left[\because \sin A - \sin B = 2\sin\frac{A-B}{2}\cos\frac{A+B}{2} \text{ and } \cos A - \cos B = 2\sin\frac{A+B}{2}\sin\frac{A-B}{2}\right]$$

$$= \lim_{h\to 0} \frac{\sinh . \cos\frac{2x+h}{2} - 2\sin\left(x + \frac{h}{2}\right)\sinh}{h}$$

$$= \lim_{h\to 0} \frac{\sinh}{h}\left\{\cos\frac{x+h}{2} - \sin\left(x + \frac{h}{2}\right)\right\} \qquad \left[\because \lim_{h\to 0}\frac{\sinh}{h} = 1\right]$$

$$= \cos x - \sin x$$

Derivatives Ex 30.2 Q3(vii)

We have,

$f(x) = x^2 e^x$

$$\because f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{x^2 e^x e^h + h^2 . e^x . e^h + 2xhe^x e^h - x^2 e^x}{h}$$

$$= \lim_{h\to 0} x^2 e^x \frac{(e^h - 1)}{h} + e^x e^h \frac{(h^2 + 2xh)}{h} \qquad \left[\because \frac{e^h - 1}{h} - 1\right]$$

$$\therefore x^2 e^x + e^x (0 + 2x)$$

$$= x^2 e^x + 2xe^x$$

$$= e^x (x^2 + 2x)$$

Derivatives Ex 30.2 Q3(viii)

We have,

$f(x) = e^{x^2+1}$

$$\therefore f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{e^{(x+h)^2} + 1 - e^{x^2+1}}{h}$$

$$= \lim_{h\to 0} \frac{e^{x^2+h^2+2xh+1} - e^{x^2+1}}{h}$$

$$= \lim_{h\to 0} \frac{e^{x^2+1}\left(e^{2xh}.e^{h^2} - 1\right)}{h}$$

$$= \lim_{h\to 0} \frac{e^{x^2+1}\left(e^{2xh+h^2} - 1\right)}{2xh+h^2} \times \frac{2xh+h^2}{h}$$

$\because h \to 0$

$\Rightarrow 2xh + h^2 = 0$

and $\lim_{\theta\to 0} \frac{e^\theta - 1}{\theta} = 1$

$= \lim_{h\to 0} \; e^{x^2+1}.1 \times 2x + h$

$= 2xe^{x^2+1}$

Derivatives Ex 30.2 Q3(ix)

We have,

$f(x) = e\sqrt{2x}$

$$\because f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{e\sqrt{2(x+h)} - e\sqrt{2x}}{h}$$

$$= \lim_{h\to 0} \frac{e\sqrt{2x}\left(e^{\sqrt{2(x+h)}-\sqrt{2x}} - 1\right)}{h}$$

$$= \lim_{h\to 0} \; e\sqrt{2x} \frac{\left(e^{\sqrt{2(x+h)}-\sqrt{2x}} - 1\right)}{\sqrt{2(x+h)} - \sqrt{2x}} \times \frac{\sqrt{2(x+h)} - \sqrt{2x}}{h}$$

Multiplying Numerator and Denominator by $\sqrt{2(x+h)} - \sqrt{2x}$

$\because h \to 0 \Rightarrow \sqrt{2(x+h)} - \sqrt{2x} \Rightarrow 0$

and $\lim_{\theta\to 0} \frac{e^\theta - 1}{\theta} = 1$

$$\therefore \lim_{h\to 0} \; e^{\sqrt{2x}} \times \frac{\sqrt{2(x+h)} - \sqrt{2x}}{h}$$

Again Multiplying Numerator and Denominator by $\sqrt{2(x+h)} + \sqrt{2x}$

$$\therefore \lim_{h\to 0} \; e^{\sqrt{2x}} \times \frac{\sqrt{2(x+h)} - \sqrt{2x}}{h} \times \frac{\sqrt{2(x+h)} + \sqrt{2x}}{\sqrt{2(x+h)} + \sqrt{2x}}$$

$$= e^{\sqrt{2x}} \times \frac{1}{2\sqrt{2x}}$$

Derivatives Ex 30.2 Q3(x)

We have,

$f(x) = e^{\sqrt{ax+b}}$

$$\because f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{e^{\sqrt{a(x+h)+b}} - e^{\sqrt{ax+b}}}{h}$$

$$= \lim_{h\to 0} e^{\sqrt{a(x+h)+b}} \frac{\left(e^{\sqrt{a(x+h)+b}-\sqrt{ax+b}} - 1\right)}{h}$$

$$= \lim_{h\to 0} e^{\sqrt{ax+b}} \times \frac{e^{\sqrt{a(x+h)+b}-\sqrt{ax+b}} - 1}{\sqrt{a(x+h)+b} - \sqrt{ax+b}} \times \frac{\sqrt{a(x+h)+b} - \sqrt{ax+b}}{h}$$

Multiplying Numerator and Denominator by $\sqrt{a(x+h)+b} - \sqrt{ax+b}$

$\because h \to 0$

$\therefore \sqrt{a(x+h)+b} - \sqrt{ax+b} = 0$

and $\lim_{\theta\to 0} \frac{e^{\theta} - 1}{\theta} = 1$

$$= \lim_{h\to 0} e^{\sqrt{ax+b}} \times 1 \times \frac{\sqrt{a(x+h)+b} - \sqrt{ax+b}}{\sqrt{a(x+h)+b} + \sqrt{ax+b}} \times \frac{\sqrt{a(x+h)+b} + \sqrt{ax+b}}{h}$$

Again multiplied Numerator and Denominator by $\sqrt{a(x+h)+b} + \sqrt{ax+b}$

$$= \lim_{h\to 0} e^{\sqrt{ax+b}} \times \frac{a(x+h)+b-(ax+b)}{h} \times \frac{1}{\left(\sqrt{a(x+h)+b} + \sqrt{ax+b}\right)}$$

$$= \frac{e^{\sqrt{ax+b}} \times a}{2\sqrt{ax+b}}$$

$$= \frac{ae^{\sqrt{ax+b}}}{2\sqrt{ax+b}}$$

Derivatives Ex 30.2 Q3(xi)

$$f(x) = a^{\sqrt{x}} = e^{\sqrt{x}\log a}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{e^{\sqrt{x+h}\log a} - e^{\sqrt{x}\log a}}{h}$$

$$= \lim_{h \to 0} e^{\sqrt{x}\log a} \frac{e^{\sqrt{x+h}\log a - \sqrt{x}\log a} - 1}{h}$$

$$= \lim_{h \to 0} e^{\sqrt{x}\log a} \frac{e^{\left(\sqrt{x+h} - \sqrt{x}\right)\log a} - 1}{h}$$

Multiply numerator and denominator by $\left(\sqrt{x+h} - \sqrt{x}\right)\log a$

$$f'(x) = \lim_{h \to 0} e^{\sqrt{x}\log a} \frac{e^{\left(\sqrt{x+h} - \sqrt{x}\right)\log a} - 1}{h\left(\sqrt{x+h} - \sqrt{x}\right)\log a}\left(\sqrt{x+h} - \sqrt{x}\right)\log a$$

$$= e^{\sqrt{x}\log a} \lim_{h \to 0} \frac{e^{\left(\sqrt{x+h} - \sqrt{x}\right)\log a} - 1}{\left(\sqrt{x+h} - \sqrt{x}\right)\log a} \lim_{h \to 0} \log a \frac{\left(\sqrt{x+h} - \sqrt{x}\right)}{h}$$

$$= e^{\sqrt{x}\log a} \lim_{h \to 0} \log a \frac{\left(\sqrt{x+h} - \sqrt{x}\right)}{h}$$

Multiply numerator and denominator by $\left(\sqrt{x+h} + \sqrt{x}\right)$

$$f'(x) = e^{\sqrt{x}\log a} \lim_{h \to 0} \log a \frac{\left(\sqrt{x+h} - \sqrt{x}\right)}{h\left(\sqrt{x+h} + \sqrt{x}\right)}\left(\sqrt{x+h} + \sqrt{x}\right)$$

$$= e^{\sqrt{x}\log a} \lim_{h \to 0} \log a \frac{h}{h\left(\sqrt{x+h} + \sqrt{x}\right)}$$

$$= e^{\sqrt{x}\log a} \frac{\log a}{2\sqrt{x}}$$

$$= \frac{a^{\sqrt{x}}}{2\sqrt{x}} \log_e a$$

Derivatives Ex 30.2 Q3(xii)

We have,

$$f(x) = 3^{x^2} = e^{x^2 \log 3}$$

$$\because f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{e^{(x+h)^2 \log 3} - e^{x^2 \log 3}}{h}$$

$$= \lim_{h \to 0} e^{x^2 \log 3} \frac{\left[e^{\left((x+h)^2 - x^2\right)^{\log 3}} - 1\right]}{h}$$

$$= \lim_{h \to 0} e^{x^2 \log 3} \frac{\left[e^{(x+h)^2 - x^2}\right]^{\log 3} - 1}{(x+h)^2 - x^2} \times \frac{(x+h)^2 - x^2}{h}$$

Multiplying Numerator and Denominator by $(x+h)^2 - x^2$

$$\therefore \lim_{h \to 0} e^{x^2 \log 3} \times \frac{(x+h+x)(x+h-x)}{h}$$

$$= e^{x^2} \log 3 \quad \times 2x$$

$$= 2x\, e^{x^2} \log 3$$

$$= 2x\, 3^{x^2} \log 3$$

Derivatives Ex 30.2 Q4(i)

We have,

$f(x) = \tan^2 x$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\tan^2(x+h) - \tan^2 x}{h}$$

$$= \lim_{h\to 0} \frac{\{\tan(x+h) + \tan x\}\{\tan(x+h) - \tan x\}}{h} \quad \left[\because \tan^2 A - \tan^2 B = (\tan A + \tan B)(\tan A - \tan B)\right]$$

$$= \lim_{h\to 0} \frac{\frac{\sin(x+h+x)}{\cos(x+h)\cos x} \times \frac{\sin(x+h-x)}{\cos(x+h)\cos x}}{h}$$

$$= \lim_{h\to 0} \frac{\sin(2x+h)}{h.\cos(x+h)\cos x} \times \frac{\sinh}{\cos(x+h)\cos x}$$

$$= \lim_{h\to 0} \frac{\sinh}{h} \times \frac{\sin 2x}{\cos^2 x.\cos^2(x+h)}$$

$$= \lim_{h\to 0} \frac{\sin 2x}{\cos^2 x.\cos^2 x} \quad \left[\because \lim_{h\to 0} \frac{\sinh}{h} = 1\right]$$

$$= \lim_{h\to 0} \frac{2\sin x.\cos x}{\cos^2 x} \times \frac{1}{\cos^2 x} \quad \left[\sin 2x = 2\sin x\cos x\right]$$

$$= 2\tan x.\sec^2 x$$

Derivatives Ex 30.2 Q4(ii)

We have,

$f(x) = \tan(2x+1)$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\tan\{2(x+h)+1\} - \tan(2x+1)}{h}$$

$$= \lim_{h\to 0} \frac{\sin(2x+2h+1-2x-1)}{h.\cos\{2(x+h)+1\}\cos(2x+1)} \quad \left[\because \tan A - \tan B = \frac{\sin(A-B)}{\cos A.\cos B}\right]$$

$$= \lim_{h\to 0} \frac{2.\sin 2h}{2h.\cos(2x+2h+1)\cos(2x+1)}$$

Multiplying both, Numerator and Denominator by 2.

$$\therefore \lim_{h\to 0} 2\left(\frac{\sin 2h}{2}\right) \times \frac{1}{\cos(2x+2h+1)\cos(2x+1)}$$

$$= \frac{2}{\cos^2(2x+1)} \quad \left[\because \lim_{h\to 0} \frac{\sin 2h}{2} = 1\right]$$

$$= 2\sec^2(2x+1) \quad \left[\because \sec^2 x = \frac{1}{\cos^2 x}\right]$$

$$= 2\sec^2(2x+1)$$

Derivatives Ex 30.2 Q4(iii)

We have,

$f(x) = \tan 2x$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\tan 2(x+h) - \tan 2x}{h}$$

$$= \lim_{h\to 0} \frac{\sin(2x+2h-2x)}{h.\cos(2x+2h)\cos 2x} \qquad \left[\because \tan A - \tan B = \frac{\sin(A-B)}{\cos A.\cos B}\right]$$

$$= \lim_{h\to 0} \frac{\sin 2h}{h.\cos(2x+2h)\cos 2x}$$

$$= \lim_{h\to 0} \left(\frac{\sin 2h}{2h}\right) \times \frac{1 \times 2}{\cos(2h+2x)\cos 2x}$$

$$= \frac{2}{\cos 2x.\cos 2x} \qquad \left[\because \lim_{h\to 0} \frac{\sin 2h}{2h} = 1\right]$$

$$= 2\sec^2 2x \qquad \left[\because \frac{1}{\cos^2 x} = \sec^2 x\right]$$

Derivatives Ex 30.2 Q4(iv)

We have,

$f(x) = \sqrt{\tan x}$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\sqrt{\tan(x+h)} - \sqrt{\tan x}}{h}$$

Multiplying Numerator and Denominator by $\sqrt{\tan(x+h)} + \sqrt{\tan x}$

$$= \lim_{h\to 0} \frac{\tan(x+h) - \tan x}{h\left(\sqrt{\tan(x+h)} + \sqrt{\tan x}\right)}$$

$$= \lim_{h\to 0} \frac{\sin(x+h-x)}{h.\cos(x+h)\cos x\left\{\sqrt{\tan(x+h)} + \sqrt{\tan x}\right\}}$$

$$= \lim_{h\to 0} \frac{\sinh}{h} \times \frac{1}{\cos(x+h)\cos x\left\{\sqrt{\tan(x+h)} + \sqrt{\tan x}\right\}}$$

$$= \lim_{h\to 0} \frac{1}{\cos^2 x.2\sqrt{\tan x}} \qquad \left[\lim_{h\to 0} \frac{\sinh}{h} = 1\right]$$

$$= \frac{1}{2}\frac{\sec^2 x}{\sqrt{tanx}} \qquad \left[\because \frac{1}{\cos^2 x} = \sec^2 x\right]$$

Derivatives Ex 30.2 Q5(i)

let $f(x) = \sin\sqrt{2x}$. Then $f(x+h) = \sin\sqrt{2(x+h)}$

$$\frac{d(f(x))}{dx} = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0}\frac{\sin\sqrt{2(x+h)} - \sin\sqrt{2x}}{h}$$

$$= \lim_{h\to 0}\frac{2\sin\left(\frac{\sqrt{2(x+h)}-\sqrt{2x}}{2}\right)\cos\left(\frac{\sqrt{2(x+h)}+\sqrt{2x}}{2}\right)}{h}$$

$$= \lim_{h\to 0}\frac{\sin\left(\frac{\sqrt{2(x+h)}-\sqrt{2x}}{2}\right)}{\left(\frac{\sqrt{2(x+h)}-\sqrt{2x}}{2}\right)}\frac{\left(\sqrt{2(x+h)}-\sqrt{2x}\right)\left(\sqrt{2(x+h)}+\sqrt{2x}\right)}{\left(\sqrt{2(x+h)}+\sqrt{2x}\right)h}\cos\left(\frac{\sqrt{2(x+h)+2x}}{2}\right)$$

$$= \lim_{h\to 0}\frac{\sin\left(\frac{\sqrt{2(x+h)}-\sqrt{2x}}{2}\right)}{\left(\frac{\sqrt{2(x+h)}-\sqrt{2x}}{2}\right)}\lim_{h\to 0}\frac{2(x+h)-2x}{\left(\sqrt{2(x+h)}+\sqrt{2x}\right)h}\lim_{h\to 0}\cos\left(\frac{\sqrt{2(x+h)+2x}}{2}\right)$$

$$= 1\times\frac{2}{2\sqrt{2x}}\cos\left(\sqrt{2x}\right)$$

$$= \frac{\cos\left(\sqrt{2x}\right)}{\sqrt{2x}}$$

Derivatives Ex 30.2 Q5(ii)

We have,

$$f(x) = \cos\sqrt{x}$$

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0}\frac{\cos\sqrt{x+h}-\cos\sqrt{x}}{h}$$

$$= \lim_{h\to 0}-2\sin\frac{\left(\frac{\sqrt{x+h}+\sqrt{x}}{2}\right)\sin\left(\frac{\sqrt{x+h}-\sqrt{x}}{2}\right)}{h}$$

$$= \lim_{h\to 0}-2\sin\frac{\left(\frac{\sqrt{x+h}-\sqrt{x}}{2}\right)\left(\sqrt{x+h}-\sqrt{x}\right)\left(\sqrt{x+h}+\sqrt{x}\right)}{\left(\frac{\sqrt{x+h}-\sqrt{x}}{2}\right)\left(\sqrt{x+h}+\sqrt{x}\right)h}\times\sin\left(\frac{\sqrt{x+h}+\sqrt{x}}{2}\right)$$

Multiplied Numerator and Denominator by $\left(\sqrt{x+h}-\sqrt{x}\right)$

$$= \lim_{h\to 0}\frac{-\sin\left(\frac{\sqrt{x+h}-\sqrt{x}}{2}\right)}{\left(\frac{\sqrt{x+h}-\sqrt{x}}{2}\right)}\times\frac{x+h-x}{\left(\sqrt{x+h}+\sqrt{x}\right)h}\times\sin\left(\frac{\sqrt{x+h}+\sqrt{x}}{2}\right)$$

$$= \lim_{h\to 0}-1\frac{h}{h\left(\sqrt{x+h}+\sqrt{x}\right)}\times\sin\left(\frac{\sqrt{x+h}+\sqrt{x}}{2}\right) \qquad \left[\because \lim_{\theta\to 0}\frac{\sin\theta}{\theta}=1\right]$$

$$= \lim_{h\to 0}\frac{-1}{\left(\sqrt{x+h}+\sqrt{x}\right)}\sin\frac{\sqrt{x+h}+\sqrt{x}}{2}$$

$$= \frac{-\sin\sqrt{x}}{2\sqrt{x}}$$

Derivatives Ex 30.2 Q5(iii)

We have,

$$f(x) = \tan\sqrt{x}$$

$$\because f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$\therefore f'(x) = \lim_{h\to 0} \frac{\tan\sqrt{(x+h)} - \tan\sqrt{x}}{h}$$

$$= \lim_{h\to 0} \frac{\sin\sqrt{x+h} - \sqrt{x}}{h.\cos\sqrt{x+h}\,\cos\sqrt{x}} \qquad \left[\because \tan A - \tan B = \frac{\sin(A-B)}{\cos A.\cos B}\right]$$

$$= \lim_{h\to 0} \frac{\sin\sqrt{x+h} - \sqrt{x}}{(x+h-x)\cos\sqrt{x}.\cos\sqrt{x+h}}$$

$$= \lim_{h\to 0} \frac{\sin\left(\sqrt{x+h} - \sqrt{x}\right)}{\left(\sqrt{x+h} - \sqrt{x}\right)\left(\sqrt{x+h} + \sqrt{x}\right)\cos\sqrt{x}.\cos\sqrt{x+h}}$$

$$= \lim_{h\to 0} \frac{\sin\left(\sqrt{x+h} - \sqrt{x}\right)}{\left(\sqrt{x+h} - \sqrt{x}\right)} \times \frac{1}{\left(\sqrt{x+h} + \sqrt{x}\right)\cos\sqrt{x}.\cos\sqrt{x+h}}$$

$$= 1 \times \frac{1}{2\sqrt{x}\,\cos\sqrt{x}\,\cos\sqrt{x} +} \qquad \left[\because \lim_{h\to 0} = \frac{\sin\sqrt{x+h} - \sqrt{x}}{\sqrt{x+h} - \sqrt{x}} = 1\right]$$

$$= \frac{1}{2\sqrt{x}\,\cos^2 x}$$

$$= \frac{\sec^2 x}{2\sqrt{x}}$$

Derivatives Ex 30.2 Q5(iv)

We have,

$f(x) = \tan x^2$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\tan(x+h)^2 - \tan x^2}{h}$$

$$= \lim_{h\to 0} \frac{\frac{\sin(x+h)^2}{\cos(x+h)^2} - \frac{\sin x^2}{\cos x^2}}{h}$$

$$= \lim_{h\to 0} \frac{\frac{\sin(x+h)^2 \cos x^2 - \cos(x+h)^2 \sin x^2}{\cos(x+h)^2 \cos x^2}}{h}$$

$$= \lim_{h\to 0} \frac{\sin\left((x+h)^2 - x^2\right)}{h.\cos(x+h)^2.\cos x^2}$$

$$= \lim_{h\to 0} \frac{\sin\left(x^2 + h^2 + 2hx - x^2\right)}{h.\cos(x+h)^2.\cos x^2}$$

$$= \lim_{h\to 0} \frac{\sin\left(h^2 + 2hx\right)}{h.\cos(x+h)^2.\cos x^2}$$

$$= \lim_{h\to 0} \frac{\sin h}{h} \times \frac{(h+2x)}{\cos(x+h)^2.\cos x^2}$$

$$= 1.\frac{2x}{\cos^2(x)^2} \quad \left[\because \lim_{h\to 0} \frac{\sin h}{h} = 1\right]$$

$$= 2x\sec^2 x^2$$

Derivatives Ex 30.2 Q6(i)

We have,

$f(x) = (-x)$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{-(x+h) + x}{h}$$

$$= \lim_{h\to 0} \frac{-x - h + x}{h}$$

$$= -1$$

Derivatives Ex 30.2 Q6(ii)

We have,

$$f(x) = (-x)^{-1}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{-1}{x+h} + \frac{1}{x}}{h}$$

$$= \lim_{h \to 0} \frac{-x + x + h}{hx(x+h)}$$

$$= \lim_{h \to 0} \frac{1}{x^2 + xh}$$

$$= \frac{1}{x^2}$$

Derivatives Ex 30.2 Q6(iii)

We have,

$$f(x) = \sin(x+1)$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\sin(x+h+1) - \sin(x+1)}{h}$$

$$= \lim_{h \to 0} \frac{2\cos\left(\frac{x+h+1+x+1}{2}\right)\sin\left(\frac{x+h+1-x-1}{2}\right)}{h}$$

$$= \lim_{h \to 0} \frac{2\cos\left(\frac{2x+2+h}{2}\right)\sin\frac{h}{2}}{h} \qquad \left[\because \sin A - \sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2}\right]$$

$$= \lim_{h \to 0} \cos\left(\frac{2x+2+h}{2}\right)\left(\frac{\sin\frac{h}{2}}{\frac{h}{2}}\right)$$

$$= \cos\left(\frac{2(x+1)}{2}\right) \qquad \left[\because \lim_{h \to 0} \frac{\sin\frac{h}{2}}{\frac{h}{2}} = 1\right]$$

$$= \cos(x+1)$$

Derivatives Ex 30.2 Q6(iv)

We have,

$$f(x) = \cos\left(x - \frac{\pi}{8}\right)$$

$$\because f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\cos\left(x + h - \frac{\pi}{8}\right) - \cos\left(x - \frac{\pi}{8}\right)}{h}$$

$$= \lim_{h \to 0} \frac{-2\sin\left(\frac{2x + h - \frac{2\pi}{8}}{2}\right)\sin\left(\frac{h + x - \frac{\pi}{8} - x + \frac{\pi}{8}}{2}\right)}{h} \quad \left[\because \cos C - \cos D = -2\sin\frac{C+D}{2}\sin\frac{C-D}{2}\right]$$

$$= \lim_{h \to 0} \frac{-2\sin\left(\frac{2x + h - \frac{2\pi}{8}}{2}\right) \times \sin\left(\frac{h}{2}\right)}{2 . \frac{h}{2}}$$

$$= \lim_{h \to 0} \frac{-2\sin\left(\frac{2x + h - \frac{2\pi}{8}}{2}\right)}{2.} \times \lim_{h \to 0} \frac{\sin\left(\frac{h}{2}\right)}{\frac{h}{2}}$$

$$= \lim_{h \to 0} \sin\left(\frac{2x + h - \frac{2\pi}{8}}{2}\right) \quad \left[\because \lim_{h \to 0} \frac{\sin\frac{h}{2}}{\frac{h}{2}} = 1\right]$$

$$= \sin\left(\frac{2x - \frac{2\pi}{8}}{2}\right)$$

$$= -\sin\left(x - \frac{\pi}{8}\right)$$