RD Sharma Solutions Class 11 Maths Chapter 3 Ex 3.2

Functions Ex 3.1 Q18

$f: X \to R$ given by $f(x) = x^3 + 1$

$f(-1) = (-1)^3 + 1 = -1 + 1 = 0$

$f(0) = (0)^3 + 1 = 0 + 1 = 1$

$f(3) = (3)^3 + 1 = 27 + 1 = 28$

$f(9) = (9)^3 + 1 = 81 + 1 = 82$

$f(7) = (7)^3 + 1 = 343 + 1 = 344$

Set of ordered pairs are $\{(-1,0),(0,1),(3,28),(9,82),(7,344)\}$

Functions Ex 3.2 Q1

We have,

$$f(x) = x^2 - 3x + 4$$

Now,

$$f(2x+1) = (2x+1)^2 - 3(2x+1) + 4$$
$$= 4x^2 + 1 + 4x - 6x - 3 + 4$$
$$= 4x^2 - 2x + 2$$

It is given that

$$f(x) = f(2x+1)$$
$$\Rightarrow \quad x^2 - 3x + 4 = 4x^2 - 2x + 2$$
$$\Rightarrow \quad 0 = 4x^2 - x^2 - 2x + 3x + 2 - 4$$
$$\Rightarrow \quad 3x^2 + x - 2 = 0$$
$$\Rightarrow \quad 3x^2 + 3x - 2x - 2 = 0$$
$$\Rightarrow \quad 3x(x+1) - 2(x+1) = 0$$
$$\Rightarrow \quad (x+1)(3x-2) = 0$$
$$\Rightarrow \quad x + 1 = 0 \quad \text{or} \quad 3x - 2 = 0$$
$$\Rightarrow \quad x = -1 \quad \text{or} \quad x = \frac{2}{3}$$

Functions Ex 3.2 Q2

We have,

$$f(x) = (x-a)^2 (x-b)^2$$

Now,

$$f(a+b) = (a+b-a)^2 (a+b-b)^2$$
$$= b^2 a^2$$
$$\Rightarrow \quad f(a+b) = a^2 b^2$$

Functions Ex 3.2 Q3

We have,

$$y = f(x) = \frac{ax-b}{bx-a}$$
$$\Rightarrow \quad y = \frac{ax-b}{bx-a}$$
$$\Rightarrow \quad y(bx-a) = ax - b$$
$$\Rightarrow \quad xyb - ay = ax - b$$
$$\Rightarrow \quad xyb - ax = ay - b$$
$$\Rightarrow \quad x(by-a) = ay - b$$
$$\Rightarrow \quad x = \frac{ay-b}{by-a}$$
$$\Rightarrow \quad x = f(y)$$

Hence, proved

Functions Ex 3.2 Q4

We have,

$$f(x) = \frac{1}{1-x}$$

Now,

$$f\{f(x)\} = f\left\{\frac{1}{1-x}\right\}$$
$$= \frac{1}{1 - \frac{1}{1-x}}$$
$$= \frac{1}{\frac{1-x-1}{1-x}}$$
$$= \frac{1-x}{-x}$$
$$= \frac{x-1}{x}$$

$\therefore \quad f[f\{x\}] = f\left\{\frac{x-1}{x}\right\}$
$$= \frac{1}{1 - \left(\frac{x-1}{x}\right)}$$
$$= \frac{1}{\frac{x-x+1}{x}}$$
$$= \frac{x}{1}$$
$$= x$$

$\therefore \quad f[f(x)] = x$ Hence, proved.

Functions Ex 3.2 Q5

We have,

$$f(x) = \frac{x+1}{x-1}$$

Now,

$$f[f(x)] = f\left(\frac{x+1}{x-1}\right)$$
$$= \frac{\left(\frac{x+1}{x-1}\right)+1}{\left(\frac{x+1}{x-1}\right)-1}$$
$$= \frac{\frac{x+1+x-1}{x-1}}{\frac{x+1-1(x-1)}{x-1}}$$
$$= \frac{\frac{2x}{x-1}}{\frac{x+1-x+1}{x-1}}$$
$$= \frac{2x}{2}$$
$$= x$$

$\therefore \quad f[f(x)] = x$ Hence, proved.

Functions Ex 3.2 Q6

We have,

$$f(x) = \begin{cases} x^2, & \text{when } x < 0 \\ x, & \text{when } 0 \le x < 1 \\ \frac{1}{x}, & \text{when } x \ge 1 \end{cases}$$

(a) $f(1/2) = \frac{1}{2}$

(b) $f(-2) = (-2)^2 = 4$

(c) $f(1) = \frac{1}{1} = 1$

(d) [illegible]

(d) $f(\sqrt{3}) = \frac{}{\sqrt{3}}$

(e) $f\left(\sqrt{-3}\right) =$ does not exist because $\sqrt{-3} \notin \text{domain}(f)$.

Functions Ex 3.2 Q7

We have,

$$f(x) = x^3 - \frac{1}{x^3} \qquad \text{---(i)}$$

Now,

$$f\left(\frac{1}{x}\right) = \left(\frac{1}{x}\right)^3 - \frac{1}{\left(\frac{1}{x}\right)^3}$$

$$= \frac{1}{x^3} - \frac{1}{\frac{1}{x^3}}$$

$$\Rightarrow \quad f\left(\frac{1}{x}\right) = \frac{1}{x^3} - x^3 \qquad \text{---(ii)}$$

Adding equation (i) and equation (ii), we get

$$f(x) + f\left(\frac{1}{x}\right) = \left(x^3 - \frac{1}{x^3}\right) + \left(\frac{1}{x^3} - x^3\right)$$

$$= x^3 - \frac{1}{x^3} + \frac{1}{x^3} - x^3$$

$$= 0$$

$\therefore \quad f(x) + f\left(\frac{1}{x}\right) = 0$ Hence, proved.

Functions Ex 3.2 Q8

We have,

$$f(x) = \frac{2x}{1+x^2}$$

Now,

$$f(\tan\theta) = \frac{2(\tan\theta)}{1+\tan^2\theta}$$

$$= \sin 2\theta \qquad \left[\because \sin 2\theta = \frac{2\tan\theta}{1+\tan^2\theta}\right]$$

$\therefore \quad f(\tan\theta) = \sin 2\theta$ Hence, proved.

Functions Ex 3.2 Q9

$$\frac{x-1}{x+1}$$

$$f\left(\frac{1}{x}\right) = \frac{\frac{1}{x}-1}{\frac{1}{x}+1} = \frac{\frac{1-x}{x}}{\frac{1+x}{x}} = \frac{1-x}{1+x} = -f(x)$$

ii. $f(x) = \frac{x-1}{x+1}$

$$f\left(-\frac{1}{x}\right) = \frac{-\frac{1}{x}-1}{-\frac{1}{x}+1} = \frac{\frac{-1-x}{x}}{\frac{-1+x}{x}} = \frac{-1-x}{-1+x} = -\frac{1}{\frac{1+x}{x-1}} = -\frac{1}{f(x)}$$

Functions Ex 3.2 Q10

We have,

$$f(x) = \left(a - x^n\right)^{1/n}, \; a > 0$$

Now,

$$f(f(x)) = f\left(a - x^n\right)^{1/n}$$

$$= \left[a - \left\{\left(a - x^n\right)^{1/n}\right\}^n\right]^{1/n}$$

$$= \left[a - \left(a - x^n\right)\right]^{1/n}$$

$$= \left[a - a + x^n\right]^{1/n}$$

$$= \left[a^n - a^n + x^n\right]^{1/n}$$

$$= \left(x^n\right)^{1/n}$$

$$= (x)^{n \times \frac{1}{n}}$$

$$= x$$

$\therefore \quad f\left(f(x)\right) = x$ Hence, proved.

Functions Ex 3.2 Q11

We have,

$$af(x) + bf\left(\frac{1}{x}\right) = \frac{1}{x} - 5 \qquad \text{---(i)}$$

$$\Rightarrow \quad af\left(\frac{1}{x}\right) + bf(x) = \frac{1}{\frac{1}{x}} - 5$$

$$= x - 5$$

$$\Rightarrow \quad af\left(\frac{1}{x}\right) + bf(x) = x - 5 \qquad \text{---(ii)}$$

Adding equations (i) and (ii), we get

$$af(x) + bf(x) + bf\left(\frac{1}{x}\right) + af\left(\frac{1}{x}\right) = \frac{1}{x} - 5 + x - 5$$

$$\Rightarrow \quad (a+b)f(x) + f\left(\frac{1}{x}\right)(a+b) = \frac{1}{x} + x - 10$$

$$\Rightarrow \quad f(x) + f\left(\frac{1}{x}\right) = \frac{1}{a+b}\left[\frac{1}{x} + x - 10\right] \qquad \text{---(iii)}$$

Subtracting equation (ii) from equation (i), we get

$$af(x) - bf(x) + bf\left(\frac{1}{x}\right) - af\left(\frac{1}{x}\right) = \frac{1}{x} - 5 - x + 5$$

$$\Rightarrow \quad (a-b)f(x) - f\left(\frac{1}{x}\right)(a-b) = \frac{1}{x} - x$$

$$\Rightarrow \quad f(x) - f\left(\frac{1}{x}\right) = \frac{1}{a-b}\left[\frac{1}{x} - x\right]$$

Adding equations (iii) and (iv), we get

$$2f(x) = \frac{1}{a+b}\left[\frac{1}{x} + x - 10\right] + \frac{1}{a-b}\left[\frac{1}{x} - x\right]$$

$$\Rightarrow \quad 2f(x) = \frac{(a-b)\left[\frac{1}{x} + x - 10\right] + (a+b)\left[\frac{1}{x} - x\right]}{(a+b)(a-b)}$$

$$\Rightarrow \quad 2f(x) = \frac{\frac{a}{x} + ax - 10a - \frac{b}{x} - bx + 10b + \frac{a}{x} - ax + \frac{b}{x} - bx}{a^2 - b^2}$$

$$\Rightarrow \quad 2f(x) = \frac{\frac{2a}{x} - 10a + 10b - 2bx}{a^2 - b^2}$$

$$\Rightarrow \quad f(x) = \frac{1}{a^2 - b^2} \times \frac{1}{2}\left[\frac{2a}{x} - 10a + 10b - 2bx\right]$$

$$= \frac{1}{a^2 - b^2}\left[\frac{a}{x} - 5a + 5b - bx\right]$$

$$\therefore \quad f(x) = \frac{1}{a^2 - b^2}\left[\frac{a}{x} - bx - 5a + 5b\right]$$

$$= \frac{1}{a^2 - b^2}\left[\frac{a}{x} - bx\right] - \frac{5(a-b)}{a^2 - b^2}$$

$$= \frac{1}{a^2 - b^2}\left[\frac{a}{x} - bx\right] - \frac{5(a-b)}{(a-b)(a+b)}$$

$$= \frac{1}{a^2 - b^2}\left[\frac{a}{x} - bx\right] - \frac{5}{a+b}$$