RD Sharma Solutions Class 11 Maths Chapter 22 Ex 22.1

Brief Review of Cartesian System of Rectangular co-ordinates Ex 22.1 Q1

It is given that O is the origin.
Then,

$$OQ^2 = x_2^2 + y_2^2,$$

$$OP^2 = x_1^2 + y_1^2$$

and, $PQ^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$

Using cosine fromula in □OPQ, we have

$$PQ^2 = OP^2 + OQ^2 - 2.(OP)(OQ)\cos\alpha$$

$$\Rightarrow \quad (x_2 - x_1)^2 + (y_2 - y_1)^2 = x_2^2 + y_2^2 + x_1^2 + y_1^2 - 2(OP).(OQ)\cos\alpha$$

$$\Rightarrow \quad x_2^2 + x_1^2 - 2x_2x_1 + y_2^2 + y_1^2 - 2y_2y_1 = x_2^2 + y_2^2 + x_1^2 + y_1^2 - 2\,OP.OQ\cos\alpha$$

$$\Rightarrow \quad -2x_1x_2 - 2y_1y_2 = -2OP.OQ\cos\alpha$$

$$\Rightarrow \quad x_1x_2 + y_1y_2 = OP.\ OQ\cos\alpha$$

$$\Rightarrow \quad OP.\,OQ\cos\alpha = x_1x_2 + y_1y_2$$

Hence, proved.

Brief Review of Cartesian System of Rectangular co-ordinates Ex 22.1 Q2

We know that

$$\cos B = \frac{a^2+c^2-b^2}{2ac},$$

where $a = BC, b = CA$ and $C = AB$ are the sides of the triangle ABC.

we have,

$$a = BC = \sqrt{(9-2)^2+(2+1)^2} = \sqrt{49+9} = \sqrt{58}$$

$$b = CA = \sqrt{(0-9)^2+(0+2)^2} = \sqrt{81+4} = \sqrt{85}$$

and, $c = AB = \sqrt{(2-0)^2+(-1-0)^2} = \sqrt{4+1} = \sqrt{5}$

$$\therefore \quad \cos B = \frac{a^2+c^2-b^2}{2ac}$$

$$= \frac{58+5-85}{2\times\sqrt{58}\times\sqrt{5}}$$

$$= \frac{63-85}{2\sqrt{290}}$$

$$= \frac{-22}{2\sqrt{290}} = \frac{-11}{\sqrt{290}}$$

Hence, $\cos B = \frac{-11}{\sqrt{290}}$.

Brief Review of Cartesian System of Rectangular co-ordinates Ex 22.1 Q3

$A(6,3), B(-3,5), C(4,-2), D(x,3x)$

$$\text{or } (\square DBC) = \frac{1}{2}\left[x_1(y_2 - y_1) + x_2(y_3 - y_2) + x_3(y_1 - y_2)\right]$$

$$= \frac{1}{2}\left[-3(-2 - 3x) + 4(3x - 5) + x(5 + 2)\right]$$

$$= \frac{1}{2}\left[6 + 9x + 12x - 20 + 5x + 2x\right]$$

$$= \frac{1}{2}\left[28x - 14\right]$$

$$= 7\left[2x - 1\right]$$

$$\text{or } (\square ABC) = \frac{1}{2}\left[6(5+2) - 3(-2-3) + 4(3-5)\right]$$

$$= \frac{1}{2}\left[42 + 15 - 8\right]$$

$$= \frac{49}{2}$$

$$\frac{\text{or } (\square DBC)}{\text{or } (\square ABC)} = \frac{1}{2}$$

$$\frac{7(2x - 1)}{\frac{49}{2}} = \frac{1}{2}$$

$$\frac{14(2x - 1)}{49} = \frac{1}{2}$$

$$\frac{28x - 14}{49} = \frac{1}{2}$$

$$56x - 28 = 49$$

$$56x = 28 + 49$$

$$56x = 77$$

$$x = \frac{11}{8}$$

Brief Review of Cartesian System of Rectangular co-ordinates Ex 22.1 Q4

It is given that $A(2,0), B(9,1), C(11,6)$ and $D(4,4)$ are the vertices of a quadrilateral.

Now,

Coordinates of the mid-point of AC are $\left(\frac{2+11}{2}, \frac{0+6}{2}\right) = \left(\frac{13}{2}, 3\right)$

Coordinates of the mid-point of BD are $\left(\frac{9+4}{2}, \frac{1+4}{2}\right) = \left(\frac{13}{2}, \frac{5}{2}\right)$

Thus, AC and BD do not have the same mid-point. Hence $ABCD$ is not a paralleogram.

$\therefore$ $ABCD$ is not a rhombus.

Brief Review of Cartesian System of Rectangular co-ordinates Ex 22.1 Q5

Let $A(-36,7)$, $B(20,7)$ and $C(0,-8)$ be the vertices of the triangle ABC.

Now,

$$a = BC = \sqrt{(0-20)^2+(-8-7)^2}$$

$$= \sqrt{400+225}$$

$$= \sqrt{625}$$

$$= 25,$$

$$b = AC = \sqrt{(0+36)^2+(-8-7)^2}$$

$$= \sqrt{1296+225}$$

$$= \sqrt{1521}$$

$$= 39$$

and, $$c = AB = \sqrt{(20+36)^2+(7-7)^2}$$

$$= \sqrt{(56)^2}$$

$$= 56$$

The coordinates of the centre of the circle are

$$\left(\frac{ax_1+bx_2+cx_3}{a+b+c}, \frac{ay_1+by_2+cy_3}{a+b+c}\right)$$

or, $$\left[\frac{25\times(-36)+39\times 20+56\times 0}{25+39+56}, \frac{25\times 7+39\times 7+56\times(-8)}{25+39+56}\right]$$

or, $$\left[\frac{-900+780}{120}, \frac{175+273-448}{120}\right]$$

or, $$\left[\frac{-120}{120}, \frac{0}{120}\right]$$

or, $$(-1,0)$$

Hence, the coordinates of the centre of the circle are $(-1,0)$.

Brief Review of Cartesian System of Rectangular co-ordinates Ex 22.1 Q6

It is given that ABC is an equilatral triangle.

$\therefore \quad AB = BC = AC = 2a$

Area of equilatral triangle

$$= \frac{\sqrt{3}}{4}(\text{side})^2$$

$$= \frac{\sqrt{3}}{4} \times (2a)^2$$

$$= \frac{\sqrt{3}}{4} \times 4 \times a^2$$

$$= \sqrt{3}\,a^2$$

But, area of triangle $= \frac{1}{2} \times \text{Base} \times \text{Height}$.

$\Rightarrow \quad \frac{1}{2} \times \text{Base} \times \text{Height} = \sqrt{3}\,a^2$

$\Rightarrow \quad \frac{1}{2} \times BC \times OA = \sqrt{3}\,a^2$

$\Rightarrow \quad \frac{1}{2} \times 2a \times OA = \sqrt{3}\,a^2$

$\Rightarrow \quad OA = \sqrt{3}\,a$

$\therefore$ Coordinates of A are $\left(\sqrt{3}\,a, 0\right)$ or $OA\left(-\sqrt{3}\,a, 0\right)$

Clearly, the coordinates of B and C are $(0, -a)$ and $(0, a)$ respectively.

Hence, the vertices of the triangle are $(0, a), (0, -a)$ and $\left(-\sqrt{3}\,a, 0\right)$ or $(0, a), (0, -a)$ and $\left(\sqrt{3}\,a, 0\right)$.

Brief Review of Cartesian System of Rectangular co-ordinates Ex 22.1 Q7

It is given that $P(x_1, y_1)$ and $Q(x_2, y_2)$ are two points

(i) PQ is parallel to the y-axis.

$\therefore \quad x_1 = x_2 \text{ .... } (1)$

$$\therefore \quad PQ = \left|\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}\right|$$

$$= \left|\sqrt{(x_2 - x_x)^2 + (y_2 - y_1)^2}\right| \quad \text{[Using equation 1]}$$

$$= \left|\sqrt{(y_2 - y_1)^2}\right|$$

$$= |y_2 - y_1|$$

(ii) PQ is parallel to the x-axis.

$\therefore \quad y_1 = y_2 \text{ .... } (2)$

$$\therefore \quad PQ = \left|\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}\right|$$

$$= \left|\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}\right| \quad \text{[Using equation 2]}$$

$$= \left|\sqrt{(x_2 - x_1)^2}\right|$$

$$= \left|\sqrt{x_2 - x_1}\right|$$

$$\therefore \quad PQ = |x_2 - x_1|$$

Brief Review of Cartesian System of Rectangular co-ordinates Ex 22.1 Q8

It is given that C lie on the x-axis. *Let* coordinates of C be $(x, 0)$.

Now, C *is* equidistant from the points $A(7, 6)$ and $B(3, 4)$.

$\therefore \quad AC = BC \quad$ [given]

$\Rightarrow \quad AC^2 = BC^2$

$$\Rightarrow \quad \left[\sqrt{(x-7)^2 + (0-6)^2}\right]^2 = \left[\sqrt{(x-3)^2 + (0-4)^2}\right]^2$$

$$\Rightarrow \quad (x-7)^2 + (-6)^2 = (x-3)^2 + (-4)^2$$

$$\Rightarrow \quad x^2 + 49 - 14x + 36 = x^2 + 9 - 6x + 16$$

$$\Rightarrow \quad 49 + 36 - 36 - 16 - 9 = x^2 - x^2 - 6x + 14x$$

$\Rightarrow \quad 85 - 25 = 8x$

$\Rightarrow \quad 60 = 8x$

$\Rightarrow \quad 8x = 60$

$$\Rightarrow \quad x = \frac{60}{8} = \frac{15}{2}$$

Hence, coordinates of c are $\left(\frac{15}{2}, 0\right)$.

RD Sharma Solutions Class 11 Maths Chapter 22 Ex 22.2

Brief Review of Cartesian System of Rectangular co-ordinates Ex 22.2 Q1

Let $P(h,k)$ be any point on the locus and let $A(2,4)$ and $B(0,k)$. Then,

$$PA = PB$$

$$\Rightarrow \quad PA^2 = PB^2$$

$$\Rightarrow \quad \left[\sqrt{(2-h)^2+(4-k)^2}\right]^2 = \left[\sqrt{(0-h)^2+(k-k)^2}\right]^2$$

$$\Rightarrow \quad (2-h)^2+(4-k)^2 = (0-h)^2+(0)^2$$

$$\Rightarrow \quad 4+h^2-4h+16+k^2-8k = h^2$$

$$\Rightarrow \quad k^2-8k-4h+20 = 0$$

Hence, locus of (h,k) is $y^2-8y-4x+20=0$

Let $P(h,k)$ be any point on the locus and let $AC(2,4)$ and $B(0,k)$ be the given points.

Brief Review of Cartesian System of Rectangular co-ordinates Ex 22.2 Q2

Let $P(h,k)$ be any point on the locus and let $A(2,0)$ and $B(1,3)$. Then,

$$\frac{PA}{BP} = \frac{5}{4}$$

$$\Rightarrow \quad \frac{PA^2}{BP^2} = \frac{25}{16}$$

$$\Rightarrow \quad \frac{\left[\sqrt{(h-2)^2+(k-0)^2}\right]^2}{\left[\sqrt{(h-1)^2+(k-3)^2}\right]^2} = \frac{25}{16}$$

$$\Rightarrow \quad \frac{(h-2)^2+k^2}{(h-1)^2+(k^2-3)^2} = \frac{25}{16}$$

$$\Rightarrow \quad \frac{h^2+4-4h+k^2}{h^2+1-2h+k^2+9-6k} = \frac{25}{16}$$

$$\Rightarrow \quad \frac{(h^2-4h+k^2+4)}{h^2+k^2-2h-6k+10} = \frac{25}{16}$$

$$\Rightarrow \quad 16(h^2-4h+k^2+4) = 25(h^2+k^2-2h-6k+10)$$

$$\Rightarrow \quad 16h^2-64h+16k^2+64 = 25h^2+25k^2-50h-150k+250$$

$$\Rightarrow \quad 25h^2-16h^2+25k^2-16k^2-50h+64h-150k+250-64 = 0$$

$$\Rightarrow \quad 9h^2+9k^2+14h-150k+186 = 0$$

Hence, locus of (h,k) is $9x^2+9y^2+14x-150y+186=0$

Brief Review of Cartesian System of Rectangular co-ordinates Ex 22.2 Q3

Let $P(h,k)$ be any point on the locus and let $A(ae,0)$ and $B(-ae,0)$ be the given points.
By the given condition

$$PA - PB = 2a$$

$$\Rightarrow \quad PA = 2a + PB$$

$$\Rightarrow \quad \sqrt{(ae-h)^2 + (0-k)^2} = 2a + \sqrt{(-ae-h)^2 + (0-k)^2}$$

$$\Rightarrow \quad (ae-h)^2 + k^2 = \left(2a + \sqrt{(ae+h)^2 + k^2}\right)^2 \qquad \text{[Taking square on both sides]}$$

$$\Rightarrow \quad (ae)^2 + h^2 - 2aeh + k^2 = 4a^2 + (ae+h)^2 + k^2 + 2 \times 2a \times \sqrt{(ae+h)^2 + k^2}$$

$$\Rightarrow \quad h^2 + k^2 + (ae)^2 - 2aeh = 4a^2 + (ae)^2 + h^2 + 2hae + k^2 + 4a\sqrt{(ae+h)^2 + k^2}$$

$$\Rightarrow \quad -4a^2 - 2aeh - 2aeh = 4a\sqrt{(ae+h)^2 + k^2}$$

$$\Rightarrow \quad -4a^2 - 4aeh = 4a\sqrt{(ae+h)^2 + k^2}$$

$$\Rightarrow \quad -4\left[a^2 + aeh\right] = 4a\sqrt{(ae+h)^2 + k^2}$$

$$\Rightarrow \quad -\left[a^2 + aeh\right] = a\sqrt{(ae+h)^2 + k^2}$$

$$\Rightarrow \quad -a\left[a + eh\right] = a\sqrt{(ae+h)^2 + k^2}$$

$$\Rightarrow \quad -\left[a + eh\right] = \sqrt{(ae+h)^2 + k^2}$$

$\Rightarrow \quad -[a+eh] = \sqrt{(ae+h)^2+k^2}$

$\Rightarrow \quad (a+eh)^2 = \left(\sqrt{(ae+h)^2+k^2}\right)^2$ [Taking square on both sides]

$\Rightarrow \quad a^2+(eh)^2 2hae = (ae+h)^2+k^2$

$\Rightarrow \quad a^2+(eh)^2+2hae = (ae)^2+h^2+2hae+k^2$

$\Rightarrow \quad a^2+e^2h^2 = a^2e^2+h^2+k^2$

$\Rightarrow \quad e^2h^2-h^2-k^2 = a^2e^2-a^2$

$\Rightarrow \quad h^2(e^2-1)-k^2 = a^2(e^2-1)$

$\Rightarrow \quad \frac{h^2(e^2-1)}{a^2(e^2-1)} - \frac{k^2}{a^2(e^2-1)} = 1$

$\Rightarrow \quad \frac{h^2}{a^2} - \frac{k^2}{b^2} = 1$, Where $b^2 = a^2(e^2-1)$

$\therefore$ The locus of (h,k) is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Hence proved.

Brief Review of Cartesian System of Rectangular co-ordinates Ex 22.2 Q4

Let $P(h,k)$ be any point on the locus and let $A(0,2)$ and $B(0,-2)$ be the given points.
By the given condition $PA+PB=6$

$\Rightarrow \quad \sqrt{(h-0)^2+(k-2)^2} + \sqrt{(h-0)^2+(k+2)^2} = 6$

$\Rightarrow \quad \sqrt{h^2(k-2)h^2} = 6 - \sqrt{h^2+(k+2)^2}$

$\Rightarrow \quad h^2+(k-2)h^2 = 36 - 12\sqrt{h^2+(k+2)^2} + h^2 + (k+2)^2$

$\Rightarrow \quad -8k-36 = -12\sqrt{h^2+(k+2)^2}$

$\Rightarrow \quad (2k+9) = 3\sqrt{h^2+(k+2)^2}$

$\Rightarrow \quad (2k+9)^2 = 9\left(h^2+(k+2)^2\right)$

$\Rightarrow \quad 4k^2+36k+81 = 9h^2+9k^2+36k+36$

$\Rightarrow \quad 9h^2+5k^2 = 45$

Hence, locus of (h,k) is $9x^2+5y^2=45$.

Brief Review of Cartesian System of Rectangular co-ordinates Ex 22.2 Q5

Let $P(h,k)$ be any point on the locus and let $A(1,3)$ and $B(h,0)$. Then,

$PA = PB$

$\Rightarrow \quad PA^2 = PB^2$

$\Rightarrow \quad (1-h)^2 + (3-k)^2 = (h-h)^2 + (0-k)^2$

$\Rightarrow \quad 1 + h^2 - 2h + 9 + k^2 - 6k = 0 + k^2$

$\Rightarrow \quad h^2 - 2h - 6k + 10 = 0$

Hence, locus of (h,k) is $x^2 - 2x - 6y + 10 = 0$

Brief Review of Cartesian System of Rectangular co-ordinates Ex 22.2 Q6

Let $P(h,k)$ be any point on the locus and let $O(0,0)$ be the origin.
By the given condition

$OP = 3k \qquad [\because k \text{ is the diffance of point from x-axis}]$

$\Rightarrow \quad OP^2 = 9k^2$

$\Rightarrow \quad \left(\sqrt{(0-h)^2 + (0-k)^2}\right)^2 = 9k^2$

$\Rightarrow \quad h^2 + k^2 = 9k^2$

$\Rightarrow \quad h^2 = 9k^2 - k^2$

$\Rightarrow \quad h^2 = 8k^2$

Hence, locus of (h,k) is $x^2 = 8y^2$

Brief Review of Cartesian System of Rectangular co-ordinates Ex 22.2 Q7

Let $P(h,k)$ be any point on the locus. Then,
Area $(PAB) = 9$ sq units

$$\Rightarrow \quad \frac{1}{2}\left|x_1(y_2-y_3)+x_2(y_3-y_1)+x^3(y_1-y_2)\right| = 9$$

$$\Rightarrow \quad \left|5(-2-k)+3(k-3)+h(3+2)\right| = 18$$

$$\Rightarrow \quad \left|-10-5k+3k-9+5h\right| = 18$$

$$\Rightarrow \quad \left|5h-2k-19\right| = 18$$

$$\Rightarrow \quad 5h-2k-19 = \pm 18$$

$$\Rightarrow \quad 5h-2k-19 \mp 18 = 0$$

$$\Rightarrow \quad 5h-2k-37 = 0 \quad \text{or,} \quad 5h-2k-1 = 0$$

Hence, the locus of (h,k) is

$$5x-2y-37=0 \quad \text{or,} \quad 5x-2y-1=0.$$

Brief Review of Cartesian System of Rectangular co-ordinates Ex 22.2 Q8

Let $P(h,k)$ be the variable point and let $A(2,0)$ and $B(-2,0)$ be the given points.
Then $\angle APB = \pi/2$

$$\Rightarrow \quad AB^2 = PA^2 + PB^2$$

$$\Rightarrow \quad (2+2)^2+0 = (2-h)^2+(0-k^2)+(-2-h)^2+(0-k)^2$$

$$\Rightarrow \quad 16 = 4+h^2-4h+k^2+4+h^2+4h+k^2$$

$$\Rightarrow \quad 16 = 2h^2+2k^2+8$$

$$\Rightarrow \quad 2h^2+2k^2+8-16=0$$

$$\Rightarrow \quad 2h^2+2k^2-8=0$$

$$\Rightarrow \quad h^2+k^2-4=0$$

Hence, the locus of (h,k) is $x^2+y^2=4$

Brief Review of Cartesian System of Rectangular co-ordinates Ex 22.2 Q9

Let $P(h,k)$ be any point on the locus. Then,

Area (PAB) = 8 sq units

$$\Rightarrow \quad \frac{1}{2}\left|x_1(y_2-y_3)+(y_3-y_1)+x_3(y_1-y_2)\right| = 8$$

$$\Rightarrow \quad \frac{1}{2}\left|-1(3-k)+2(k-1)+h(1-3)\right| = 8$$

$$\Rightarrow \quad \frac{1}{2}\left|-3+k+2k-2-2h\right| = 8$$

$$\Rightarrow \quad \frac{1}{2}\left|-2h+3k-5\right| = 8$$

$$\Rightarrow \quad \left|-2h+3k-5\right| = 16$$

$$\Rightarrow \quad -2h+3k-5 = \pm 16$$

$$\Rightarrow \quad 2h-3k+5 \pm 16 = 0$$

$$\Rightarrow \quad 2h-3k+21 = 0 \quad \text{or,} \quad 2h-3k-11 = 0$$

Hence, the locus of (h,k) is

$$2x-3y+21 = 0 \quad \text{or,} \quad 2x-3y-11 = 0$$

Brief Review of Cartesian System of Rectangular co-ordinates Ex 22.2 Q10

Let the two perpendicular lines be the coordinate axes. Let AB be a rod length l. Let the coordinates of A and B be $(a,0)$ and $(0,b)$ respectively. As the rod slides the value of a and b change. so, a and b are two variables.
Let $P(h,k)$ be the point on the locus. Then,

$$h = \frac{2 \times a + 1 \times 0}{2+1}$$

$$\Rightarrow \quad h = \frac{2a}{3}$$

$$\Rightarrow \quad a = \frac{3h}{2}$$

$$\text{and } k = \frac{2 \times 0 + b \times 1}{2+1}$$

$$\Rightarrow \quad k = \frac{b}{3}$$

$$\Rightarrow \quad b = 3k$$

from $\triangle AOB$, we have

$$AB^2 = OA^2 + OB^2$$

$$\Rightarrow \quad l^2 = \left[(a-0)^2 + (0,0)\right]^2 + \left[(0-0)^2 + (b-0)^2\right]$$

$$\Rightarrow \quad l^2 = a^2 + b^2$$

$$\Rightarrow \quad a^2 + b^2 = l^2$$

$$\Rightarrow \quad \left(\frac{3h}{2}\right)^2 + (3k)^2 = l^2$$

$$\Rightarrow \quad \frac{9h^2}{4} + 9k^2 = l^2$$

$$\Rightarrow \quad \frac{h^2}{4} + k^2 = \frac{l^2}{9}$$

Hence, the locus of (h,k) is $\frac{x^2}{4} + y^2 = \frac{l^2}{9}$

Brief Review of Cartesian System of Rectangular co-ordinates Ex 22.2 Q11

Gicen, line is

$$x \cos\alpha + y \sin\alpha = p$$

$$\frac{x}{\frac{p}{\cos\alpha}} + \frac{y}{\frac{p}{\sin\alpha}} = 1$$

Intercepts on x axis is $\frac{p}{\cos\alpha}$ and y - axis is $\frac{p}{\sin\alpha}$

Let $P(x, y)$ be the mid point of AB.

$$(x, y) = \left(\frac{\frac{p}{\cos\alpha} + 0}{2}, \frac{\frac{p}{\sin\alpha} + 0}{2} \right) = \left(\frac{p}{2\cos\alpha}, \frac{p}{2\sin\alpha} \right)$$

$$\therefore x = \frac{p}{2\cos\alpha},\ y = \frac{p}{2\sin\alpha}$$

$$2\cos\alpha = \frac{p}{x},\quad 2\sin\alpha = \frac{p}{y}$$

Square both sides,

$$4\cos^2\alpha = \frac{p^2}{x^2} \text{ - - - - (1)}$$

and

$$4\sin^2\alpha = \frac{p^2}{y^2} \text{ - - - - - (2)}$$

$[(1) + (2)]$

$$4\cos^2\alpha + 4\sin^2\alpha = \frac{p^2}{x^2} + \frac{p^2}{y^2}$$

$$4 = \frac{p^2(x^2 + y^2)}{x^2 y^2}$$

$$4x^2y^2 = p^2(x^2 + y^2)$$

Brief Review of Cartesian System of Rectangular co-ordinates Ex 22.2 Q12

Let $P(h, k)$ be the point on the locus and let the coordinates of a are (a, b). Then,

$$h = \frac{a + 0}{2} \text{ and } \frac{b + 0}{2} = k \qquad [\because P \text{ is the mid-point of } Q \text{ and the origino}]$$

Brief Review of Cartesian System of Rectangular co-ordinates Ex 22.3 Q1

We have,

$$(x-a)^2+(y-b)^2=r^2 \quad \ldots\ldots\text{(i)}$$

Substituting $x = X+(a-c)$, $y = Y+b$ in the equation (i), we get

$$[X+a-c-a]^2+[Y+b-b]^2=r^2$$

$$\Rightarrow \quad [X-c]^2+[Y]^2=r^2$$

$$\Rightarrow \quad X^2+c^2-2Xc+Y^2=r^2$$

$$\Rightarrow \quad X^2+y^2-2cX=r^2-c^2$$

Hence, the required equation is $X^2+y^2-2cX=r^2-c^2$

Brief Review of Cartesian System of Rectangular co-ordinates Ex 22.3 Q2

We have,

$$(a-b)\left(x^2+y^2\right)-2abx=0$$

Substituting $x = X+\frac{ab}{a-b}$, $y = Y$

in the given equation, we get

$$(a-b)\left[\left(X+\frac{ab}{a-b}\right)+Y^2\right]-2ab\left[X+\frac{ab}{a-b}\right]=0$$

$$\Rightarrow \quad (a-b)\left[X^2+\left(\frac{ab}{a-b}\right)^2+2\frac{Xab}{a-b}+Y^2\right]-2abX-2\frac{(ab)^2}{a-b}=0$$

$$\Rightarrow \quad (a-b)\left[\frac{X^2(a-b)^2+(ab)h^2+2Xab(a-b)+Y^2(a-b)^2}{(a-b)^2}\right]-\frac{2abX(a-b)+2(ab)^2}{a-b}=0$$

$$\Rightarrow \quad \frac{X^2(a-b)^2+(ab)^2+2ab(a-b)+Y^2(a-b)^2}{a-b}=\frac{2ab(a-b)+2(ab)^2}{a-b}$$

$$\Rightarrow \quad x^2(a-b)^2+Y^2(a-b)^2+(ab)^2+2ab(a-b)=2ab(a-b)+2(ab)^2$$

$$\Rightarrow \quad (a-b)^2\left(X^2+Y^2\right)=(ab)^2$$

$$\Rightarrow \quad (a-b)^2\left(X^2+Y^2\right)=a^2b^2$$

Brief Review of Cartesian System of Rectangular co-ordinates Ex 22.3 Q3(i)

We have,

$$x^2 + xy - 3x - y + 2 = 0$$

Substituting $x = X + 1,\ Y + 1$ in the equation, we get

$$(X+1)^2 + (X+1)(Y+1) - 3(X+1) - (Y+1) + 2 = 0$$

$$\Rightarrow \quad X^2 + 1 + 2X + XY + X + Y + 1 - 3X - 3 - Y - 1 + 2 = 0$$

$$\Rightarrow \quad X^2 + XY = 0$$

Brief Review of Cartesian System of Rectangular co-ordinates Ex 22.3 Q3(ii)

We have,

$$x^2 - y^2 - 2x + 2y = 0$$

Substituting $x = X + 1, \quad y = Y + 1$ in the equation, we get

$$(X+1)^2 - (Y-1)^2 - 2(X+1) + 2(Y+1) = 0$$

$$\Rightarrow \quad X^2 + 1 + 2X - (Y^2 + 1 + 2Y) - 2X - 2 + 2Y + 2 = 0$$

$$\Rightarrow \quad X^2 + 1 - Y^2 - 1 - 2Y + 2Y = 0$$

$$\Rightarrow \quad X^2 - Y^2 = 0$$

Brief Review of Cartesian System of Rectangular co-ordinates Ex 22.3 Q3(iii)

We have,

$$xy - x - y + 1 = 0$$

Substituting $x = X + 1, \quad y = Y + 1$ in the equation, we get

$$(X+1)(Y+1) - (X+1) - (Y+1) + 1 = 0$$

$$\Rightarrow \quad XY + X + Y + 1 - X - 1 - Y - 1 + 1 = 0$$

$$\Rightarrow \quad XY = 0$$

Brief Review of Cartesian System of Rectangular co-ordinates Ex 22.3 Q3(iv)

We have,

$$xy - y^2 - x + y = 0$$

Substituting $x = X + 1, \quad y = Y + 1$ in the equation, we get

$$(X+1)(Y+1) - (Y+1) - (X+1) + (Y+1) = 0$$

$$\Rightarrow \quad XY + Y + Y + 1 - (Y^2 + 1 + 2Y) - X - 1 + Y + 1 = 0$$

$$\Rightarrow \quad XY + 2Y - Y^2 - 1 - 2Y + 1 = 0$$

$$\Rightarrow \quad XY - Y^2 = 0$$

Brief Review of Cartesian System of Rectangular co-ordinates Ex 22.3 Q4

We have,

$$x^2 + xy - 3x - y + 2 = 0 \;\ldots\ldots\; (i)$$

Let the origin be shifted to (h, k). Then $x = X + h$ and $y = Y + k$.

Substituting $x = X + h,\; y = Y + k$
in the equation (i), we get

$$(X + h)^2 + (X + h)(Y + k) - 3(X + h) - (Y + k) + 2 = 0$$

$$\Rightarrow \quad X^2 + h^2 + 2Xh + XY + Xk + Yh + hk - 3X - 3h - Y - k + 2 = 0$$

$$\Rightarrow \quad X^2 + XY + 2Xh + Xk + Yh - Y3 - X + h^2 + hk - 3h - k + 2 = 0$$

$$\Rightarrow \quad X^2 + (2Xh + Xk - 3X) + XY + (Yh - Y) + (h^2 + hk - 3h - k + 2) = 0$$

$$\Rightarrow \quad X^2 + (2h + k - 3)X + XY + (h - 1)Y + (h^2 + hk - 3h - k + 2) = 0$$

For this equation to be free from first degree and the constant term, we must have,

$$2h + k - 3 = 0 \;\ldots\ldots\ldots\ldots\; (ii)$$

$$h - 1 = 0$$

$$\Rightarrow \quad h = 1 \;\ldots\ldots\ldots\; (iii)$$

and

$$h^2 + hk - 3h - k + 2 = 0 \;\ldots\ldots\; (iv)$$

Putting $h = 1$ in equation (ii), we get

$$2 + k - 3 = 0$$

$$\Rightarrow \quad k = 1$$

Putting $h = 1$ and $k = 1$ in equation (iv), we get

$$(1)^2 + 1 - 3 - 1 + 2 = 0$$

Hence, the value of h and k satisfies the equation (iv)

$\therefore$ The origin is shifted at the point $(1, 1)$.

Brief Review of Cartesian System of Rectangular co-ordinates Ex 22.3 Q5

Let the vertices of a triangle be $A(2,3)$, $B(5,7)$ and $C(-3,-1)$.

Then, area of ΔABC is given by

$$\Delta = \frac{1}{2}\left|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right|$$

$$= \frac{1}{2}\left|2(7+1) + 5(-1-3) - 3(-3-7)\right|$$

$$= \frac{1}{2}\left|2 \times 8 + 5 \times (-4) - 3 \times (-4)\right|$$

$$= \frac{1}{2}\left|16 - 20 + 12\right|$$

$$= \frac{8}{2}$$

$$= 4$$

$\Rightarrow \quad \Delta = 4 sq$ unit

It is given that the origin is shifted at $(-1,3)$. Then new coordinates of the vertices are

$$A_1 = (2-3, 3+3) = (-1, 6)$$

$$B_1 = (5-1, 7+3) = (4, 10)$$

and $\quad C_1 = (-3-1, -1+3) = (-4, 2)$

Therefore, the area of the triangle in the new coordinate system is given by

$$\Delta_1 = \frac{1}{2}\left|\left[-1(10-2) + 4(2-6) - 4(6-10)\right]\right|$$

$$= \frac{1}{2}\left|\left[-1 \times 8 + 4 \times (-4) - 4 \times (-4)\right]\right|$$

$$= \frac{1}{2}\left|\left[-8 - 16 + 16\right]\right|$$

$$= \frac{1}{2}\left|-8\right|$$

$$= \frac{8}{2}$$

$\Rightarrow \quad \Delta_1 = 4 \ldots\ldots\ldots\ldots(2)$

From (i) and (ii), we get

$$\Delta = \Delta_1$$

Hence, the area of a triangle is invariant under the translation of the axes.

Brief Review of Cartesian System of Rectangular co-ordinates Ex 22.3 Q6(i)

We have,

$$x^2 + xy - 3y^2 - y + 2 = 0 \ldots\ldots(i)$$

Substituting $x = X + 1$, $y = Y + 1$ in equation (i), we get

$$(X+1)^2 + (X+1)(Y+1) - 3(Y+1)^2 - (Y+1) + 2 = 0$$

$$\Rightarrow \quad X^2 + 1 + 2X + XY + X + Y + 1 - 3(Y^2 + 1 + 2Y) - Y - 1 + 2 = 0$$

$$\Rightarrow \quad X^2 + XY + 3X + 3 - 3Y^2 - 3 - 6Y = 0$$

$$\Rightarrow \quad X^2 - 3Y^2 + XY + 3X - 6Y = 0$$

Brief Review of Cartesian System of Rectangular co-ordinates Ex 22.3 Q6(ii)

We have,

$$xy - y^2 - x + y = 0 \ldots\ldots(i)$$

Substituting $x = X + 1$, $y = Y + 1$ in equation (i), we get

$$(X+1)(Y+1) - (Y+1)^2 - (X+1) + (Y+1) = 0$$

$$\Rightarrow \quad XY + X + Y + 1 - (Y^2 + 1 + 2Y) - X - 1 + Y + 1 = 0$$

$$\Rightarrow \quad XY + 2Y + 1 - Y^2 - 1 - 2Y = 0$$

$$\Rightarrow \quad XY - Y^2 = 0$$

Brief Review of Cartesian System of Rectangular co-ordinates Ex 22.3 Q6(iii)

We have,

$$xy - x - y + 1 = 0 \ldots\ldots\text{(i)}$$

Substituting $x = X + 1$, $y = Y + 1$
in equation (i), we get

$$(X+1)(Y+1) - (X+1) - (Y+1) + 1 = 0$$

$$\Rightarrow \quad XY + X + Y + 1 - X - 1 - Y - 1 + 1 = 0$$
$$\Rightarrow \quad XY = 0$$

Brief Review of Cartesian System of Rectangular co-ordinates Ex 22.3 Q6(iv)

We have,

$$x^2 - y^2 - 2x - 2y = 0 \ldots\ldots\text{(i)}$$

Substituting $x = X + 1$, $y = Y + 1$
in equation (i), we get

$$(X+1)^2 - (Y+1)^2 - 2(X+1) + 2(Y+1) = 0$$

$$\Rightarrow \quad X^2 + 1 + 2X - \left(Y^2 + 1 + 2Y\right) - 2X - 2 + 2Y + 2 = 0$$
$$\Rightarrow \quad X^2 + 1 - Y^2 - 1 - 2Y + 2X = 0$$

$$\Rightarrow \quad X^2 + 2X + 1 - \left(Y^2 + 2Y + 1\right)$$
$$\Rightarrow \quad (X+1)^2 - (Y+1)^2$$
$$\Rightarrow \quad x^2 - y^2 = 0$$

Brief Review of Cartesian System of Rectangular co-ordinates Ex 22.3 Q7(i)

Let the origin be shifted to (h, k). Then, $x = X + h$ and $y = Y + k$

Substituting $x = X + h$, $y = Y + k$
in the equation $y^2 + x^2 - 4x - 8y + 3 = 0$, we get

$$(Y+k)^2 + (X+h)^2 - 4(X-h) - 8(Y+k) + 3 = 0$$

$$\Rightarrow \quad Y^2 + k^2 + 2Yk + X^2 + h^2 + 2Xh - 4X - 4h - 8Y - 8k + 3 = 0$$
$$\Rightarrow \quad Y^2 + X^2 + 2Yk - 8Y + 2Xh - 4X + k^2 + h^2 - 4h - 8k + 3 = 0$$
$$\Rightarrow \quad Y^2 + X^2 + (2k - 8)Y + (2h - 4)X + \left(k^2 + h^2 - 4h - 8k + 3\right) = 0$$

For this equation to be free from the term of first degree, we must have

$$2k - 8 = 0 \text{ and } 2h - 4 = 0$$
$$\Rightarrow \quad k = 4 \text{ and } h = 2$$

Hence, the origin is shifted at the point $(2, 4)$.

Brief Review of Cartesian System of Rectangular co-ordinates Ex 22.3 Q7(ii)

Let the origin be shifted to (h, k). Then, $x = X + h$ and $y = Y + k$
Substituting $x = X + h$, $y = Y + k$
in the equation $x^2 + y^2 - 5x + 2y - 5 = 0$, we get

$$(X+h)^2 + (Y+k)^2 - 5(X+h) + 2(Y+k) - 5 = 0$$

$$\Rightarrow \quad X^2 + h^2 + 2Xh + Y^2 + k^2 + 2Yk - 5X - 5h + 2Y + 2k - 5 = 0$$
$$\Rightarrow \quad X^2 + Y^2 + 2Yk + 2Y + 2Xh - 5X + h^2 + k^2 - 5h + 2k - 5 = 0$$
$$\Rightarrow \quad X^2 + Y^2 + (2k + 2)Y + (2h - 5)X + h^2 + k^2 - 5h + 2k - 5 = 0$$

For this equation to be free from the term of first degree, we must have

$$2k + 2 = 0 \text{ and } 2h - 5 = 0$$
$$\Rightarrow \quad k = -1 \text{ and } h = \frac{5}{2}$$

Hence, the origin is shifted at the point $\left(\frac{5}{2}, -1\right)$.

Brief Review of Cartesian System of Rectangular co-ordinates Ex 22.3 Q7(iii)

Let the origin be shifted to (h,k). Then, $x = X + h$ and $y = Y + k$

Substituting $x = X + h, \quad y = Y + k$

in the equation $x^2 + 12x + 4 = 0$, we get

$$(X+h)^2 - 12(X+h)^2 + 4 = 0$$

$$\Rightarrow \quad X^2 + h^2 + 2Xh - 12X - 12h + 4 = 0$$

$$\Rightarrow \quad X^2 + (2h - 12)X + h^2 - 12h + 4 = 0$$

For this equation to be free from term of first degree, we must have

$$2h - 12 = 0$$

$$\Rightarrow \quad h = \frac{12}{2}$$

$$\Rightarrow \quad h = 6$$

Hence, the origin is shifted at the point $(6,k)\,K \in R$.

Brief Review of Cartesian System of Rectangular co-ordinates Ex 22.3 Q8

Let the co-ordinate of the vertex be A(4,6) B(7,10) and C(1,-2)

Now area of the ΔABC is given by

$$\Delta = \frac{1}{2}\left|\left(x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right)\right|$$

$$= \frac{1}{2}\left|\left(4(10+2)+7(-2-6)+1(6-10)\right)\right|$$

$$= \frac{1}{2}\left|(48-56-4)\right|$$

$$= 6$$

After transforming the origin to (-2,1), the co-ordinate of the vertex will be

A(2,7),B(5,11) and C(-1,-1). Now the area will be

$$\Delta_1 = \frac{1}{2}\left|\left(x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right)\right|$$

$$= \frac{1}{2}\left|\left(2(11+1)+5(-1-7)-1(7-11)\right)\right|$$

$$= \frac{1}{2}\left|(24-40+4)\right|$$

$$= 6$$

Here $\Delta = \Delta_1$

Hence proved.

RD Sharma Solutions Class 11 Maths Chapter 22 Ex 22.3

Brief Review of Cartesian System of Rectangular co-ordinates Ex 22.3 Q1

We have,

$$(x-a)^2+(y-b)^2=r^2 \quad \ldots\ldots\text{(i)}$$

Substituting $x=X+(a-c)$, $y=Y+b$ in the equation (i), we get

$$[X+a-c-a]^2+[Y+b-b]^2=r^2$$

$$\Rightarrow \quad [X-c]^2+[Y]^2=r^2$$

$$\Rightarrow \quad X^2+c^2-2Xc+Y^2=r^2$$

$$\Rightarrow \quad X^2+y^2-2cX=r^2-c^2$$

Hence, the required equation is $X^2+y^2-2cX=r^2-c^2$

Brief Review of Cartesian System of Rectangular co-ordinates Ex 22.3 Q2

We have,

$$(a-b)\left(x^2+y^2\right)-2abx=0$$

Substituting $x=X+\frac{ab}{a-b}$, $y=Y$

in the given equation, we get

$$(a-b)\left[\left(X+\frac{ab}{a-b}\right)+Y^2\right]-2ab\left[X+\frac{ab}{a-b}\right]=0$$

$$\Rightarrow \quad (a-b)\left[X^2+\left(\frac{ab}{a-b}\right)^2+2\frac{Xab}{a-b}+Y^2\right]-2abX-2\frac{(ab)^2}{a-b}=0$$

$$\Rightarrow \quad (a-b)\left[\frac{X^2(a-b)^2+(ab)h^2+2Xab(a-b)+Y^2(a-b)^2}{(a-b)^2}\right]-\frac{2abX(a-b)+2(ab)^2}{a-b}=0$$

$$\Rightarrow \quad \frac{X^2(a-b)^2+(ab)^2+2ab(a-b)+Y^2(a-b)^2}{a-b}=\frac{2ab(a-b)+2(ab)^2}{a-b}$$

$$\Rightarrow \quad x^2(a-b)^2+Y^2(a-b)^2+(ab)^2+2ab(a-b)=2ab(a-b)+2(ab)^2$$

$$\Rightarrow \quad (a-b)^2\left(X^2+Y^2\right)=(ab)^2$$

$$\Rightarrow \quad (a-b)^2\left(X^2+Y^2\right)=a^2b^2$$

Brief Review of Cartesian System of Rectangular co-ordinates Ex 22.3 Q3(i)

We have,

$$x^2 + xy - 3x - y + 2 = 0$$

Substituting $x = X + 1,\ Y + 1$ in the equation, we get

$$(X+1)^2 + (X+1)(Y+1) - 3(X+1) - (Y+1) + 2 = 0$$

$$\Rightarrow \quad X^2 + 1 + 2X + XY + X + Y + 1 - 3X - 3 - Y - 1 + 2 = 0$$

$$\Rightarrow \quad X^2 + XY = 0$$

Brief Review of Cartesian System of Rectangular co-ordinates Ex 22.3 Q3(ii)

We have,

$$x^2 - y^2 - 2x + 2y = 0$$

Substituting $x = X + 1,\ \ y = Y + 1$ in the equation, we get

$$(X+1)^2 - (Y-1)^2 - 2(X+1) + 2(Y+1) = 0$$

$$\Rightarrow \quad X^2 + 1 + 2X - \left(Y^2 + 1 + 2Y\right) - 2X - 2 + 2Y + 2 = 0$$

$$\Rightarrow \quad X^2 + 1 - Y^2 - 1 - 2Y + 2Y = 0$$

$$\Rightarrow \quad X^2 - Y^2 = 0$$

Brief Review of Cartesian System of Rectangular co-ordinates Ex 22.3 Q3(iii)

We have,

$$xy - x - y + 1 = 0$$

Substituting $x = X + 1,\ \ y = Y + 1$ in the equation, we get

$$(X+1)(Y+1) - (X+1) - (Y+1) + 1 = 0$$

$$\Rightarrow \quad XY + X + Y + 1 - X - 1 - Y - 1 + 1 = 0$$

$$\Rightarrow \quad XY = 0$$

Brief Review of Cartesian System of Rectangular co-ordinates Ex 22.3 Q3(iv)

We have,

$$xy - y^2 - x + y = 0$$

Substituting $x = X + 1,\ \ y = Y + 1$ in the equation, we get

$$(X+1)(Y+1) - (Y+1) - (X+1) + (Y+1) = 0$$

$$\Rightarrow \quad XY + Y + Y + 1 - \left(Y^2 + 1 + 2Y\right) - X - 1 + Y + 1 = 0$$

$$\Rightarrow \quad XY + 2Y - Y^2 - 1 - 2Y + 1 = 0$$

$$\Rightarrow \quad XY - Y^2 = 0$$

Brief Review of Cartesian System of Rectangular co-ordinates Ex 22.3 Q4

We have,

$$x^2 + xy - 3x - y + 2 = 0 \;\ldots\ldots (i)$$

Let the origin be shifted to (h, k). Then $x = X + h$ and $y = Y + k$.

Substituting $x = X + h,\;\; y = Y + k$
in the equation (i), we get

$$(X + h)^2 + (X + h)(Y + k) - 3(X + h) - (Y + k) + 2 = 0$$

$$\Rightarrow \quad X^2 + h^2 + 2Xh + XY + Xk + Yh + hk - 3X - 3h - Y - k + 2 = 0$$

$$\Rightarrow \quad X^2 + XY + 2Xh + Xk + Yh - Y3 - X + h^2 + hk - 3h - k + 2 = 0$$

$$\Rightarrow \quad X^2 + (2Xh + Xk - 3X) + XY + (Yh - Y) + (h^2 + hk - 3h - k + 2) = 0$$

$$\Rightarrow \quad X^2 + (2h + k - 3)X + XY + (h - 1)Y + (h^2 + hk - 3h - k + 2) = 0$$

For this equation to be free from first degree and the constant term, we must have,

$$2h + k - 3 = 0 \;\ldots\ldots\ldots\ldots (ii)$$
$$h - 1 = 0$$
$$\Rightarrow \; h = 1 \;\ldots\ldots\ldots (iii)$$

and

$$h^2 + hk - 3h - k + 2 = 0 \;\ldots\ldots\ldots (iv)$$

Putting $h = 1$ in equation (ii), we get

$$2 + k - 3 = 0$$
$$\Rightarrow \quad k = 1$$

Putting $h = 1$ and $k = 1$ in equation (iv), we get

$$(1)^2 + 1 - 3 - 1 + 2 = 0$$

Hence, the value of h and k satisfies the equation (iv)

The origin is shifted at the point $(1, 1)$.

Brief Review of Cartesian System of Rectangular co-ordinates Ex 22.3 Q5

Let the vertices of a triangle be $A(2,3)$, $B(5,7)$ and $C(-3,-1)$

Then, area of ΔABC is given by

$$\Delta = \frac{1}{2}\left|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right|$$

$$= \frac{1}{2}\left|2(7+1) + 5(-1-3) - 3(-3-7)\right|$$
$$= \frac{1}{2}\left|2 \times 8 + 5 \times (-4) - 3 \times (-4)\right|$$
$$= \frac{1}{2}\left|16 - 20 + 12\right|$$
$$= \frac{8}{2}$$
$$= 4$$

$\Rightarrow \quad \Delta = 4 sq$ unit

It is given that the origin is shifted at $(-1,3)$. Then new coordinates of the vertices are

$$A_1 = (2-3, 3+3) = (-1, 6)$$
$$B_1 = (5-1, 7+3) = (4, 10)$$
and $\quad C_1 = (-3-1, -1+3) = (-4, 2)$

Therefore, the area of the triangle in the new coordinate system is given by

$$\Delta_1 = \frac{1}{2}\left|\left[-1(10-2) + 4(2-6) - 4(6-10)\right]\right|$$
$$= \frac{1}{2}\left|\left[-1 \times 8 + 4 \times (-4) - 4 \times (-4)\right]\right|$$
$$= \frac{1}{2}\left|\left[-8 - 16 + 16\right]\right|$$
$$= \frac{1}{2}\left|-8\right|$$
$$= \frac{8}{2}$$

$\Rightarrow \quad \Delta_1 = 4 \ldots\ldots\ldots\ldots (2)$

From (i) and (ii), we get

$$\Delta = \Delta_1$$

Hence, the area of a triangle is invariant under the translation of the axes.

Brief Review of Cartesian System of Rectangular co-ordinates Ex 22.3 Q6(i)

We have,

$$x^2 + xy - 3y^2 - y + 2 = 0 \ldots\ldots (i)$$

Substituting $x = X + 1$, $y = Y + 1$
in equation (i), we get

$$(X+1)^2 + (X+1)(Y+1) - 3(Y+1)^2 - (Y+1) + 2 = 0$$

$\Rightarrow \quad X^2 + 1 + 2X + XY + X + Y + 1 - 3\left(Y^2 + 1 + 2Y\right) - Y - 1 + 2 = 0$

$\Rightarrow \quad X^2 + XY + 3X + 3 - 3Y^2 - 3 - 6Y = 0$

$\Rightarrow \quad X^2 - 3Y^2 + XY + 3X - 6Y = 0$

Brief Review of Cartesian System of Rectangular co-ordinates Ex 22.3 Q6(ii)

We have,

$$xy - y^2 - x + y = 0 \ldots\ldots (i)$$

Substituting $x = X + 1$, $y = Y + 1$
in equation (i), we get

$$(X+1)(Y+1) - (Y+1)^2 - (X+1) + (Y+1) = 0$$

$\Rightarrow \quad XY + X + Y + 1 - \left(Y^2 + 1 + 2Y\right) - X - 1 + Y + 1 = 0$

$\Rightarrow \quad XY + 2Y + 1 - Y^2 - 1 - 2Y = 0$

$\Rightarrow \quad XY - Y^2 = 0$

Brief Review of Cartesian System of Rectangular co-ordinates Ex 22.3 Q6(iii)

We have,

$$xy - x - y + 1 = 0 \ldots\ldots (i)$$

Substituting $x = X + 1, \ y = Y + 1$
in equation (i), we get

$$(X+1)(Y+1) - (X+1) - (Y+1) + 1 = 0$$

$\Rightarrow \quad XY + X + Y + 1 - X - 1 - Y - 1 + 1 = 0$

$\Rightarrow \quad XY = 0$

Brief Review of Cartesian System of Rectangular co-ordinates Ex 22.3 Q6(iv)

We have,

$$x^2 - y^2 - 2x - 2y = 0 \ldots\ldots (i)$$

Substituting $x = X + 1, \ y = Y + 1$
in equation (i), we get

$$(X+1)^2 - (Y+1)^2 - 2(X+1) + 2(Y+1) = 0$$

$\Rightarrow \quad X^2 + 1 + 2X - \left(Y^2 + 1 + 2Y\right) - 2X - 2 + 2Y + 2 = 0$

$\Rightarrow \quad X^2 + 1 - Y^2 - 1 - 2Y + 2X = 0$

$\Rightarrow \quad X^2 + 2X + 1 - \left(Y^2 + 2Y + 1\right)$

$\Rightarrow \quad (X+1)^2 - (Y+1)^2$

$\Rightarrow \quad x^2 - y^2 = 0$

Brief Review of Cartesian System of Rectangular co-ordinates Ex 22.3 Q7(i)

Let the origin be shifted to (h, k). Then, $x = X + h$ and $y = Y + k$

Substituting $x = X + h, \quad y = Y + k$
in the equation $y^2 + x^2 - 4x - 8y + 3 = 0$, we get

$$(Y+k)^2 + (X+h)^2 - 4(X-h) - 8(Y+k) + 3 = 0$$

$\Rightarrow \quad Y^2 + k^2 + 2Yk + X^2 + h^2 + 2Xh - 4X - 4h - 8Y - 8k + 3 = 0$

$\Rightarrow \quad Y^2 + X^2 + 2Yk - 8Y + 2Xh - 4X + k^2 + h^2 - 4h - 8k + 3 = 0$

$\Rightarrow \quad Y^2 + X^2 + (2k - 8)Y + (2h - 4)X + \left(k^2 + h^2 - 4h - 8k + 3\right) = 0$

For this equation to be free from the term of first degree, we must have

$2k - 8 = 0$ and $2h - 4 = 0$

$\Rightarrow \quad k = 4$ and $h = 2$

Hence, the origin is shifted at the point $(2, 4)$.

Brief Review of Cartesian System of Rectangular co-ordinates Ex 22.3 Q7(ii)

Let the origin be shifted to (h, k). Then, $x = X + h$ and $y = Y + k$
Substituting $x = X + h, \quad y = Y + k$
in the equation $x^2 + y^2 - 5x + 2y - 5 = 0$, we get

$$(X+h)^2 + (Y+k)^2 - 5(X+h) + 2(Y+k) - 5 = 0$$

$\Rightarrow \quad X^2 + h^2 + 2Xh + Y^2 + k^2 + 2Yk - 5X - 5h + 2Y + 2k - 5 = 0$

$\Rightarrow \quad X^2 + Y^2 + 2Yk + 2Y + 2Xh - 5X + h^2 + k^2 - 5h + 2k - 5 = 0$

$\Rightarrow \quad X^2 + Y^2 + (2k + 2)Y + (2h - 5)X + h^2 + k^2 - 5h + 2k - 5 = 0$

For this equation to be free from the term of first degree, we must have

$2k + 2 = 0$ and $2h - 5 = 0$

$\Rightarrow \quad k = -1$ and $h = \frac{5}{2}$

Hence, the origin is shifted at the point $\left(\frac{5}{2}, -1\right)$.

Brief Review of Cartesian System of Rectangular co-ordinates Ex 22.3 Q7(iii)

Let the origin be shifted to (h, k). Then, $x = X + h$ and $y = Y + k$

Substituting $x = X + h, \quad y = Y + k$

in the equation $x^2 + 12x + 4 = 0$, we get

$$(X+h)^2 - 12(X+h)^2 + 4 = 0$$

$$\Rightarrow \quad X^2 + h^2 + 2Xh - 12X - 12h + 4 = 0$$

$$\Rightarrow \quad X^2 + (2h - 12)X + h^2 - 12h + 4 = 0$$

For this equation to be free from term of first degree, we must have

$$2h - 12 = 0$$

$$\Rightarrow \quad h = \frac{12}{2}$$

$$\Rightarrow \quad h = 6$$

Hence, the origin is shifted at the point $(6, k)\, K \in R$.

Brief Review of Cartesian System of Rectangular co-ordinates Ex 22.3 Q8

Let the co-ordinate of the vertex be A(4,6) B(7,10) and C(1,-2)

Now area of the ΔABC is given by

$$\Delta = \frac{1}{2}\left|\left(x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right)\right|$$

$$= \frac{1}{2}\left|\left(4(10+2) + 7(-2-6) + 1(6-10)\right)\right|$$

$$= \frac{1}{2}\left|(48 - 56 - 4)\right|$$

$$= 6$$

After transforming the origin to (-2,1), the co-ordinate of the vertex will be

A(2,7),B(5,11) and C(-1,-1). Now the area will be

$$\Delta_1 = \frac{1}{2}\left|\left(x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right)\right|$$

$$= \frac{1}{2}\left|\left(2(11+1) + 5(-1-7) - 1(7-11)\right)\right|$$

$$= \frac{1}{2}\left|(24 - 40 + 4)\right|$$

$$= 6$$

Here $\Delta = \Delta_1$

Hence proved.

RD Sharma Solutions Class 11 Maths Chapter 23 Ex 23.2

Straight Lines Ex 23.2 Q1

Let the equation of the line be:

$y - y_1 = m(x - x_1)$

Now,

$m = 0$ [$\because$ Parallel lines have equal slopes, the slope of x-axis is 0]

$(x_1, y_1) = (3, -5)$

$\therefore y - y_1 = m(x - x_1)$

$y - (-5) = 0(x - 3)$

$y + 5 = 0$

Straight Lines Ex 23.2 Q2

The slope of x-axis is 0, any line perpendicular to it will have

slope $= \frac{-1}{0}$

Also the required line is passing through the point $(-2, 0)$

(because it is given it has x-intercept is -2)

The required equation of line is

$y - y_1 = m(x - x_1)$

where $m = \frac{-1}{0}, (x_1 y_1) \Rightarrow (-2, 0)$

$y - 0 = \frac{-1}{0}(x - (-2))$

$y - 0 = \frac{-1}{0}(x + 2)$

$-(x + 2) = 0$

$x + 2 = 0$

$x = -2$

Straight Lines Ex 23.2 Q3

The slope of x-axis is 0

Any line parallel to x-axis will also have the same slope.

therefore $m = 0$

Also line has y-intercept, ie. $(0, b)$

$\Rightarrow (0, -2) \Rightarrow (x_1 y_1)$

The required equation of the line is $y - y_1 = m(x - x_1)$

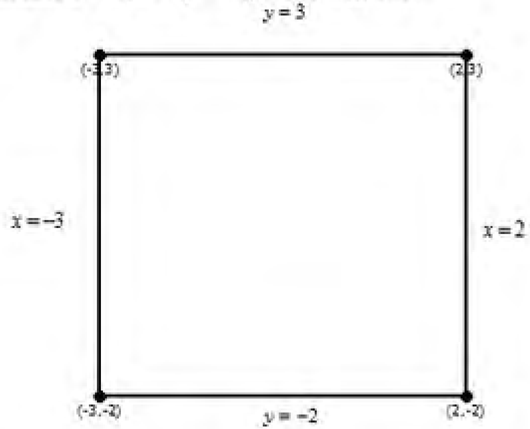
$y - (-2) = 0(x - 0)$

$y + 2 = 0$

$y = -2$

Straight Lines Ex 23.2 Q4

The figure with the lines $x=-3, x=2, y=-2, y=3$ is as follows:



From the figure, the co-ordinates of the vertices of the square are $(2,3),(-3,3),(-3,-2),(2,-2)$.

Straight Lines Ex 23.2 Q5

Slope of a line parallel to x-axis = 0

Since the line passes through $(4,3)$,

The required equation of the line parallel to x-axes is

$y - y_1 = m(x - x_1)$

$y - (3) = 0(x - 4)$

$y - 3 = 0$

$y = 3$

Slope of a line perpendicular to x-axis = $\frac{-1}{0}$

The required equation of the line perpendicular to x-axis is

$y - y_1 = m(x - x_1)$

$y - 3 = \frac{-1}{0}(x - 4)$

$x - 4 = 0$

$x = 4$

Straight Lines Ex 23.2 Q6

Let $x = \lambda$ be the line equidistant from

$x = -2$ and $x = 6$

so $\left|\frac{-2-\lambda}{\sqrt{1}}\right| = \left|\frac{\lambda-6}{\sqrt{1}}\right|$

$-2-\lambda = \lambda - 6$

$4 = 2\lambda$

$\therefore \lambda = 2$

$\therefore$ The line equidistant from $x = -2$ and $x = 6$ is $x = 2$

Straight Lines Ex 23.2 Q7

A line which is equidistant from two other lines, must have the same slope.

The slope of $y = 10$ and $y = -2$ is 0, ie line parallel to x-axis.

The required line is also parallel to $y = 10$ and $y = -2$

$\therefore \quad m = 0$

Also, the required line will pass from the mid-point of the line joining (0, -2) and (0,10)

Coordinates of this point will be $(0, \frac{10-2}{2}) = (0, \frac{8}{2}) = (0, 4)$

$\therefore$ The equation of the require line is:

$y-4=0(x-x_1)$

$\Rightarrow y = 4$

RD Sharma Solutions Class 11 Maths Chapter 23 Ex 23.3

Straight Lines Ex 23.3 Q1

The equation of the line having slope m and y-intercept (0, c) is given by:

$$y = mx + c$$

Now, $m = \tan\left(150^0\right) = \frac{-1}{\sqrt{3}}$

and

y-intercept is $(0, 2)$

The required equation of line is

$$y = mx + c$$

$$\Rightarrow \quad y = \frac{-x}{\sqrt{3}} + 2$$

$$\Rightarrow \quad \sqrt{3}y - 2\sqrt{3} + x = 0$$

$$\Rightarrow \quad x + \sqrt{3}y = 2\sqrt{3}$$

Straight Lines Ex 23.3 Q2

(i) With slope 2 and y intercept 3

$m = 2$, point is $(0, 3)$

The required equation of line is

$$y = mx + c$$

$$\Rightarrow \quad y = 2x + 3$$

(ii) slope $= \frac{-1}{3}$, y intercept $= (0, -4)$

$m = \frac{-1}{3}$, $c = -4$

The required equation of line is $y = mx + c$

$$\Rightarrow \quad y = \frac{-1}{3}x - 4$$

$$\Rightarrow \quad 3y + x = -12$$

(iii) $m = -2$, $c = -3$

The required equation of line is

$y - y_1 = m(x - x_1)$

Since the line cuts the $x - axis$ *at* $(-3, 0)$ *with slope* -2, *we have*,

$y - 0 = -2(x + 3)$

$\Rightarrow y = -2x - 6$

$\Rightarrow 2x + y + 6 = 0$

Straight Lines Ex 23.3 Q3

The given lines are $x = 0, y = 0$.

The equation of the bisectors of the angles between $x = 0$ and $y = 0$ are:

$$\frac{x}{\sqrt{(1)^2 + (0)^2}} = \pm \frac{y}{\sqrt{(0)^2 + (1)^2}}$$

$x = \pm y$

$x \pm y = 0$

Straight Lines Ex 23.3 Q4

$\theta = \tan^{-1} 3 \Rightarrow m = \tan\theta = 3$

Intercept in negative direction of $y - axis$ is $(0, -4)$

Hence, required equation of line is

$$y = mx + c$$

$$\Rightarrow \quad y = 3x - 4$$

Straight Lines Ex 23.3 Q5

Here, y intercept, c = -4

The required line is parallel to line joining (2,-5) and (1,2)
Let m be the slope of the required line, then

m = slope of (2,-5) and (1,2)

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-(-5)}{1-2} = \frac{7}{-1} = -7$$

∴ the required equation of line is

$y = mx + c$

$y = -7x - 4$

$7x + y + 4 = 0$

Straight Lines Ex 23.3 Q6

The required equation of line is $y = mx + c$
Here, c = 3

Let m be slope of the required line.

Then,
m × slope of given line = -1

$$\text{Slope of given line} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5-2}{3-4} = \frac{3}{-1} = -3$$

$$\Rightarrow \quad m = \frac{1}{3}$$

So, the required equation is:

$y = mx + c$

$y = \frac{1}{3}x + 3$

$x - 3y + 9 = 0$

Straight Lines Ex 23.3 Q7

The required equation of line is $y = mx + c$
Here, $c = -3$

Let m be slope of the required line.

Then,
$m \times$ slope of given line $= -1$

Slope of line joining $(4,3)$ and $(-1,1)$ $= \frac{y_2 - y_1}{x_2 - x_1} = \frac{1-3}{-1-4} = \frac{-2}{-5} = \frac{2}{5}$

$\Rightarrow \quad m = -\frac{5}{2}$

So, the required equation is:

$y = mx + c$

$y = -\frac{5}{2}x - 3$

$y + 3 = \frac{-5x}{2}$

$2y + 5x + 6 = 0$

Straight Lines Ex 23.3 Q8

The required equation of line is

$y - y_1 = m(x - x_1)$

where $m = \tan 30^0 = \frac{1}{\sqrt{3}}$

point is $(x_1 y_1) = (0,2)$

$\Rightarrow y - 2 = \frac{1}{\sqrt{3}}(x - 0)$

$x - \sqrt{3}y + 2\sqrt{3} = 0$

RD Sharma Solutions Class 11 Maths Chapter 23 Ex 23.4

Straight Lines Ex 23.4 Q1

Let the required equation of the line be

$y - y_1 = m(x - x_1)$

Now,

$m = \text{slope} = -3$

$(x_1 y_1) = (6, 2)$

$\therefore\ y - y_1 = m(x - x_1)$

$\Rightarrow y - 2 = -3(x - 6)$

$\Rightarrow y - 2 = -3x + 18$

$\Rightarrow 3x + y = +20$

$\Rightarrow 3x + y - 20 = 0$

$\therefore$ The equation of the given line is $3x + y - 20 = 0$.

Straight Lines Ex 23.4 Q2

Let the required equation of the line be

$y - y_1 = m(x - x_1)$

Now,
The line is inclined at an angle of 45^0 with x-axis
$\therefore m = \tan 45^0 = 1$

$(x_1 y_1) = (-2, 3)$

$\therefore y - y_1 = m(x - x_1)$
$\Rightarrow y - 3 = 1(x - (-2))$
$\Rightarrow y - 3 = x + 2$
$\Rightarrow x - y = -5$
$\therefore$ Equation of required line is $x - y + 5 = 0$

Straight Lines Ex 23.4 Q3

The required equation of the line is
$y - y_1 = m(x - x_1)$
$(x_1, y_1) = (0,0)$ and slope is m
Therefore, $y - y_1 = m(x - x_1)$
$y - 0 = m(x - 0)$
$y = mx$

Straight Lines Ex 23.4 Q4

The required equation of the line is
$y - y_1 = m(x - x_1)$
Since the line makes an angle 75° with $x - axis$
$m = \tan 75^\circ = 3.73$
$(x_1, y_1) = (2, 2\sqrt{3})$
Therefore, $y - y_1 = m(x - x_1)$
$y - 2\sqrt{3} = (2 + \sqrt{3})(x - 2)$
$y - 2\sqrt{3} = (2 + \sqrt{3})x - 7.46$
$(2 + \sqrt{3})x - y - 4 = 0$

Straight Lines Ex 23.4 Q5

Let $\sin\theta = \frac{3}{4}$

Then,

$\Rightarrow m = \text{slope} = \tan\theta = \frac{3}{4}$

The equation of straight line with slope m and passing through (1, 2) is

$y - y_1 = m(x - x_1)$

$y - 2 = \frac{3}{4}(x - 1)$

$4y - 8 = 3x - 3$

$3x - 4y = -5$

$3x - 4y + 5 = 0$

Straight Lines Ex 23.4 Q6

The required equation of the line is

$y - y_1 = m(x - x_1)$

Since the line makes an angle 60^0 with the positive direction of y axis, it makes 30^0 with the positive direction of x axis.

$\therefore m = \tan 30^0 = \frac{1}{\sqrt{3}}$ (angle with y-axis)

A point on the line is $(x_1 y_1) = (3, -2)$

Therefore, the equation of the line is:

$y - y_1 = m(x - x_1)$

$y - (-2) = \frac{1}{\sqrt{3}}(x - 3)$

$x - \sqrt{3}y - 3 - 2\sqrt{3} = 0$

Straight Lines Ex 23.4 Q7

Equation of the line passing through (x_1, y_1)
and making angle θ with the x-axis is,
$(y - y_1) = \tan\theta (x - x_1)$

For the first line: $(x_1, y_1) = (0, 2), \theta = \frac{\pi}{3}$
$(y - y_1) = \tan\theta (x - x_1)$
$(y - 2) = \left(\tan\frac{\pi}{3}\right)(x - 0)$
$y - 2 = \sqrt{3}x$
$\sqrt{3}x - y + 2 = 0$

For the second line: $(x_1, y_1) = (0, 2), \theta = \frac{2\pi}{3}$
$(y - y_1) = \tan\theta (x - x_1)$
$(y - 2) = \left(\tan\frac{2\pi}{3}\right)(x - 0)$
$y - 2 = -\sqrt{3}x$
$\sqrt{3}x + y - 2 = 0$

The line parallel to $\sqrt{3}x - y + 2 = 0$
and cutting y-axis at a distance of 2 units below the origin.
$y = \sqrt{3}x - 2$
$\sqrt{3}x + y - 2 = 0$

The line parallel to $\sqrt{3}x + y - 2 = 0$
and cutting y-axis at a distance of 2 units below the origin.
$y = -\sqrt{3}x - 2$
$\sqrt{3}x + y + 2 = 0$

Straight Lines Ex 23.4 Q8

If a line is equally inclined to axis, then
$\theta = 45^0$ or $\theta = 135^0 \Rightarrow m = \tan\theta = \pm 1$

Since, y intercept, $c = 5$

$\therefore$ We get the solution of the line as:
$y = mx + c$
$y = \pm 1x + 5$
$y - x = 5$ or $y + x = 5$

Straight Lines Ex 23.4 Q9

The line passes through the point (2,0).

Also its inclination to $y-$axis is $135°$.

That is, the inclination of the given line with the $x-$axis is $180° - 135°$.

That is, the slope of the given line is $45°$

The equation of the line having slope 'm' and passing through the point (x_1, y_1) is $y - y_1 = m(x - x_1)$

Therefore, the required equation is

$y - 0 = \tan 45°(x-2)$

$\Rightarrow y = 1 \times (x-2)$

$\Rightarrow y = x - 2$

$\Rightarrow x - y - 2 = 0$

Straight Lines Ex 23.4 Q10

The coordinates of the point which divides the join of the points (2,3) and (-5,8) in the ratio 3:4 is given by (x, y) where,

$$x = \frac{lx_2 + mx_1}{l+m} = \frac{3(-5)+4(2)}{3+4} = \frac{-15+6}{7} = \frac{-9}{7}$$

$$y = \frac{ly_2 + my_1}{l+m} = \frac{3(8)+4(3)}{3+4} = \frac{24+12}{7} = \frac{36}{7}$$

Slope of the line joining the points (2,3) and (-5,8) $= \frac{8-3}{-5-2} = \frac{5}{-7} = \frac{-5}{7}$

$\therefore$ Slope of line perpendicular to line $= m = \frac{7}{5}$

The required equation is:

$$y - y_1 = m(x - x_1)$$

$$y - \frac{36}{7} = \frac{7}{5}\left(x - \left(\frac{-9}{7}\right)\right)$$

$$49x - 35y + 229 = 0$$

Straight Lines Ex 23.4 Q11

Let the perpendicular drawn from $P(4,1)$ on line joining $A(2,-1)$ and $B(6,5)$ divide in the ratio k:1 at the point R.

Using section formula, coordinates of R are:

$$x = \frac{6k+2}{k+1} \text{ and } y = \frac{5k-1}{k+1} \quad ---(1)$$

PR is perpendicular to AB

$\therefore$ (slope of PR) $\times$ (slope of AB) = -1

$$\Rightarrow \left(\frac{y-1}{x-4}\right) \times \left(\frac{5-(-1)}{6-2}\right) = -1$$

$$\Rightarrow \frac{\frac{5k-1}{k+1} - 1}{\frac{6k+2}{k+1} - 4} \times \frac{6}{4} = -1$$

$$\Rightarrow \frac{5k-1-k-1}{6k+2-4k-4} = \frac{-4}{6}$$

$$\Rightarrow \frac{4k-2}{2k-2} = \frac{-2}{3}$$

$$\Rightarrow 3(2k-1) = -2(k-1)$$

$$\Rightarrow 6k-3 = -2k+2$$

$$\Rightarrow 8k = 5$$

$$\Rightarrow k = \frac{5}{8}$$

ratio is 5 : 8

$\therefore$ R divides AB in the ratio 5 : 8

Straight Lines Ex 23.4 Q12

AD, BE and CF are the three altitudes of the triangle

We know,

Slope of AD × Slope of BC = -1; AD passes through A(2,-2)
Slope of BE × Slope of AC = -1; AD passes through B(1,1)
Slope of CF × Slope of AB = -1; AD passes through C(-1,0)

Slope of BC $= \frac{0-1}{-1-1} = \frac{-1}{-2} = \frac{1}{2}$ $\Rightarrow$ Slope of AD = -2

Slope of AC $= \frac{0-(-2)}{-1-2} = \frac{2}{-3} = \frac{-2}{3}$ $\Rightarrow$ Slope of BE $= \frac{3}{2}$

Slope of AB $= \frac{1+2}{1-2} = \frac{3}{-1} = -3$ $\Rightarrow$ Slope of CF $= \frac{1}{3}$

So, for AD, we have

$y - y_1 = m(x - x_1)$

$\Rightarrow y - (-2) = -2(x - 2)$

$\Rightarrow y + 2 = -2x + 4$

$\Rightarrow 2x + y - 2 = 0$

And, for BE, we have

$y - y_1 = m(x - x_1)$

$\Rightarrow y - 1 = \frac{3}{2}(x - 1)$

$\Rightarrow 2y - 3x + 1 = 0$

And, for CF, we have

$y - y_1 = m(x - x_1)$

$\Rightarrow y - 0 = \frac{1}{3}(x + 1)$

$\Rightarrow x - 3y + 1 = 0$

Straight Lines Ex 23.4 Q13

The right bisector PQ of AB bisects AB at C and is perpendicular to AB.

The co-ordinates of C are $= \left(\frac{3-1}{2}, \frac{4+2}{2}\right) = (1,3)$

And slope of $PQ = \frac{-1}{\text{slope of } AB} = \frac{-1}{2-4}(-1-3) = \frac{4}{-2} = -2$

The equation of PQ is

$(y - 3) = -2(x - 1)$

$y - 3 = -2x + 2$

$y + 2x = 5$

Straight Lines Ex 23.4 Q14

The line passes through the point $(-3, 5)$

So $(x_1, y_1) = (-3, 5)$

The line is perpendicular to the line joining $(2,5)$ and $(-3,6)$.

$$\Rightarrow m = \frac{-1}{\text{slope of line joining } (2,5) \text{ and } (-3,6)} = \frac{-1}{\frac{y_2 - y_1}{x_2 - x_1}} = \frac{-1}{\frac{6-5}{-3-2}} = \frac{-1}{\frac{-1}{5}}$$

$\therefore m = 5$

Hence, equation of straight line is

$y - y_1 = m(x - x_1)$

$y - 5 = 5(x(-3))$

$y - 5 = 5x + 15$

$5x - y + 20 = 0$

Straight Lines Ex 23.4 Q15

The right bisector PQ of AB bisects AB at C and is also perpendicular to AB.

Slope of $AB = \frac{3-0}{2-1} = 3$

Now,

$(\text{slope of } AB) \times (\text{slope of } PQ) = -1$

$\therefore$ slope of $PQ = \frac{-1}{3}$

Co-ordinates of c are $= \left(\frac{1+2}{2}, \frac{3+0}{2}\right) = \left(\frac{3}{2}, \frac{3}{2}\right)$

$\therefore$ Equation of right bisector PQ is

$\left(y - \frac{3}{2}\right) = \frac{-1}{3}\left(x - \frac{3}{2}\right)$

$6y - 9 = -2x + 3$

$x + 3y = 6$

Straight Lines Ex 23.4 Q16

Equation of the line passing through (x_1, y_1) and making angle θ with the x-axis is,

$$(y - y_1) = \tan\theta(x - x_1)$$

Here $(x_1, y_1) = (1, 2)$, angle with y-axis is 30°

$\therefore$ angle with x-axis is $\theta = 90^\circ - 30^\circ = 60^\circ$

$$(y - y_1) = \tan\theta(x - x_1)$$

$$(y - 2) = \left(\tan 60^\circ\right)(x - 1)$$

$$y - 2 = \sqrt{3}x - \sqrt{3}$$

$$\sqrt{3}x - y + 2 - \sqrt{3} = 0$$

RD Sharma Solutions Class 11 Maths Chapter 23 Ex 23.5

Straight Lines Ex 23.5 Q1(i)

Here,

$(x_1 y_1) = (0,0)$

$(x_2 y_2) = (2,-2)$

The equation of the given straight line is:

$$y - y_1 = m(x - x_1)$$

$$\Rightarrow \quad y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$\Rightarrow \quad y - 0 = \frac{-2 - 0}{2 - 0}(x - 0)$$

$$\Rightarrow \quad y = \frac{-2x}{2}$$

$$\Rightarrow \quad y = -x$$

$\therefore$ The equation of the line joining the points $(0,0)$ and $(2,-2)$ is $y = -x$

Straight Lines Ex 23.5 Q1(ii)

Let $A(a,b) = (x_1 y_1)$

$B(a + c\sin\alpha, b + c\cos\alpha) = (x_2 y_2)$

Then equation of line AB is

$$\Rightarrow \quad y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$\Rightarrow \quad y - b = \frac{b + c\cos\alpha - b}{a + c\sin\alpha - a}(x - a)$$

$$\Rightarrow \quad y - b = \frac{c\cot\alpha}{c\sin\alpha}(x - a)$$

$$\Rightarrow \quad y - b = \cot\alpha(x - a)$$

$\therefore$ The equation of the line joining the points (a,b) and $(a + c\sin\alpha, b + c\cos\alpha)$ is $y - b = \cot\alpha(x - a)$

Straight Lines Ex 23.5 Q1(iii)

Let $A(a,-a)$ be $(x_1 y_1)$

$B(b,0)$ be $(x_2 y_2)$

Then equation of line AB is

$$\Rightarrow \quad y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$\Rightarrow \quad y - (-a) = \frac{0 - (-a)}{b - 0}(x - 0)$$

$$\Rightarrow \quad y + a = \frac{a}{b}(x - 0)$$

$$\Rightarrow \quad ax - by = ab$$

$\therefore$ The equation of the line joining the points $(0,-a)$ and $(b,0)$ is $ax - by = ab$

Straight Lines Ex 23.5 Q1(iv)

Let $A(a,b)$ be (x_1y_1)

$B(a+b,a-b)$ be (x_2y_2)

Then equation of line AB is

$$\Rightarrow \quad y-y_1=\frac{y_2-y_1}{x_2-x_1}(x-x_1)$$

$$\Rightarrow \quad y-b=\frac{a-b-b}{a+b-a}(x-a)$$

$$\Rightarrow \quad y-b=\frac{a-2b}{b}(x-a)$$

$$\Rightarrow \quad by-b^2=ax-a^2-2bx+2ba$$

$$\Rightarrow \quad (a-2b)x-by+b^2-a^2+2ab=0$$

$\therefore$ The equation of the line joining the points (a,b) and $(a+b,a-b)$ is $(a-2b)x-by+b^2-a^2+2ab=0$

Straight Lines Ex 23.5 Q1(v)

Let $A(x_1y_1)$ be $\left(at_1,\frac{a}{t_1}\right)$

$B(x_2y_2)$ be $\left(at_2,\frac{a}{t_2}\right)$

Then equation of line AB is

$$\Rightarrow \quad y-y_1=\frac{y_2-y_1}{x_2-x_1}(x-x_1)$$

$$\Rightarrow \quad y-\frac{a}{t_1}=\frac{\frac{a}{t_2}-\frac{a}{t_1}}{at_2-at_1}(x-at_1)$$

$$\Rightarrow \quad y-\frac{a}{t_1}=\frac{a(t_1-t_2)}{at_1t_2(t_2-t_1)}(x-at_1)$$

$$\Rightarrow \quad y-\frac{a}{t_1}=\frac{-1}{t_1t_2}(x-at_1)$$

$$\Rightarrow \quad t_1t_2y+x=a(t_1+t_2)$$

$\therefore$ The equation of the line joining the points $\left(at_1,\frac{a}{t_1}\right)$ and $\left(at_2,\frac{a}{t_2}\right)$ is $t_1t_2y+x=a(t_1+t_2)$

Straight Lines Ex 23.5 Q1(vi)

Let $A(x_1 y_1)$ be $(a\cos\alpha, a\sin\alpha)$

$B(x_2 y_2)$ be $(a\cos\beta, a\sin\beta)$

$$\Rightarrow \quad y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$\Rightarrow \quad y - a\sin\alpha = \frac{a\sin\beta - a\sin\alpha}{a\cos\beta - a\cos\alpha}(x - a\cos\alpha)$$

$$\Rightarrow \quad y - a\sin\alpha = \frac{a\left(-2\sin\left(\frac{\beta-\alpha}{2}\right)\right)\cos\beta\left(\frac{\beta+\alpha}{2}\right)}{a\left(-2\sin\frac{\beta-\alpha}{2}\right)\sin\left(\frac{\beta+\alpha}{2}\right)}(x - a\cos\alpha)$$

$$\Rightarrow \quad y - a\sin\alpha = \frac{\cos\left(\frac{\alpha+\beta}{2}\right)}{\sin\left(\frac{\alpha+\beta}{2}\right)}(x - a\cos\alpha)$$

$$\Rightarrow \quad x\cos\left(\frac{\alpha+\beta}{2}\right) + y\sin\frac{\alpha+\beta}{2} = a\cos\frac{\alpha-\beta}{2}$$

$\therefore$ The equation of the line joining the points $(a\cos\alpha, a\sin\alpha)$ and $(a\cos\beta, a\sin\beta)$ is

$$x\cos\left(\frac{\alpha+\beta}{2}\right) + y\sin\frac{\alpha+\beta}{2} = a\cos\frac{\alpha-\beta}{2}$$

Straight Lines Ex 23.5 Q2(i)

Let $A(1,4)$, $B(2,-3)$, $C(-1,-2)$

Then equation of AB is

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y - 4 = \frac{-3-4}{2-1}(x-1)$$

$$y - 4 = \frac{-7}{1}(x-1)$$

$$7x + y = 11$$

Equation of side BC is

$$y - y_2 = \frac{y_3 - y_2}{x_3 - x_2}(x - x_2)$$

$$y - (-3) = \frac{-2-(-3)}{-1-2}(x-2)$$

$$y + 3 = \frac{1}{-3}(x-2)$$

$$x + 3y + 7 = 0$$

Equation of side AC is

$$y - y_1 = \frac{y_3 - y_1}{x_3 - x_1}(x - x_1)$$

$$y - 4 = \frac{-2-4}{-1-1}(x-1)$$

$$y - 4 = 3(x-1)$$

$$y - 3x = 1$$

Straight Lines Ex 23.5 Q2(ii)

Let $A(0,1), B(2,0), C(-1,-2)$

then equation of side AB is

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$
$$y - 1 = \frac{0 - 1}{2 - 0}(x - 0)$$
$$y - 1 = \frac{-1}{2}(x)$$
$$x + 2y = 2$$

Equation of side BC is

$$y - y_2 = \frac{y_3 - y_2}{x_3 - x_2}(x - x_1)$$
$$y - 0 = \frac{-2 - 0}{-1 - 2}(x - 2)$$
$$y = \frac{2}{3}(x - 2)$$
$$2x - 3y = 4$$

Equation of side AC is

$$y - y_1 = \frac{y_3 - y_1}{x_3 - x_1}(x - x_1)$$
$$y - 1 = \frac{-2 - 1}{-1 - 0}(x - 0)$$
$$y - 1 = 3(x - 0)$$

$$y - 3x = 1$$

Straight Lines Ex 23.5 Q3

Let $A(-1,6)$ be $(x_1 y_1)$

$B(-3,-9)$ be $(x_2 y_2)$

$C(5,-8)$ be $(x_3 y_3)$

Median is a line segment which joins a vertex to the mid-point of the side opposite to it.
Let D, E and F be the mid points of sides AB, BC, and CA.

Then, using mid point formula $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$ we can find the coordinates of D, E and F as:

$$D = \left(\frac{-3+5}{2}, \frac{-9-8}{2}\right) = \left(-1, \frac{-17}{2}\right)$$

$$E = \left(\frac{-1+5}{2}, \frac{6-8}{2}\right) = (2,-1)$$

$$F = \left(\frac{-1-3}{2}, \frac{6-9}{2}\right) = \left(-2, \frac{-3}{2}\right)$$

Equation of median AD is

$$y-y_1 = \frac{y_2-y_1}{x_2-x_1}(x-x_1)$$

$$y-6 = \frac{\frac{-17}{2}-6}{1-(-1)}(x+1) = \frac{-29}{4}(x+1) \qquad \left[A(-1,6), D\left(1, \frac{-17}{2}\right)\right]$$

$$29x+4y+5=0$$

Equation of median BE is

$$y-y_1 = \frac{y_2-y_1}{x_2-x_1}(x-x_1)$$

$$y-(-9) = \frac{-1-(-9)}{2-(-3)}(x-(-3)) \qquad [B(-3,-9), E(2,-1)]$$

$$y+9 = \frac{8}{5}(x+3)$$

$$5y+45 = 8x+24$$

$$8x-5y-21=0$$

Equation of median CF is

$$y-y_1 = \frac{y_2-y_1}{x_2-x_1}(x-x_1)$$

$$y-(-8) = \frac{\frac{-3}{2}-(-8)}{-2-5}(x-5) \qquad \left[C(5,-8), F\left(-2, \frac{-3}{2}\right)\right]$$

$$y+8 = \frac{-3+16}{2\times(-7)}(x-5)$$

$$y+8 = \frac{-13}{14}(x-5)$$

$$13x+14y+47=0$$

Straight Lines Ex 23.5 Q4

The rectangle ABCD will have diagonals AC and BD
AC passes through A(a,b) and C(a',b').
Thus equation of AC is:

$$\frac{y-y_1}{y_2-y_1}=\frac{x-x_1}{x_2-x_1}$$
$$\Rightarrow \frac{y-b}{b'-b}=\frac{x-a}{a'-a}$$
$$\Rightarrow (y-b)(a'-a)=(x-a)(b'-b)$$
$$\Rightarrow y(a'-a)-a'b+ab=x(b'-b)-ab'+ab$$
$$\Rightarrow y(a'-a)=x(b'-b)-ab'+a'b$$
$$\Rightarrow y(a'-a)-x(b'-b)=a'b-ab'$$

BD passes through B(a',b) and D(a,b').
Thus equation of BD is:

$$\frac{y-y_1}{y_2-y_1}=\frac{x-x_1}{x_2-x_1}$$
$$\Rightarrow \frac{y-b}{b'-b}=\frac{x-a'}{a-a'}$$
$$\Rightarrow (y-b)(a-a')=(x-a')(b'-b)$$
$$\Rightarrow -y(a'-a)-ab+a'b=x(b'-b)-a'b'+a'b$$
$$\Rightarrow a'b'-ab=x(b'-b)+y(a'-a)$$
$$\Rightarrow x(b'-b)+y(a'-a)=a'b'-ab$$

Straight Lines Ex 23.5 Q5

Equation of BC

$$y-y_1=\frac{y_2-y_1}{x_2-x_1}(x-x_1)$$
$$y-1=\frac{0-1}{2-0}(x-0) \quad [\because B(0,1), C(2,0)]$$
$$2y-2=-x$$
$$x+2y=2$$

D is mid point of BC

So,

$$D=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)=\left(\frac{0+2}{2},\frac{1+0}{2}\right)=\left(1,\frac{1}{2}\right)$$

$\therefore$ Equation of the median AD :

$$y-y_1=\frac{y_2-y_1}{x_2-x_1}(x-x_1)$$
$$y-(-2)=\frac{\frac{1}{2}-(-2)}{1-(-1)}(x-(-1))=\frac{\frac{5}{2}}{2}(x+1) \qquad \left[\because A(-1,-2), D\left(1,\frac{1}{2}\right)\right]$$
$$4y+8=5x+5$$
$$5x-4y-3=0$$

Straight Lines Ex 23.5 Q6

The equation of the line passing through points $(-2,-2)$ and $(8,2)$ is

$$y+2=\frac{2+2}{8+2}(x+2)$$

$$2x-5y-6=0$$

Clearly, $(3,0)$ satisfies this equation which means that the line passing through $(-2,-2)$ and $(8,2)$ also passes through $(3,0)$.

Hence three points are collinear.

Straight Lines Ex 23.5 Q7

Let AB be the line segment

Let P be any point which divides the line segment in the ratio 2:3

then using section formula

$$x=\frac{lx_2+mx_1}{l+m},\ y=\frac{ly_2+my_1}{l+m}$$

where $l:m::2:3$

$$\Rightarrow x=\frac{2(8)+3(3)}{2+3}=\frac{16+9}{5}=\frac{25}{5}=5$$

$$y=\frac{2(9)+3(-1)}{2+3}=\frac{18-3}{5}=\frac{15}{5}=3$$

Now P must lie on the line, where P is $(5,3)$

$$y-x+2=0$$

$$\Rightarrow 3-(5)+2=0$$

$$-2+2=0$$

$$0=0$$

Hence, Proved

Straight Lines Ex 23.5 Q8

The line that bisects the distance between the points $A(a,b)$, $B(a'b')$ and between $C(-a,b)$, $D(a'-b')$ means a line passing through the mid-point of AB and CD

mid point of AB is $\left(\frac{a+a'}{2},\frac{b+b'}{2}\right)$

mid point of CD is $\left(\frac{-a+a'}{2},\frac{b-b'}{2}\right)$

Equation is $y-y_1=m(x-x_1)$

$$y-y_1=\frac{y_2-y_1}{x_2-x_1}(x-x_1)$$

$$y-\left(\frac{b+b'}{2}\right)=\frac{\left(\frac{b-b'}{2}\right)-\left(\frac{b+b'}{2}\right)}{\left(\frac{-a+a'}{2}-\frac{a+a'}{2}\right)}\left(x-\left(\frac{a+a'}{2}\right)\right)$$

$$y-\left(\frac{b+b'}{2}\right)=\frac{\frac{b}{2}-\frac{b'}{2}-\frac{b}{2}-\frac{b'}{2}}{\frac{-a}{2}+\frac{a'}{2}-\frac{a}{2}-\frac{a'}{2}}\left(x-\left(\frac{a+a'}{2}\right)\right)$$

$$y-\left(\frac{b+b'}{2}\right)=\frac{+b'}{a}\left(x-\left(\frac{a+a'}{2}\right)\right)$$

$$2ay-2b'x=ab-a'b'$$

Straight Lines Ex 23.5 Q9

In what ratio is the line joining the points $(2,3)$ and $(4,-5)$ divided by the line passing through the points $(6,8)$ and $(-3,-2)$.

Let the equation of line AB joining the points $(6,8)$ and $(-3,-2)$ be

$y - y_1 = m(x - x_1)$

$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

$y - 8 = \frac{-2-8}{-3-6}(x-6)$

$y - 8 = \frac{10}{9}(x-6)$

$9y - 10x = 12 \quad$ ---(1)

Suppose the line joining $(2,3)$ and $(4,-5)$ is divided by the line $9y - 10x = 12$ in the ratio $k:1$ at the point (x, y), then

$x = \frac{k(4)+1(2)}{k+1}, y = \frac{k(-5)+1(3)}{k+1}$

Substituiting in equation (i), we get:

$\frac{9(-5k+3)}{k+1} - 10\left(\frac{4k+2}{k+1}\right) = 12$

$\Rightarrow -45k + 27 - 40k - 20 = 12k + 12$

$\Rightarrow 97k = 5$

$\Rightarrow k = \frac{5}{97}$

Straight Lines Ex 23.5 Q10

The quadrilateral $ABCD$ has diagonals AC and BD.
The required equation is

Since, $A(-2,6), C(10,4)$, the equation for AC is:

$y - 6 = \frac{4-6}{10-(-2)}(x-(-2)) \qquad \left[y - y_1 = \frac{y_2-y_1}{x_2-x_1}(x-x_1)\right]$

$y - 6 = \frac{-12}{6}(x+2)$

$y - 6 = \frac{-(x+2)}{6}$

$6y - 36 = -x - 2$

$x + 6y - 34 = 0$

Since, $B(1,2), D(7,8)$, the equation for BD is:

$y - y_1 = \frac{y_2-y_1}{x_2-x_1}(x-x_1)$

$y - 2 = \frac{8-2}{7-1}(x-1)$

$y - 2 = \frac{6}{6}(x-1)$

$y - 2 = x - 1$

$x - y + 1 = 0$

Straight Lines Ex 23.5 Q11

$L_1 = 124.942,\ C_1 = 20$

$L_2 = 125.134,\ C_2 = 110$

Equation of line passing through

(L_1, C_1) and (L_2, C_2)

$$L - L_1 = \left(\frac{L_2 - L_1}{C_2 - C_1}\right)(C - C_1)$$

$$L - 124.942 = \left(\frac{125.134 - 124.942}{110 - 20}\right)(C - 20)$$

$$L - 124.942 = \frac{0.192}{90}(C - 20)$$

$$L - 124.942 = \frac{192}{90000}(C - 20)$$

$$L - 124.942 = \frac{4}{1875}(C - 20)$$

$$L = \frac{4}{1875}C + 124.942 - 4 \times \frac{20}{1875}$$

$$\Rightarrow L = \frac{4}{1875}C + 124.899$$

Straight Lines Ex 23.5 Q12

Assuming x be the price per litre and y be the quantity of the milk.

sold at this price.

So, the line representing the relationship passes through $(14,980)$ and $(16,1220)$.

So its equation is

$$y - 980 = \frac{1220 - 980}{16 - 14}(x - 14)$$

$$y - 980 = 120(x - 14)$$

$$120x - y - 700 = 0$$

When $x = 17, 120 \times 17 - y - 700 = 0$

$$y = 1340$$

Straight Lines Ex 23.5 Q13

Let AD be the bisector of $\angle A$

Then, $BD : DC = AB : AC$

Now,

$$|AB| = \sqrt{(4-0)^2 + (3-0)^2} = 5$$

$$|AC| = \sqrt{(4-2)^2 + (3-3)^2} = 2$$

$$\Rightarrow \frac{AB}{AC} = \frac{BD}{DC} = \frac{5}{2}$$

$\Rightarrow D$ divides BC in the ratio $5 : 2$

So, coordinates of D are $\left(\frac{5 \times 2 + 0}{5+2}, \frac{5 \times 3 + 0}{5+2}\right) = \left(\frac{10}{7}, \frac{15}{7}\right)$

$\therefore$ The equation of AD is

$$y - 3 = \left(\frac{\frac{15}{7} - 3}{\frac{10}{7} - 4}\right)(x - 4)$$

$$y - 3 = \left(\frac{15 - 21}{10 - 28}\right)(x - 4)$$

$$\Rightarrow y - 3 = \frac{1}{3}(x - 4)$$

$$\Rightarrow 3(y - 3) = x - 4$$

$$\Rightarrow x - 3y + 9 - 4 = 0$$

$$\Rightarrow x - 3y + 5 = 0$$

Straight Lines Ex 23.5 Q14

The required straight line passes through $(0,0)$ and trisects the part of the line $3x+y=12$ that lies between the axes of coordinates.

The line $3x+y=12$ has $A(4,0)$ and $B(0,12)$ as x and y intercepts.

Let P and Q be the points of trisection of AB.

Since P divides AB in the ratio 1:2, coordinates of P are:

$$P=\frac{1(0)+2(4)}{1+2},\frac{1(12)+2(0)}{1+2}=\left(\frac{8}{3},4\right)$$

Since Q divides BA in the ratio 1:2, coordinates of Q are:

$$Q=\frac{2(0)+1(4)}{1+2},\frac{1(0)+2(12)}{1+2}=\left(\frac{4}{3},8\right)$$

Equation of line through $(0,0)$ and $P\left(\frac{8}{3},4\right)$ is:

$$y-0=\frac{4-0}{\frac{8}{3}-0}(x-0)$$

$$y-0=\frac{12}{8}x$$

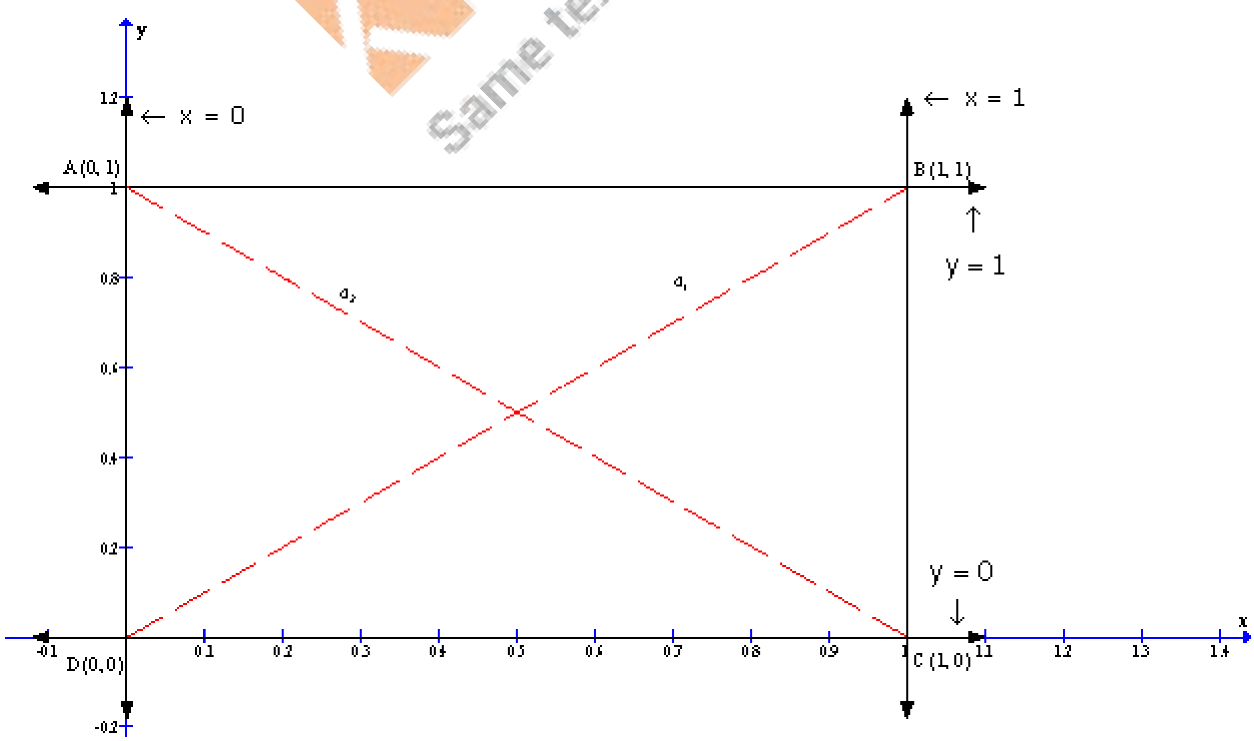
$$2y=3x$$

Equation of line through $(0,0)$ and $Q\left(\frac{4}{3},8\right)$ is:

$$y-0=\frac{8-0}{\frac{4}{3}-0}(x-0)=6x$$

$$y=6x$$

Straight Lines Ex 23.5 Q15



When we draw all the given equations of lines on the graph we get the points of intersection A(0, 1), B(1,1), C(1,0) and D(0,0).

Let d_1 be the diagonal fomed by joining the points B and D.
Let d_2 be the diagonal fomed by joining the points A and C.

Equation of the diagonal d_1 is given by,

$$(y - 1) = \frac{(0-1)}{(0-1)}(x-1)$$

$$(y-1) = 1(x-1)$$

$$y = x$$

Equation of the diagonal d_2 is given by,

$$(y - 1) = \frac{(0-1)}{(1-0)}(x-0)$$

$$(y-1) = -1(x)$$

$$y + x = 1$$

$\therefore$ The equations of the diagonals are $y = x$ and $y + x = 1$.

RD Sharma Solutions Class 11 Maths Chapter 23 Ex 23.6

Straight Lines Ex 23.6 Q1

(i)

If $(a,0)$ and $(0,b)$ are the intercepts of a line then the intercept form of equation is

$$\frac{x}{a}+\frac{y}{b}=1$$

Here, $a=3, b=2$

$\therefore$ The required equation is

$$\frac{x}{3}+\frac{y}{2}=1$$

$\Rightarrow 2x+3y=6$

(ii) If $(a,0)$ and $(0,b)$ are the intercepts of a line then the intercept form of equation is

$$\frac{x}{a}+\frac{y}{b}=1$$

Here, $a=-5, b=6$

$\therefore$ The required equation is

$$\frac{x}{-5}+\frac{y}{6}=1$$

$\Rightarrow 6x-5y=-30$

Straight Lines Ex 23.6 Q2

The equation of straight line in the intercept form is

$$\frac{x}{a}+\frac{y}{b}=1 \quad ---(1)$$

If (1) passes through the point $(1,-2)$ and has equal intercepts $(a=b=k)$, we get,

$$\frac{1}{k}+\frac{(-2)}{k}=1$$

$$\frac{1}{k}-\frac{2}{k}=1$$

$1-2=k$

$k=-1$

$\Rightarrow a=b=-1$

Putting in (1)

$$\frac{x}{-1}+\frac{y}{-1}=1$$

$$x+y=-1$$

Straight Lines Ex 23.6 Q3

(i) Intercepts are equal and positive

$\Rightarrow a=b=k$

The equation of straight line is

$$\frac{x}{a}+\frac{y}{b}=1 \quad ---(1)$$

Since this line passes through (5, 6) and a=b=k, we get:

$$\frac{5}{k}+\frac{6}{k}=1$$

$$k=1$$

3 $\therefore \frac{x}{11}+\frac{y}{11}=1$

$\Rightarrow x+y=11$

(ii) Intercepts are equal but opposite in sign

Let, $a=k, b=-k$

Putting in(1), we get,

$$\frac{5}{k}+\frac{6}{-k}=1$$

$$\frac{5}{k}-\frac{6}{k}=1$$

$\Rightarrow \quad k=-1$

thus from (1)

$x-y=-1$

Straight Lines Ex 23.6 Q4

The equation of the given line is,

$ax + by + 8 = 0$

$\Rightarrow -\frac{x}{\frac{8}{a}} - \frac{y}{\frac{8}{b}} = 1$

It cuts the axes at $A\left(\frac{-8}{a}, 0\right)$ and $B\left(0, \frac{-8}{b}\right)$.

The equation of the given line is,

$2x - 3y + 6 = 0$

$\Rightarrow \frac{-x}{3} + \frac{y}{2} = 1$

It cuts the axes at $C(-3, 0)$ and $D(0, 2)$.

The intercepts of both the lines are opposite in sign

$\Rightarrow \left(\frac{-8}{a}, 0\right) = -(-3, 0) \quad and \quad \left(0, \frac{-8}{b}\right) = -(0, 2)$

$\Rightarrow \frac{-8}{a} = 3 \quad and \quad \frac{-8}{b} = -2$

$\Rightarrow a = \frac{-8}{3} \quad and \quad b = 4$

Straight Lines Ex 23.6 Q5

Let the intercepts on the axes be $(a,0)$ and $(0,a)$.

Then,

$a \times a = 25$

$a^2 = 25$

$a = 5$

(Ignoring negative sign because it is given that the intercepts are positive)

$\Rightarrow a = b = 5$ (given the intercepts are equal)

$\therefore$ Putting in equation of straight line

$$\frac{x}{a} + \frac{y}{b} = 1$$

$$\frac{x}{5} + \frac{y}{5} = 1$$

$$x + y = 5$$

Straight Lines Ex 23.6 Q6

The equation of the given line is,

$$\frac{x}{a} + \frac{y}{b} = 1$$

It cuts the axes at $A(a,0)$ and $B(0,b)$.

The portion of AB intercepted between the axis is 5:3.

$$\therefore h = \frac{3 \times a + 5 \times 0}{8} \text{ and } k = \frac{3 \times 0 + 5 \times b}{8}$$

$$\Rightarrow p = \left(\frac{3a}{8}, \frac{5b}{8}\right)$$

The line is passing through the point $(-4,3)$

$$\Rightarrow \frac{3a}{8}=-4 \qquad \frac{5b}{8}=3$$

$$\Rightarrow a=\frac{-32}{3} \qquad b=\frac{24}{5}$$

$\therefore$ The equation of the given line is,

$$\frac{x}{\frac{-32}{3}}+\frac{y}{\frac{24}{5}}=1$$

$$\frac{-3x}{32}+\frac{5y}{24}=1$$

$$9x-20y+96=0$$

Straight Lines Ex 23.6 Q7

The line intercepted by the axes are $(a,0)$ and $(0,b)$, if this line segment is bisected at point (α,β)

then $\frac{a+0}{2}=\alpha, \frac{0+b}{2}=\beta$ (Using mid point formula)

$a=2\alpha, b=2\beta$

The equation of straight line in the intercept form is

$$\frac{x}{a}+\frac{y}{b}=1$$

$$\frac{x}{2\alpha}+\frac{y}{2\beta}=1$$

Straight Lines Ex 23.6 Q8

Suppose $P=(3,4)$ divides the line joining the points $A(a,0)$ and $B(0,b)$ in the ration 2:3.

Then,

$$3=\frac{2(0)+3(a)}{2+3} \Rightarrow 3=\frac{3a}{5} \Rightarrow a=5$$

$$4=\frac{2(b)+3(0)}{2+3} \Rightarrow 4=\frac{2b}{5} \Rightarrow b=10$$

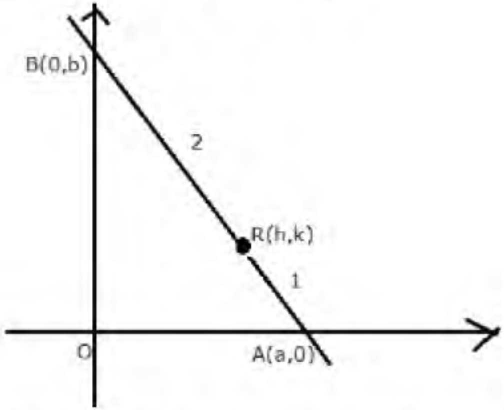
$\therefore$ A is $(5,0)$, B is $(0,10)$

Equation of line AB is

$$\frac{x}{5}+\frac{y}{10}=1$$

$$2x+y=10$$

Straight Lines Ex 23.6 Q9



Point (h,k) divides the line segment in the ratio 1:2

Thus, using section point formula, we have

$$h = \frac{2 \times a + 1 \times 0}{1 + 2}$$

and

$$k = \frac{2 \times 0 + 1 \times b}{1 + 2}$$

Therefore, we have,

$$h = \frac{2a}{3} \text{ and } k = \frac{b}{3}$$

$$\Rightarrow a = \frac{3h}{2} \text{ and } b = 3k$$

Thus, the corresponding points of A and B are $\left(\frac{3h}{2}, 0\right)$ *and* $(0, 3k)$

Thus, the equation of the line joining the points A and B is

$$\frac{y - 3k}{3k - 0} = \frac{x - 0}{0 - \frac{3h}{2}}$$

$$\Rightarrow -\frac{3h}{2}(y - 3k) = x \times 3k$$

$$\Rightarrow -3hy + 9hk = 6kx$$

$$\Rightarrow 2kx + hy = 3kh$$

RD Sharma Solutions Class 11 Maths Chapter 23 Ex 23.7

Straight lines Ex 23.7 Q1(i)

$P = 5, \alpha = 60^\circ$

$x\cos\alpha + y\sin\alpha = P$

$\Rightarrow \quad x\cos 60^\circ + y\sin 60^\circ = 5$

$\Rightarrow \quad x \times \frac{1}{2} + y \times \frac{\sqrt{3}}{2} = 5$

$\Rightarrow \quad x + \sqrt{3}y = 10$

Straight lines Ex 23.7 Q1(ii)

$P = 4, \alpha = 150^\circ$

$x\cos\alpha + y\sin\alpha = P$

$\Rightarrow \quad x\cos 150^\circ + y\sin 150^\circ = 4$

$\Rightarrow \quad -x \times \frac{\sqrt{3}}{2} + y \times \frac{1}{2} = 4$

$\Rightarrow \quad -\sqrt{3}x + y = 8$

Straight lines Ex 23.7 Q1(iii)

$P = 8, \alpha = 225^\circ$

$x\cos\alpha + y\sin\alpha = P$

$$\Rightarrow \quad x\cos 225^\circ + y\sin 225^\circ = 8$$

$$\Rightarrow \quad -x \times \frac{1}{\sqrt{2}} - y \times \frac{1}{\sqrt{2}} = 8$$

$$\Rightarrow \quad x + y + 8\sqrt{2} = 0$$

Straight lines Ex 23.7 Q1(iv)

$P = 8, \alpha = 300^\circ$

$x\cos\alpha + y\sin\alpha = P$

$$\Rightarrow \quad x\cos 300^\circ + y\sin 300^\circ = 8$$

$$\Rightarrow \quad x \times \frac{1}{2} - y \times \frac{\sqrt{3}}{2} = 8$$

$$\Rightarrow \quad x - \sqrt{3}y = 16$$

Straight lines Ex 23.7 Q2

Given, Inclination of perpendicular line (L) passing through origin is 30°

$\Rightarrow$ Slope=Tan $30^\circ = \frac{1}{\sqrt{3}}$

Slope of perpedicular line (M) which is perpendicular to line L is $-\sqrt{3}$

So equation of line M is $y=-\sqrt{3}x+c$

Given perpendicular distance from origin to line M is 4

$$4=\frac{c}{2} \Rightarrow c=8$$

So equation of line M is $y=-\sqrt{3}x+8$

Straight lines Ex 23.7 Q3

Here,

$p = 4$ and $\alpha = 15^\circ$

The equation of line is

$x\cos\alpha + y\sin\alpha = p \quad ---(1)$

$x\cos 15^\circ + y\sin 15^\circ = 4$

$\cos 15^\circ = \cos(45-30) = \cos 45\cos 30 + \sin 45\sin 30$

$\left(\because \cos(\theta-\phi) = \cos\theta\cos\phi + \sin\theta\sin\phi\right)$

$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} \times \frac{1}{2}$

$= \frac{1}{2\sqrt{2}}\left(\sqrt{3}+1\right)$

$\sin 15^\circ = \sin(45-30) = \sin 45\cos 30 - \cos 45\sin 30$

$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \times \frac{1}{2} = \frac{1}{2\sqrt{2}}\left(\sqrt{3}-1\right)$

Putting in (1)

$x \times \frac{1}{2\sqrt{2}}\left(\sqrt{3}+1\right) + y \times \frac{1}{2\sqrt{2}}\left(\sqrt{3}-1\right) = 4$

$x\left(\sqrt{3}+1\right) + y\left(\sqrt{3}-1\right) = 8\sqrt{2}$

Straight lines Ex 23.7 Q4

Here $P = 3$

and $\alpha = \tan^{-1}\left(\frac{5}{12}\right)$

$\Rightarrow \cos\alpha = \frac{12}{13}, \sin\alpha = \frac{5}{13}$

Equation of straight line is:

$x\cos\alpha + y\sin\alpha = P$

$x\left(\frac{12}{13}\right) + y\left(\frac{5}{13}\right) = 3$

$12x + 5y = 39$

Straight lines Ex 23.7 Q5

Here $P = 2, \ \sin\alpha = \frac{1}{3}$

$\Rightarrow \ \cos\alpha = \frac{2\sqrt{2}}{3}$

The equation of straight line is

$x\cos\alpha + y\sin\alpha = P$

$x\left(\frac{2\sqrt{2}}{3}\right) + y\left(\frac{1}{3}\right) = 2$

$2\sqrt{2}x + y = 6$

Straight lines Ex 23.7 Q6

Given:

$P = \pm 2$

$\tan\alpha = \frac{5}{12}$

The equation of line is

$x\cos\alpha + y\sin\alpha = \pm P$

$x\frac{12}{13} + y\frac{5}{13} = \pm 2$

$12x + 5y \pm 26 = 0$

Straight lines Ex 23.7 Q7

Here,

P = perpendicular distance from origin = 7

Angle made with y-axis is 150°,

∴ Angle made with x-axis is 30°

$\cos\alpha = \cos 30^\circ = \frac{\sqrt{3}}{2}$

$\sin\alpha = \sin 30^\circ = \frac{1}{2}$

The equation of line is

$x\cos\alpha + y\sin\alpha = P$

$x\left(\frac{\sqrt{3}}{2}\right) + y\left(\frac{1}{2}\right) = 7$

$\sqrt{3}x + y = 14$

Straight lines Ex 23.7 Q8

We have,

$$\sqrt{3}x + y + 2 = 0$$
$$-\sqrt{3}x - y = 2$$
$$\left(\frac{-\sqrt{3}}{2}\right)x + \left(\frac{-1}{2}\right)y = 1$$

This same as $x\cos\theta + y\sin\theta = p$

Therefore, $\cos\theta = \frac{-\sqrt{3}}{2}, \sin\theta = -\frac{1}{2}$ and $p = 1$

$\theta = 210^\circ$ and $p = 1$

$\theta = \frac{7\pi}{6}$ and $p = 1$

Straight lines Ex 23.7 Q9

Perpendicular from origin makes an angle of 30° with y-axis, thus making 60° woth x-axis

Area of triangle is $= 96\sqrt{3}$

$$\frac{1}{2} \times 2P \times \frac{2p}{\sqrt{3}} = 96\sqrt{3}$$

$$p^2 = \frac{96\sqrt{3} \times \sqrt{3}}{2} = 48 \times 3 = 2 \times 2 \times 2 \times 2 \times 3 \times 3$$

$$p = 12$$

$$x\cos\alpha + y\sin\alpha = p$$

$$x\cos 60^\circ + y\sin 60^\circ = 12$$

$$x \times \frac{1}{2} + y\frac{\sqrt{3}}{2} = 12$$

$$x + \sqrt{3}y = 24$$

RD Sharma Solutions Class 11 Maths Chapter 23 Ex 23.8

Straight lines Ex 23.8 Q1

The equation of line through $(1,2)$ and making an angle of 60^0 with the x-axis is

$$\frac{x-1}{\cos 60^0} = \frac{y-2}{\sin 60^0} = r$$

$$\frac{x-1}{\frac{1}{2}} = \frac{y-2}{\frac{\sqrt{3}}{2}} = r$$

Where r is the distance of any point on the line from $A(1,2)$.

The coordinates of P on the line are

$$\left(1+\frac{1}{2}r, 2+\frac{\sqrt{3}}{2}r\right)$$

and

P lies on $x+y=6$

$$\therefore 1+\frac{r}{2}+2+\frac{\sqrt{3}r}{2} = 6$$

$$\text{or } r = \frac{6}{1+\sqrt{3}} = 3\left(\sqrt{3}-1\right)$$

Hence length $AP = 3\left(\sqrt{3}-1\right)$

Straight lines Ex 23.8 Q2

The equation of line is

$$\frac{x-3}{\cos\frac{\pi}{6}} = \frac{y-4}{\sin\frac{\pi}{6}} = \pm r$$

$$\text{or } x = \pm\frac{\sqrt{3}}{2}r+3 \text{ and } y = \pm\frac{1}{2}r+4$$

$Q\left(\pm\frac{\sqrt{3}r}{2}+3,\ \ \pm\frac{r}{2}+4\right)$ lie in $12x+5y+10=0$

$$\therefore 12\left(\pm\frac{\sqrt{3}r}{2}+3\right)+5\left(\pm\frac{r}{2}+4\right)+10=0$$

$$\pm\frac{12\sqrt{3}r}{2}+36\pm\frac{5r}{2}+20+10=0$$

$$r=\frac{\pm 132}{5+12\sqrt{3}}$$

Hence, length PQ is $\frac{132}{12\sqrt{3}+5}$

Straight lines Ex 23.8 Q3

The equation of line is

$$\frac{x-2}{\cos\alpha}=\frac{y-1}{\sin\alpha}=r$$

$$\Rightarrow \frac{x-2}{\frac{1}{\sqrt{2}}}=\frac{y-1}{\frac{1}{\sqrt{2}}}=r$$

or $x=\frac{1}{\sqrt{2}}r+2,\ y=\frac{1}{\sqrt{2}}r+1$

$B\left(\frac{r}{\sqrt{2}}+2,\ \ \frac{r}{\sqrt{2}}+1\right)$ lie on $x+2y+1=0$

$\therefore \frac{r}{\sqrt{2}} + 2 + \frac{2r}{\sqrt{2}} + 2 + 1 = 0$

$\frac{3r}{\sqrt{2}} = \pm 5$

$r = \frac{5\sqrt{2}}{3}$

The length AB is $\frac{5\sqrt{2}}{3}$ units

Straight lines Ex 23.8 Q4

The required line is parallel to $3x-4y+1=0$

$\therefore$ Slope of the line = slope of $3x - 4y + 1 = \frac{-3}{-4}$

$\tan\alpha = \frac{3}{4}$

$\Rightarrow \sin\alpha = \frac{3}{5}$ and $\cos\alpha \frac{4}{5}$

The equation of line is

$$\frac{x+4}{\cos\alpha} + \frac{y+1}{\sin\alpha} = r$$

$$\Rightarrow \frac{x-4}{\frac{4}{5}} + \frac{y+1}{\frac{3}{5}} = \pm 5$$

$$\Rightarrow x = 8 \text{ and } y = 2$$
$$\text{or}$$
$$x = 0 \text{ and } y = -4$$

$\therefore$ $(8, 2)$ and $(0, -4)$ are coordinates of two points on the line which are at a distance of 5 units from $(4, 1)$

Straight lines Ex 23.8 Q5

The equation of line is

$$\frac{x-x_1}{\cos\theta} = \frac{y-y_1}{\sin\theta} = \pm r$$

or

$x = x_1 \pm r\cos\theta$ and $y = y_1 \pm r\sin\theta$

$Q(x_1 \pm r\cos\theta,\ y_1 \pm r\sin\theta)$ lie in $ax + by + c = 0$

$$\Rightarrow a(x_1 + r\cos\theta) + b(y_1 \pm r\sin\theta) + c = 0$$

$$\Rightarrow \pm r(a\cos\theta + b\sin\theta) = -c - ax_1 - by_1$$

$$\Rightarrow -r = \left|\frac{ax_1 + by_1 + c}{a\cos\theta + b\sin\theta}\right|$$

Straight lines Ex 23.8 Q6

Equation of line is

$$\frac{x-2}{\cos 45^0} = \frac{y-3}{\sin 45^0} = r$$

$$x = \frac{r}{\sqrt{2}} + 2 \quad \text{and} \quad y = \frac{r}{\sqrt{2}} + 3$$

$$P\left(\frac{r}{\sqrt{2}} + 2, \frac{r}{\sqrt{2}} + 3\right) \text{ lie on } 2x - 3y + 9 = 0$$

$$\therefore 2\left(\frac{r + 2\sqrt{2}}{\sqrt{2}}\right) - 3\left(\frac{r + 3\sqrt{2}}{\sqrt{2}}\right) + 9 = 0$$

$$\Rightarrow 2r + 4\sqrt{2} - 3r - 9\sqrt{2} + 9\sqrt{2} = 0$$

$$\Rightarrow r = 4\sqrt{2}$$

$\therefore$ The point $(2,3)$ is at a distance of $4\sqrt{2}$ from $2x - 3y + 9 = 0$

Straight lines Ex 23.8 Q7

Equation of the required line is

$$\frac{x-3}{\cos\alpha} = \frac{y-5}{\sin\alpha} = r \quad ---(1)$$

$$\tan\alpha = \frac{1}{2} \quad \Rightarrow \cos\alpha = \frac{2}{\sqrt{5}} \text{ and } \sin\alpha = \frac{1}{\sqrt{5}}$$

$\therefore$ equation is

$$\frac{x-3}{\frac{2}{\sqrt{5}}} = \frac{y-5}{\frac{1}{5}} = r$$

$$\text{or } x = \frac{2}{\sqrt{5}}r + 3, y = \frac{1}{\sqrt{5}}r + 5$$

$$P\left(\frac{2r}{\sqrt{5}} + 3, \frac{r}{\sqrt{5}} + 5\right) \text{ lie on } 2x + 3y = 14$$

$$\therefore \frac{4r}{\sqrt{5}} + 6 + \frac{3r}{\sqrt{5}} + 15 = 1 \pm 14$$

$$\frac{7r}{\sqrt{5}} = \pm 17$$

$r = \pm\sqrt{5}$

$r = \sqrt{5} \quad \left(r \neq -\sqrt{5}\right)$

$\therefore$ Distance of $(3,5)$ from $2x + 3y = 14$ is $\sqrt{5}$ units

Straight lines Ex 23.8 Q8

Slope of the line = $\tan\alpha = \frac{3}{4}$

$\therefore \sin\alpha = \frac{3}{5}$ and $\cos\alpha = \frac{4}{5}$

$\therefore$ Equation of line is

$\frac{x-2}{\cos\alpha} = \frac{y-5}{\sin\alpha} = r$

$\Rightarrow \frac{x-2}{\frac{4}{5}} = \frac{y-5}{\frac{3}{5}} = r$

or $x = \frac{4r}{5} + 2$ and $y = \frac{3r}{5} + 5$

then $P\left(\frac{4r}{5} + 2, \frac{3r}{5} 5\right)$ lie on $3x + y + 4 = 0$

$\therefore 3\left(\frac{4r}{5} + 2\right) + \left(\frac{3r}{5} + 5\right) + 4 = 0$

$\frac{15}{5} r = \pm 15$

$r = \pm\frac{15 \times 5}{15}$

$= 5$ units

RD Sharma Solutions Class 11 Maths Chapter 23 Ex 23.9

Straight lines Ex 23.9 Q1

(i) Slope intercept form $(y = mx + c)$

$\sqrt{3}x + y + 2 = 0$

$\Rightarrow y = -\sqrt{3}x - 2$

$\Rightarrow m = -\sqrt{3},\ c = -\sqrt{2}$

y-intercept $= -2$, slope $= -\sqrt{3}$

(ii) Intercept form $\left(\frac{x}{a} + \frac{y}{b} = 1\right)$

$\sqrt{3}x + y + 2 = 0$

$\Rightarrow \sqrt{3}x + y = -2$

$\Rightarrow \frac{\sqrt{3}x}{-2} + \frac{y}{-2} = 1$

$\Rightarrow \frac{x}{\frac{-2}{\sqrt{3}}} + \frac{y}{-2} = 1$

$\Rightarrow x$ intercept $= \frac{-2}{\sqrt{3}}$, y intercept $= -2$

(iii) Normal form $(x\cos\alpha + y\sin\alpha = p)$

$\sqrt{3}x + y + 2 = 0$

$\Rightarrow -\sqrt{3}x - y = 2$

$\Rightarrow \left(\frac{-\sqrt{3}}{2}\right)x + \left(\frac{-1}{2}\right)y = 1$

$\Rightarrow \cos\alpha = \frac{-\sqrt{3}}{2} = \cos 210^0$ and $\sin\alpha = \frac{-1}{2} = \sin 210^0$

$\Rightarrow p = 1,\ \alpha = 210^0$

Straight lines Ex 23.9 Q2(i)

$x+\sqrt{3}y-4=0$

Divide the equation by 2, we get

$\frac{1}{2}x+\frac{\sqrt{3}}{2}y=2$

$x\cos 60+y\sin 60=2$

So, $p=2$ and $\omega=60$

Straight lines Ex 23.9 Q2(ii)

$x+y+\sqrt{2}=0$

$x+y=-\sqrt{2}$

Dividing each term by $\sqrt{(1)^2+(1)^2}=\sqrt{2}$

$\frac{x}{\sqrt{2}}+\frac{y}{\sqrt{2}}=-1$

$\frac{-x}{\sqrt{2}}-\frac{y}{\sqrt{2}}=1$

Comparing with $x\cos\alpha+y\sin\alpha=p$

$\cos\alpha=\frac{-1}{\sqrt{2}},\quad \sin\alpha=\frac{-1}{\sqrt{2}},\quad p=1$

Both are negative

α is in III quadrant

$\Rightarrow \alpha=\pi\frac{\pi}{4}=\frac{5\pi}{4}=225^0$

Straight lines Ex 23.9 Q2(iii)

$x - y + 2\sqrt{2} = 0$

$-x + y = 2\sqrt{2}$

Dividing each term by $\sqrt{(1)^2 + (1)^2} = \sqrt{2}$

$\frac{-x}{\sqrt{2}} + \frac{y}{\sqrt{2}} = 2$

Comparing with $x\cos\alpha + y\sin\alpha = p$

$\cos\alpha = \frac{-1}{\sqrt{2}}, \quad \sin\alpha = \frac{-1}{\sqrt{2}}, \quad p = 2$

α is in II quadrant

$\Rightarrow \alpha = \frac{\pi}{4} + \frac{\pi}{2} = \frac{3\pi}{4} = 135^0, \quad p = 2$

Straight lines Ex 23.9 Q2(iv)

$x - 3 = 0$

$x = 3$

Comparing with $x\cos\alpha + y\sin\alpha = p$

$\cos\alpha = 1$

$= \cos 0$

$\Rightarrow \alpha = 0$

$p = 3$

Straight lines Ex 23.9 Q2(v)

$y - 2 = 0$

$y = 2$

Comparing with $x\cos\alpha + y\sin\alpha = p$

$\sin\alpha = 1$

$\alpha = \frac{\pi}{2}, \quad p = 2$

Straight lines Ex 23.9 Q3

$\frac{x}{a} + \frac{y}{b} = 1$

The slope intercept form is

$y = mx + c$

$\frac{x}{a} + \frac{y}{b} = 1$

$bx + ay = ab$

$ay = -bx + ab$

$y = \frac{-bx}{a} + b$

Thus y-intercept is b.

Slope $= \frac{-b}{a}$

Straight lines Ex 23.9 Q4

The normal form is obtaned by dividing each term of the equation by $\sqrt{a^2+b^2}$,

a = coefficient of x

b = coefficient of y

$3x - 4y + 4 = 0 \quad ---(1)$

$3x - 4y = -4$

$-3x + 4y = 4$

Dividing each term by $\sqrt{(-3)^2+(4)^2} = \sqrt{9+16} = \sqrt{25} = 5$

$\frac{-3}{5}x + \frac{y}{5}y = \frac{4}{5}$

$\Rightarrow p = \frac{4}{5}$ for equation (1)

Also

$2x + 4y - 5 = 0 \quad ---(2)$

$2x + 4y = 5$

Dividing each term by $\sqrt{(2)^2+(4)^2} = \sqrt{4+16} = \sqrt{20}$

$\frac{2x}{\sqrt{20}} + \frac{4y}{\sqrt{20}} = \frac{5}{\sqrt{20}}$

$p = \frac{5}{\sqrt{20}} = \frac{5}{4.4}$ for equation (2)

Comparing P of (1) and (2)

We conclude that $3x - 4y + 4 = 0$ is nearest to origin

Straight lines Ex 23.9 Q5

Reduce $4x + 3y + 10 = 0$ to perpendicular form

$$4x + 3y = -10$$

$$-4x - 3y = 10$$

Dividing each term by $\sqrt{(-4)^2 + (-3)^2} = \sqrt{16 + 9} = \sqrt{25} = 5$

$$\frac{-4}{5}x - \frac{3}{5}y = \frac{10}{5} = 2$$

$$\Rightarrow p_1 = 2 \qquad ---(1)$$

$$5x - 12y + 26 = 0$$

$$5x - 12y = -26$$

Dividing each term by $\sqrt{(-5)^2 + (12)^2} = \sqrt{25 + 144} = \sqrt{169} = 13$

$$\frac{-5}{13}x + \frac{12}{13}y = \frac{26}{13} = 2$$

$$\Rightarrow p_2 = 2 \qquad ---(2)$$

$$7x + 24y = 50$$

Dividing each term by $\sqrt{(7)^2 + (24)^2} = \sqrt{49 + 576} = \sqrt{625} = 25$

$$\frac{7x}{25} + \frac{24}{25}y = \frac{50}{25} = 2$$

$$\Rightarrow p_3 = 2 \qquad ---(3)$$

Hence, origin is equidistant from all three lines.

Straight lines Ex 23.9 Q6

$\sqrt{3}x + y + 2 = 0$

$\sqrt{3}x + y = 2$

$-\sqrt{3}x - y = 2 ------ (1)$

So,

$\cos\theta = -\sqrt{3}, \sin\theta = -1$

$\tan\theta = \frac{1}{\sqrt{3}}$

$\theta = \left(\pi + \frac{\pi}{6}\right)$

$= 180° + 30°$

$\theta = 210°$

$p = 2 \qquad$ [From equation (1)]

Straight lines Ex 23.9 Q7

The intercept form of equation is

$\frac{x}{a} + \frac{y}{b} = 1$

$3x - 2y + 6 = 0$

$3x - 2y = -6$

$\frac{-3x}{-6} - \frac{2y}{-6} = 1$

$\frac{x}{\frac{-6}{3}} + \frac{y}{\frac{-6}{-2}} = 1$

$\frac{x}{-2} + \frac{y}{3} = 1$

$\Rightarrow x\text{-intercept} = a = -2$

$y\text{-intercept} = b = 3$

Straight lines Ex 23.9 Q8

Perpendicular distance from the origin to the line is 5, so

$$x \cos\alpha + y \sin\alpha = 5$$

$$y \sin\alpha = -x \cos\alpha + 5$$

$$y = -\frac{\cos\alpha}{\sin\alpha} x + 5$$

$$y = -\cot\alpha x + 5$$

Comparing with $y = mx + c$

$$m = -\cot\alpha$$

$$-1 = -\cot\alpha$$

$$\cot\alpha = 1$$

$$\alpha = \frac{\pi}{4}$$

So, equation of line is

$$x \cos\frac{\pi}{4} + y \sin\frac{\pi}{4} = 5$$

$$\frac{x}{\sqrt{2}} + \frac{y}{\sqrt{2}} = 5$$

$$x + y = 5\sqrt{2}$$

RD Sharma Solutions Class 11 Maths Chapter 23 Ex 23.11

Straight lines Ex 23.11 Q1(i)

If the lines are concurrent then point of intersection of any two lines satisfies the third line

$15x - 18y + 1 = 0 \quad ---(1)$

$12x + 10y - 3 = 0 \quad ---(2)$

$6x + 66y - 11 = 0 \quad ---(3)$

Solving (1) and (2)

$$x = \frac{18y - 1}{15}$$

$$12\left(\frac{18y - 1}{15}\right) + 10y - 3 = 0$$

$$216y - 12 + 150y - 45 = 0$$

$$366y = 57$$

$$y = \frac{57}{366} = \frac{19}{122}$$

$$\Rightarrow x = \frac{18y - 1}{15}$$

$$= \frac{18 \times \frac{19}{122} - 1}{15}$$

$$= \frac{18 \times 19 - 122}{122 \times 15}$$

$$= \frac{342 - 122}{1730}$$

$$= \frac{220}{1730}$$

$$= \frac{22}{173}$$

Putting x and y in (3)

$$6\left(\frac{22}{173}\right) + 66\left(\frac{19}{122}\right) - 11 = 0$$

$$6 \times 22 \times 122 + 66 \times 19 \times 173 - 11 \times 173 \times 122 = 0$$

$$0 = 0$$

Straight lines Ex 23.11 Q1(ii)

$3x - 5y - 11 = 0,\ 5x + 3y - 7 = 0,\ x + 2y = 0$

$$3x - 5y - 11 \quad ---(1)$$

$$5x + 3y - 7 = 0 \quad ---(2)$$

$$x + 2y = 0 \quad ---(3)$$

Solving (1) and (2)

$$x = -2y$$

$$5(-2y) + 3y - 7 = 0$$

$$-10y + 3y - 7 = 0$$

$$-7y = y$$

$$y = -1$$

$$\Rightarrow x = 2$$

substituting x and y in (1)

$$3(2) - 5(-1) - 11 = 0$$

$$6 + 5 - 11 = 0$$

$$0 = 0$$

Hence, the lines are concurrent

Straight lines Ex 23.11 Q1(iii)

$\frac{x}{a} + \frac{y}{b} = 1,\ \frac{x}{b} + \frac{y}{a} = 1,\ y = x$

$$bx + ay = ab,\quad ax + by = ab$$

Put $y = x$

$$bx + ax = ab,\quad ax + bx = ab$$

Hence the lines are concurrent

Straight lines Ex 23.11 Q2

The three lines are concurrent if they have the common point of intersection.

$$2x-5y+3=0 \quad \text{---(1)}$$

$$x-2y+1=0 \quad \text{---(2)}$$

Solving (1) and (2)

$$2x = 5y - 3$$

$$x = \frac{5y-3}{2}$$

$$\frac{5y-3}{2} - 2y + 1 = 0$$

$$5y - 3 - 4y + 2 = 0$$

$$y = 0$$

$$\Rightarrow \quad x = \frac{5y-3}{2} = \frac{5-3}{2} = \frac{2}{2} = 1$$

Substituting x and y is $5x - 9y + \lambda = 0$

$$5(1) - 9(1) + \lambda = 0$$

$$5 - 9 + \lambda = 0$$

$$\lambda = 4$$

Straight lines Ex 23.11 Q3

The three lines are

$$y = m_1x + c_1 \quad \text{---(1)}$$

$$y = m_2x + c_2 \quad \text{---(2)}$$

$$y = m_3x + c_3 \quad \text{---(3)}$$

Collinear or they meet at a point only when they have common point of intersection

Solving (1) and (2) for x and y

$$m_1x + c_1 = m_2x + c_2$$

$$x(m_1 - m_2) = c_2 - c_1$$

$$x = \frac{c_2 - c_1}{m_1 - m_2}$$

$$\Rightarrow \quad y = m_1x + c_1$$

$$= m_1\left(\frac{c_2 - c_1}{m_1 - m_2}\right) + c_1$$

$$= m_1c_2 - m_1c_1 + m_1c_1 - m_2c_1$$

Putting x and y in (3)

$$m_1c_2 - m_1c_1 = m_3\frac{(c_2 - c_1)}{m_1 - m_2} + c_3$$

$$m_1^2c_2 - m_1m_2c_2 - m_1m_2c_1 + m_2^2c_1 = m_3c_2 - m_3c_1 + m_1c_3 - m_2c_3$$

$$\Rightarrow \quad m_1(c_2 - c_3) + m_2(c_3 - c_1) + m_3(c_1 - c_2) = 0$$

Straight lines Ex 23.11 Q4

If the lines are concurrent, then the lines have common point of intersection.

The given line are

$$p_1x + q_1y = 1 \quad ---(1)$$

$$p_2x + q_2y = 1 \quad ---(2)$$

$$p_3x + q_3y = 1 \quad ---(3)$$

Solving (1) and (2)

$$x = \frac{1 - q_1y}{p_1}$$

$$p_2\left(\frac{1 - q_1y}{p_1}\right) + q_2y = 1$$

$$p_2 = p_2q_1y + p_1q_2y = p_1$$

$$y = \frac{p_1 - p_2}{p_1q_2 - p_2q_1} \Rightarrow x = \frac{1 - q_1\left(\frac{p_1 - p_2}{p_1q_2 - p_2q_1}\right)}{p_1}$$

Putting x, y in (3)

$$p_3[(p_1q_2 - p_2q_1) - q_1p_1 - q_1p_2][p_1q_2 - p_2q_1] + q_3p_1(p_1 - p_2) = 1$$

$$(p_1p_3q_2 - p_2p_3q_1 - p_1p_3q_1 + p_2p_3q_1)(p_1q_2 - p_2q_1) + q_3p_1^2 - q_3p_1p_2 = 1$$

$$(p_1p_3q_2 - p_1p_3q_1)(p_1q_2 - p_2q_1) + q_3p_1^2 - q_3p_1p_2 = 1$$

$$p_1^2p_3q_2^2 - p_1p_2p_3q_1q_2 - p_1^2p_3q_1q_2 + p_1p_2p_3q_1^2 + q_3p_1^2 - q_3p_1p_2 = 1 \quad ---(1)$$

Also if $(p_1q_1)(p_2q_2)(p_3q_3)$ are collinear

Then,

$$p_1(q_2 - q_3) + p_2(q_3 - q_1) + p_3(q_1 - q_3) = 0$$

From (1)

$$p_1\left[p_1p_3q_2^2 - p_2p_3q_1q_2 - p_1p_3q_1q_2 + p_2p_3q_1^2 + q_3p_1 - q_3p_2\right] = 1$$

$$p_1\left[p_3q_2(p_1q_2 - p_2q_1) - p_3q_1(p_1q_2 - p_2q_1) + q_3(p_1 - p_2)\right] = 1$$

Hence, the points are collinear

Straight lines Ex 23.11 Q5

The three lines are concurrent if they have the common point of intersection

$$(b+c)x + ay + 1 = 0$$

$$(c+a)x + by + 1 = 0$$

$$(a+b)x + cy + 1 = 0$$

Solving (1) and (2)

$$y = \frac{-1-(b+c)x}{a}$$

Putting in (2)

$$(c+a)x + b\frac{(-1-(b+c)x)}{a} + 1 = 0$$

$$acx + a^2x + b - b^2x - bcx + a = 0$$

$$x(ac + a^2 - b^2 - bc) = b - a$$

$$x(ac - bc + a^2 - b^2) = b - a$$

$$x(c(a-b) + (a-b)(a+b)) = b - a$$

$$x(c+a+b) = -1 \qquad [\text{Cancelling } (a-b) \text{ both sides}]$$

$$x = \frac{-1}{a+b+c}$$

$$y = \frac{-1 + \frac{(b+c)(-1)}{a+b+c}}{a} = \frac{-a-b-c-b-c}{a(a+b+c)}$$

Putting the value of x, y in (3);

$$(a+b)\left(\frac{-1}{a+b+c}\right) + c\left(\frac{-a-2b-2c}{a(a+b+c)}\right) + 1 = 0$$

$$-a^2 - ba - ac - 2bc - 2c^2 + a^2 + ab + ac = 0$$

$$0 = 0$$

Hence, the lines are concurrent

Straight lines Ex 23.11 Q6

If the three lines are concurrent then the point of intersection of (1) and (2) should verify the (3) line, where

$$ax + a^2y + 1 = 0 \qquad ---(1)$$

$$bx + b^2y + 1 = 0 \qquad ---(2)$$

$$cx + c^2y + 1 = 0 \qquad ---(3)$$

Solving (1) and (2)

$$x = \frac{-1 - a^2y}{a} \Rightarrow b\left(\frac{-1 - a^2y}{a}\right) + b^2y + 1 = 0$$

$$-b - a^2by + ab^2y + a = 0$$

$$y = \frac{b - a}{ab(b - a)} = \frac{1}{ab}$$

$$\Rightarrow \qquad x = \frac{1 - a^2 \times \frac{1}{ab}}{a} = \frac{1 - \frac{a}{b}}{a} = \frac{b - a}{ab}$$

Putting in (3)

$$c\left(\frac{b - a}{ab}\right) + c^2\left(\frac{1}{ab}\right) + 1 = 0$$

$$bc - ac + c^2 + ab = 0$$

$$bc + c^2 - ac + ab = 0$$

$$c(b + c) - a(c - b) = 0$$

$\Rightarrow$ Either $c = b \Rightarrow 2bc = 0 \Rightarrow 2c^2 = 0 \Rightarrow c = 0$

Straight lines Ex 23.11 Q7

If a, b, c are in $A.P.$

$b - a = c - b$

$2b = a + c$ [Common difference]

To prove that the straight lines are concurrent then they have the common point of intersection.

$$ax + 2y + 1 = 0 \quad ---(1)$$
$$bx + 3y + 1 = 0 \quad ---(2)$$
$$cx + 4y + 1 = 0 \quad ---(3)$$

Solving (1) and (2)

$$x = \frac{-1-2y}{a}$$

Put in (2)

$$b\left(\frac{-1-2y}{a}\right) + 3y + 1 = 0$$

$$y = \frac{b-a}{3a-2b} \Rightarrow x = \frac{-1-\frac{2(b-a)}{3a-2b}}{a} = \frac{-3a+2b-2b+2a}{a(3a-2b)}$$

$$x = \frac{-1}{3a-2b}$$

Putting x, y in (3)

$$c\left(\frac{-1}{3a-2b}\right) + 4\left(\frac{b-a}{3a-2b}\right) + 1 = 0$$
$$-c + 4b - 4a + 3a - 2b = 0$$
$$-a + 2b - c = 0$$
$$-a + a + c - c = 0$$
$$0 = 0$$

Hence, Proved

RD Sharma Solutions Class 11 Maths Chapter 23 Ex 23.12

Straight lines Ex 23.12 Q1

Equation of line through $(2,3)$ is

$$y - y_1 = m(x - x_1) \qquad \text{---}(1)$$

$(2,3)$ is $(x_1 y_1)$

Since the line is parallel to $3x - 4y + 5 = 0$

$\Rightarrow$ The slope will be equal

Slope of $3x - 4y + 5 = 0$

$$4y = 3x + 5$$

$$y = \frac{3}{4}x + \frac{5}{4}$$

$$\Rightarrow \quad m = \frac{3}{4}$$

Substituting m and $(x_1 y_1)$ is (1)

$$y - 3 = \frac{3}{4}(x - 2)$$

$$4y - 12 = 3x - 6$$

$$3x - 4y = -12 + 6 = -6$$

$$3x - 4y + 6 = 0$$

Straight lines Ex 23.12 Q2

Any equation passing through $(3,-2)$ and perpendicular to givven line is

$$y - y_1 = -\frac{1}{m}(x - x_1) \qquad \text{---}(1)$$

Where $(x_1 - y_1)$ is $(3,-2)$ and m is slope of line.

$\frac{-1}{m}$ is taken as lines are perpendicular

Finding slope of line $x - 3y + 5 = 0$

$$3y = x + 5$$

$$y = \frac{x}{3} + \frac{5}{3}$$

$$\Rightarrow \quad m = \frac{1}{3}$$

Substituting the value of m and $(x_1 - y_1)$ in (1)

$$y-(-2) = -\frac{1}{\frac{1}{3}}(x-3)$$

$$y+2 = -3(x-3) = -3x+9$$

$$3x + y = 7$$

Straight lines Ex 23.12 Q3

Any line which is perpendicular bisector means line is perpendicular to the given line and one end point is the mid-point of that line.

The line joining $\underset{(x_1 y_1)}{(1,3)}$ and $\underset{(x_2 y_2)}{(3,1)}$.

Has the mid-point

$$x = \frac{x_1 + x_2}{2},\ y = \frac{y_1 + y_2}{2}$$

$$\Rightarrow \quad (x_1 y_1) = \left(\frac{1+3}{2}, \frac{3+1}{2}\right) = (2,2)$$

Also slope of line is

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1-3}{3-1} = \frac{-2}{2} = -1$$

So, the slope of required line is 1 (negative reciprocal of slope)

Thus, the equation of perpendicular bisector is

$$y - y_1 = \frac{-1}{m}(x - x_1)$$

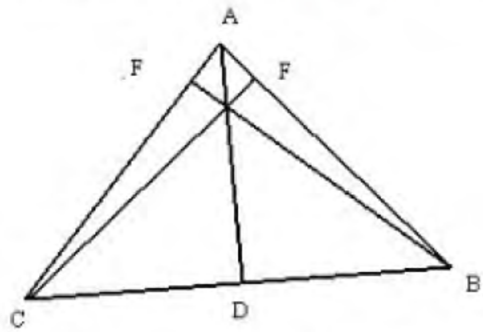
$$y - 2 = 1(x-2)$$

$$y - 2 = x - 2$$

$$y = x$$

Straight lines Ex 23.12 Q4

Let the perpendiculars of the triangle on the side AB, BC and AC be CF, AD and FB respectively.



Slope of the side AB = $\frac{4-2}{1+3} = \frac{2}{4} = \frac{1}{2}$

Corresponding slope of CF = $-\frac{1}{1/2} = -2$

[since $m_1 \times m_2 = -1$]

Equation of CF, $y - y_1 = m(x - x_1)$

$y + 3 = -2(x + 5)$ [Putting co-ordinates of C in place of x_1 and y_1]

$y + 3 = -2x - 10$

$y = -2x - 13$

Slope of the side BC = $\frac{2+3}{-3+5} = \frac{5}{2}$

Corresponding slope of AD = $-\frac{1}{5/2} = -\frac{2}{5}$

Equation of AD,

$y - y_1 = m(x - x_1)$

$y - 4 = -\frac{2}{5}(x - 1)$

$5y - 20 = -2x + 2$

$5y = -2x - 22$

Slope of the side AC = $\frac{4+3}{1+5}=\frac{7}{6}$

Corresponding slope of FB = $-\frac{1}{7/6}=-\frac{6}{7}$

Equation of FB,

$$y-y_1 = m(x-x_1)$$

$$y-2 = \frac{-6}{7}(x+3)$$

$$7y-14 = -6x-18$$

$$7y = -6x-4$$

Equation of AD, $2x+5y+22=0$
Equation of CF, $2x+y+13=0$
Equation of FB, $6x+7y+4=0$

Straight lines Ex 23.12 Q5

Required equation of line is

$$y-y_1 = m'(x-x_1) \quad ---(1)$$

Point is $(x_1 y_1) = (0,-4)$

It is perpendicular to line $\sqrt{3}x - y + 5 = 0$

$\Rightarrow$ Slope is $y = mx + c$

$$y = \sqrt{3}x + 5$$

$$m = \sqrt{3}$$

$$m' = \frac{-1}{m} = \frac{-1}{\sqrt{3}}$$

Putting m' and $(x_1 y_1)$ in (1)

$$y-(-4) = \frac{-1}{\sqrt{3}}(x-0)$$

$$y+4 = \frac{-x}{\sqrt{3}}$$

$$x + \sqrt{3}y + 4\sqrt{3} = 0$$

Straight lines Ex 23.12 Q6

Here,
Let l be line mirror and B is image of A
Let m be slope of line l
So,

$$m(\text{slope of } AB) = -1$$
$$m\left(\frac{2-1}{5-2}\right) = -1$$
$$m\left(\frac{1}{3}\right) = -1$$
$$m = -3$$

M is mid point of AB

$$M = \left(\frac{2+5}{2}, \frac{2+1}{2}\right)$$
$$M = \left(\frac{7}{2}, \frac{3}{2}\right)$$

Equation line l is,

$$y - y_1 = m(x - x_1)$$
$$y - \frac{3}{2} = (-3)\left(x - \frac{7}{2}\right)$$
$$\frac{2y-3}{2} = -3x + \frac{21}{2}$$
$$2y - 3 = -6x + 21$$
$$6x + 2y = 24$$

$$3x + y = 12$$

Straight lines Ex 23.12 Q7

Any line is given by equation

$$y-y_1 = m(x-x_1) \qquad ---(1)$$

Where $(x_1 y_1)$ is (α, β)

And m is negative reciprocal of slope of line $lm + my + n = 0$.

i.e; $y = \frac{-lx}{m} - \frac{n}{m}$

$\Rightarrow$ Slope of line $= \frac{-l}{m}$

Putting the data in (i), we get

$$y - \beta = \frac{m}{l}(x-\alpha)$$

$$ly + mx = m\alpha + l\beta$$

$$m(x-\alpha) = l(y-\beta)$$

Straight lines Ex 23.12 Q8

Let the equation of the required line be $y-y_1=m(x-x_1)$, where 'm' denotes the slope of the line and (x_1,y_1) be the point through which the line passes.
Since the x-intercept of the line is 1 on the positive direction of the x-axis therefore the line passes through (1,0)
Also, $2x-3y=5$

$$3y=2x-5$$

$$y=\frac{2x}{3}-\frac{5}{3}$$

Therefore, the slope of the given line is 2/3.

Slope of the required line $= \frac{-1}{2/3} = -\frac{3}{2}$

Therefore the equation of the required line is

$y-y_1=m(x-x_1)$

$$y-0=\frac{2}{3}(x-1)$$

$$y=-\frac{3}{2}(x-1)$$
$$2y=-3x+3$$
The equation of the required line is 3x+2y -3=0

Straight lines Ex 23.12 Q9

Slope of line through the points $\underset{(x_1, y_1)}{(a, 2a)}$, $\underset{(x_2, y_2)}{(-2, 3)}$

$$\Rightarrow \quad m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-2a}{-2-a}$$

Also, slope of line $x - ay = 1$ in the form $y = mx + c$

$$4x + 3y + 5 = 0$$
$$y = \frac{-4}{3}x - \frac{5}{3}$$
$$\Rightarrow \quad m_2 = \frac{-4}{3}$$

If two lines are perpendicular then, $m_1 m_2 = -1$

$$\left(\frac{3-2a}{-2-a}\right)\left(\frac{-4}{3}\right) = -1$$
$$-12 + 8a = 6 + 3a$$
$$5a = 18$$
$$a = \frac{18}{5}$$

RD Sharma Solutions Class 11 Maths Chapter 23 Ex 23.13

Straight lines Ex 23.13 Q1(i)

Writing the equation in the form

$$y = mx + c$$

$$3x + y + 12 = 0$$

$$y = -3x - 12$$

$\Rightarrow \quad m_1 = -3$

Also

$$x + 2y - 1 = 0$$

$$2y = 1 - x$$

$$y = \frac{1}{2} - \frac{x}{2}$$

$\Rightarrow \quad m_2 = \frac{-1}{2}$

Angle between the lines

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

$$= \left|\frac{-3 - \left(\frac{-1}{2}\right)}{1 + (-3)\left(\frac{-1}{2}\right)}\right|$$

$$= \left|\frac{-3 + \frac{1}{2}}{1 + \frac{3}{2}}\right| = \left|\frac{\frac{-6 + 12}{2}}{\frac{2 + 3}{2}}\right|$$

$$= \left|\frac{-5}{5}\right| = 1$$

$\Rightarrow \quad \text{angle} = \frac{\pi}{4}$

Straight lines Ex 23.13 Q(ii)

Finding slopes of the lines by converting the equation in the form

$$y = mx + c$$

$$3x - y + 5 = 0$$

$$\Rightarrow \quad y = 3x + 5$$

$$\Rightarrow \quad m_1 = 3$$

Also

$$x - 3y + 1 = 0$$

$$3y = x + 1$$

$$y = \frac{x}{3} + \frac{1}{3}$$

$$\Rightarrow \quad m_2 = \frac{1}{3}$$

Thus angle between the lines is

$$\tan\theta = \left|\frac{m_1 - m_2}{m_1 m_2}\right|$$

$$= \left|\frac{3 - \frac{1}{3}}{1 + 3 \times \frac{1}{3}}\right| = \left|\frac{\frac{9 - 1}{3}}{1 + 1}\right|$$

$$= \left|\frac{\frac{8}{3}}{2}\right| = \left|\frac{8}{6}\right| = \frac{4}{3}$$

$$\Rightarrow \quad \theta = \tan^{-1}\left(\frac{4}{3}\right)$$

Straight lines Ex 23.13 Q(iii)

To find angle between the lines, convert the equations in the form

$$y = mx + c$$

$$3x + 4y - 7 = 0$$

$$\Rightarrow \quad 4y = -3x + 7$$

$$y = \frac{-3}{4}x + \frac{7}{4}$$

$$\Rightarrow \quad m_1 = \frac{-3}{4}$$

Also, $4x - 3y + 5 = 0$

$$\Rightarrow \quad 3y = 4x + 5$$

$$\Rightarrow \quad y = \frac{4}{3}x + \frac{5}{3}$$

$$\Rightarrow \quad m_1 = \frac{4}{3}$$

The angle between the lines is given by $\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

$$= \left|\frac{\frac{-3}{4} - \frac{4}{3}}{1 + \frac{(-3)}{4}\left(\frac{4}{3}\right)}\right| = \left|\frac{\frac{-3}{4} - \frac{4}{3}}{1 - 1}\right|$$

$$\Rightarrow \quad \theta = \frac{\pi}{2} \text{ or } 90^0$$

Straight lines Ex 23.13 Q(iv)

To find angle convert the equation in the form $y = mx + c$

$$x - 4y = 3$$

$$\Rightarrow \quad 4y = x - 3$$

$$y = \frac{x}{4} - \frac{3}{4}$$

$$\Rightarrow \quad m_1 = \frac{1}{4}$$

Also, $6x - y = 11$

$$y = 6x - 11$$

$\Rightarrow \quad m_2 = 6$

Thus, angle between the lines is

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

$$= \left|\frac{\frac{1}{4} - 6}{1 + \frac{1}{4} \times 6}\right|$$

$$= \left|\frac{\frac{-23}{4}}{1 + \frac{3}{2}}\right| = \left|\frac{\frac{-23}{4}}{\frac{5}{2}}\right|$$

$$\theta = \tan^{-1}\left(\frac{23}{10}\right)$$

Straight lines Ex 23.13 Q(v)

Converting the equation in the form

$$y = mx + c$$

$$y = \frac{(mn + n^2)}{m^2 - mn}x + \frac{n^3}{(m^2 - mn)}$$

$\Rightarrow \quad m_1 = \dfrac{mn + n^2}{m^2 - mn}$

Also, $\quad y = \dfrac{(mn - n^2)}{nm + m^2}x + \dfrac{m^3}{nm + m^2}$

$\Rightarrow \quad m_2 = \dfrac{mn - n^2}{nm + m^2}$

Thus, angle between 2 lines is $\tan\theta$

$$\Rightarrow \quad \tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

$$= \left|\frac{\left(\frac{mn + n^2}{m^2 - mn}\right) - \left(\frac{mn - n^2}{nm + m^2}\right)}{1 + \left(\frac{mn + n^2}{m^2 - mn}\right)\left(\frac{mn - n^2}{nm + m^2}\right)}\right|$$

$$= \left|\frac{m^2n^2 + m^3n + n^3m + n^2m^2 - m^3n + m^2n^2 + n^2m^2 - mn^3}{m^3n + m^4 - m^2n^2 - m^3n + m^2n^2 - mn^3 + mn^3 - n^4}\right|$$

$$= \left|\frac{4m^2n^2}{m^4 - n^4}\right|$$

$$\Rightarrow \quad \theta = \tan^{-1}\left|\frac{4m^2n^2}{m^4 - n^4}\right|$$

Straight lines Ex 23.13 Q2

Slope of line $2x - y + 3 = 0$

is $\frac{-2}{-1} = \frac{(\text{coefficient of } x)}{(\text{coefficient of } y)} = 2$

$\therefore \quad m_1 = 2 \qquad$ ---(i)

Slope of line $x + y + 2 = 0$

is $\frac{-1}{1} = \frac{-(\text{coefficient of } x)}{(\text{coefficient of } y)}$

$\therefore \quad m_2 = -1 \qquad$ ---(ii)

Acute angle between lines

$$\theta = \tan^{-1}\left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

$$= \tan^{-1}\left|\frac{2 - (-1)}{1 - (2)(-1)}\right|$$

$$= \tan^{-1}\left|\frac{3}{1 - 2}\right| = \tan^{-1}\left|\frac{3}{1}\right| = \tan^{-1}|3|$$

Straight lines Ex 23.13 Q3

Let $ABCD$ be a quadrilateral

$$AB = \sqrt{(0-2)^2 + (2+1)^2}$$

Using distance formula

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{2^2 + 3^2} = \sqrt{4+9} = \sqrt{13}$$

$$BC = \sqrt{(2-0)^2 + (3-2)^2} = \sqrt{4+1} = \sqrt{5}$$

$$CD = \sqrt{(4-2)^2 + (0-3)^2} = \sqrt{4+9} = \sqrt{13}$$

$$DA = \sqrt{(4-2)^2 + (0+1)^2} = \sqrt{4+1} = \sqrt{5}$$

Since opposite sides $(AB$ and $CD)$ and $(BC$ and $DA)$ are equal

$\therefore$ The given quadrilateral is a parallelogram.

Straight lines Ex 23.13 Q4

The equation between the points

$$\underset{(x_1, y_1)}{(2, 0)} \text{ and } \underset{(x_2, y_2)}{(0, 3)}$$

Slope of line $= \dfrac{y_2 - y_1}{x_2 - x_1}$

$$m_1 = \frac{3-0}{0-2} = \frac{-3}{2}$$

Also, slope of line $x + y = 1$

Converting in the form $y = mx + c$

$$y = 1 - x$$

$\Rightarrow \quad m_2 = -1$

Thus, $\tan\theta$ = angle between the lines

$$\Rightarrow \quad \tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

$$= \left|\frac{\frac{-3}{2} - (-1)}{1 + \left(\frac{-3}{2}\right)(-1)}\right| = \left|\frac{\frac{-3}{2} + 1}{1 + \frac{3}{2}}\right|$$

$$= \left|\frac{\frac{-3+2}{2}}{\frac{2+3}{2}}\right| = \left|\frac{\frac{-1}{2}}{\frac{5}{2}}\right| = \frac{1}{5}$$

$$\Rightarrow \quad \theta = \tan^{-1}\left(\frac{1}{5}\right)$$

Straight lines Ex 23.13 Q5

Let l_1, be the line joining AO and
Let l_2 be the line joining BO

Then, line l_1 is $y - 0 = \left(\dfrac{0 - x_1}{0 - y_1}\right)(x - 0)$

$$yy_1 - x_1x = 0$$

Then, $m_1 = \dfrac{x_1}{y_1}$

Then line l1 is $y - 0 = \left(\dfrac{0 - x_2}{0 - y_2}\right)(x - 0)$

Then, $m_2 = \dfrac{x_2}{y_2}$

Then, $\tan\theta = \left|\dfrac{m_1 - m_2}{1 + m_1m_2}\right| = \left|\dfrac{\dfrac{x_1}{y_1} - \dfrac{x_2}{y_2}}{1 + \dfrac{x_1}{y_1}\dfrac{x_2}{y_2}}\right|$

$$= \left|\frac{x_1y_2 - y_1x_2}{y_1y_2 + x_1x_2}\right|$$

From triangle.

$$AC = \sqrt{(AB)^2 + (BC)^2}$$

$$= \sqrt{\left(m_1^2 + m_2^2 - 2m_1m_2\right) + \left(1 + m_1m_2\right)^2}$$

$$= \sqrt{m_1^2 + m_2^2 - 2m_1m_2 + 1 + m_1^2m_2^2 + 2m_1m_2}$$

$$= \sqrt{m_1^2 + m_2^2 + 1 + m_1^2m_2^2}$$

$$\cos\theta = \frac{BC}{AC} = \frac{1 + m_1m_2}{\sqrt{m_1^2 + m_2^2 + m_1^2m_2^2 - 1}}$$

$$= \frac{1 + \dfrac{x_1}{y_1}\dfrac{x_2}{y_2}}{\sqrt{\dfrac{x_1^2}{y_1^2} + \dfrac{x_2^2}{y_2^2} + \dfrac{x_1^2x_2^2}{y_1^2y_2^2} + 1}}$$

$$= \frac{\dfrac{y_1y_2 + x_1x_2}{y_1y_2}}{\sqrt{\dfrac{x_1^2y_2^2 + x_2^2y_1^2 + x_1^2x_2^2 + y_1^2y_2^2}{y_1^2y_2^2}}}$$

$$= \frac{y_1y_2 + x_1x_2}{\sqrt{x_1^2\left(y_2^2 + x_2^2\right) + y_1^2\left(y_2^2 + x_2^2\right)}}$$

$$= \frac{y_1y_2 + x_1x_2}{\sqrt{x_1^2 + y_1^2}\sqrt{y_2^2 + x_2^2}}$$

Hence proved.

RD Sharma Solutions Class 11 Maths Chapter 23 Ex 23.14

Straight lines Ex 23.14 Q1

Let ABC be the triangle of the equations whose sides AB, BC and CA are respectively
$x - 5y + 6 = 0$, $x - 3y + 2 = 0$ and $x - 2y - 3 = 0$

The coordinates of the vertices are $A(9,3)$, $B(4,2)$ and $C(13,5)$.

If the point $P(\alpha, \alpha^2)$ lies n side the ΔABC, then

(i) A and P must be on the same side of BC

(ii) B and P must be on the same side of AC.

(iii) C and P must be on the same side of AB

Now,

A and P are on the same side of BC if,

$$(9(1) + 3(-3) + 2)(\alpha^2 - 3\alpha + 2) > 0$$

$$(9 - 9 + 2)(\alpha^2 - 3\alpha + 2) > 0$$

$$\alpha^2 - 3\alpha + 2 > 0$$

$$(\alpha - 1)(\alpha - 2) > 0$$

$$\alpha \in (-\infty, 1) \ v \ (2, \infty) \qquad \text{---(i)}$$

B and P will lie on the same side of CA if,

$$(13(1) + 5(-5) + 6)(\alpha^2 - 5\alpha + 6) > 0$$

$$\Rightarrow \quad (-6)(\alpha^2 - 5\alpha + 6) > 0$$

$$\Rightarrow \quad \alpha^2 - 5\alpha + 6 < 0$$

$$\Rightarrow \quad (\alpha - 2)(\alpha - 3) < 0$$

$$\Rightarrow \quad \alpha \in (2, 3) \qquad \text{---(ii)}$$

C and P will lie on the same side of AB if,

$$(4(1) + 2(-2) - 3)(\alpha^2 - 2\alpha - 3) > 0$$

$$(-3)(\alpha^2 - 2\alpha - 3) > 0$$

$$\alpha^2 - 2\alpha - 3 < 0$$

$$(\alpha - 3)(\alpha + 1) < 0$$

$$\alpha \in (-1, 3) \qquad \text{---(iii)}$$

From i, ii, iii

$$\alpha \in [2, 3]$$

Straight lines Ex 23.14 Q2

Let ABC be the triangle. The coordinates of the vertices of the triangle ABC are marked in the following figure.

Point $P(a,2)$ lie inside or on the triangle if.

(i) A and P lie on the same side of BC.

(ii) B and P lie on the same side of AC.

(iii) C and P lie on the same side of AB.

A and P will lie on the same side of BC if.

$$(7(3)-7(-3)-8)(3a-7(2)-8)>0$$
$$(21+21-8)(3a-14-8)>0$$
$$3a-22>0$$
$$a>\frac{22}{3} \qquad \text{---(i)}$$

B and P will lie on the same side of AC if.

$$\left(4\left(\frac{18}{5}\right)-\left(\frac{2}{5}\right)-31\right)(4a-2-31)>0$$
$$4a-33>0$$
$$a>\frac{33}{4} \qquad \text{---(ii)}$$

C and P will lie on the same side of BC if.

$$\left(\frac{209}{25}+\frac{61}{25}-4\right)(a+2-4)>0$$
$$a+2>0$$
$$a>-2 \qquad \text{---(iii)}$$

From (i), (ii), (iii)

$$a\in\left(\frac{22}{3},\ \frac{33}{4}\right)$$

Straight lines Ex 23.14 Q3

Let ABC be the triangle, then coordinates of the vertices are marked in the following figure.

$P(-3,2)$ lie inside if.

(i) A and P, B and P, C and P lie on the same side of BC, AC and BA respectively.

If A and P lie on the same side of BC then,

$$(3(7)-7(-3)+8)(3(-3)-7(2)+8)>0$$
$$(21+21+8)(-9-14+8)>0$$

But, $(50)(-15)$ is not >0

$\therefore$ The point $(-3,2)$ is outside ABC.

RD Sharma Solutions Class 11 Maths Chapter 23 Ex 23.15

Straight lines Ex 23.15 Q1

Distance of a point (x_1, y_1) from $ax + by + c = 0$ is

$$= \left|\frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}}\right|$$

Here, $a = 3,\ b = -5,\ c = 7,\ x_1 = 4,\ y_1 = 5$

$$\therefore \text{ Distance } = \left|\frac{3(4) - 5(5) + 7}{\sqrt{3^2 + 5^2}}\right|$$

$$= \left|\frac{12 - 25 + 7}{\sqrt{9 + 25}}\right| = \left|\frac{6}{\sqrt{34}}\right| \text{ units.}$$

Straight lines Ex 23.15 Q2

Equation of line passing through $(\cos\theta, \sin\theta)$ and $(\cos\phi, \sin\phi)$ is

$$y - \sin\phi = \left(\frac{\sin\phi - \sin\theta}{\cos\phi - \cos\theta}\right)(x - \cos\phi)$$

$$y - \sin\phi = \left(\frac{2\cos\frac{\theta+\phi}{2}\sin\frac{\phi-\theta}{2}}{-2\sin\frac{\theta+\phi}{2}\sin\frac{\phi-\theta}{2}}\right)(x - \cos\phi)$$

$$y - \sin\phi = -\cot\left(\frac{\theta+\phi}{2}\right)(x - \cos\phi)$$

$$x\cot\left(\frac{\theta+\phi}{2}\right) + y - \sin\phi - \cos\phi\cot\left(\frac{\theta+\phi}{2}\right) = 0$$

Distance of this line from origin,

$$= \left|\frac{ax_1 + by_1 + c}{a^2 + b^2}\right|$$

$$= \left|\frac{0 + 0 - \sin\phi - \cos\phi\cot\left(\frac{\theta+\phi}{2}\right)}{\sqrt{\left(\cos\left(\frac{\theta+\phi}{2}\right)\right)^2 + 1}}\right|$$

$$= \frac{\sin\phi + \cos\phi\cot\left(\frac{\theta+\phi}{2}\right)}{\operatorname{cosec}\left(\frac{\theta+\phi}{2}\right)}$$

$$= \sin\phi \sin\left(\frac{\theta+\phi}{2}\right) + \cos\phi\, \text{co}\left(\frac{\theta+\phi}{2}\right)$$

$$= \cos\left(\frac{\theta+\phi}{2} - \phi\right)$$

$$= \cos\left(\frac{\theta+\phi-2\phi}{2}\right)$$

$$D = \cos\left(\frac{\theta-\phi}{2}\right)$$

Straight lines Ex 23.15 Q3

Line formed from joining $(a\cos\alpha, a\sin\alpha)$ and $(a\cos\beta, a\sin\beta)$

$$\Rightarrow \quad y - a\sin\beta = \frac{a\sin\beta - a\sin\alpha}{a\cos\beta - a\cos\alpha} \times x - a\cos\beta$$

$$\Rightarrow \quad y - a\sin\beta = \frac{2\sin\left(\frac{\beta-\alpha}{2}\right)\cos\left(\frac{\beta+\alpha}{2}\right)}{-2\sin\left(\frac{\beta-\alpha}{2}\right)\sin\left(\frac{\beta+\alpha}{2}\right)} \times (x - a\cos\beta)$$

$$\Rightarrow \quad y - a\sin\beta = -\cot\left(\frac{\beta+\alpha}{2}\right)(x - a\cos\beta)$$

$$\Rightarrow \quad y + \cot\left(\frac{\alpha+\beta}{2}\right)x - a\cos\beta\cot\left(\frac{\beta+\alpha}{2}\right) - a\sin\beta = 0$$

Then, the length of perpendicular

$$\Rightarrow \quad \left|\frac{0(y) + 0 - a\cos\beta\cot\left(\frac{\beta+\alpha}{2}\right) - a\sin\beta}{\sqrt{1+\cot^2\left(\frac{\alpha+\beta}{2}\right)}}\right|$$

$$\Rightarrow \quad \frac{a\cos\beta\cot\left(\frac{\alpha+\beta}{2}\right) + a\sin\beta}{\operatorname{cosec}\left(\frac{\alpha+\beta}{2}\right)}$$

$$\Rightarrow \quad a\cos\beta\cos\left(\frac{\alpha+\beta}{2}\right) + a\sin\beta\sin\left(\frac{\alpha+\beta}{2}\right)$$

$$\Rightarrow \quad a\cos\left(\frac{\alpha-\beta}{2}\right) \qquad [\text{using } \cos A\cos B + \sin A\sin B = \cos(A-B)]$$

Hence, proved.

Straight lines Ex 23.15 Q4

Let (h,k) be the point on the line $2x + 11y - 5 = 0$

$$\Rightarrow \quad 2h + 11k - 5 = 0 ------(1)$$

Let p and q be length of perpendicular from (h,k) on lines $24x + 7y - 20 = 0$ and $4x - 3y - 2 = 0$ so,

$$p = q$$

$$\frac{24h + 7k - 20}{\sqrt{(24)^2 + (7)^2}} = \frac{4h - 3k - 2}{\sqrt{(4)^2 + (-3)^2}}$$

$$\frac{24h + 7k - 20}{\sqrt{576 + 49}} = \frac{4h - 3k - 2}{\sqrt{25}}$$

$$\frac{24h + 7k - 20}{25} = \frac{4h - 3k - 2}{5}$$

$$24h + 7k - 20 = 20h - 15k - 10$$

$$4h = -22k + 10$$

$$4\left(\frac{5 - 11k}{2}\right) = -22k + 10 \qquad [\text{Using equation (1)}]$$

$10 - 22k = -22k + 10$

$LHS = RHS$

So,

Distance $24x + 7y = 20$ and $4x - 3y - 2 = 0$ from any point on the line $2x + 11y - 5 = 0$ is equal.

Straight lines Ex 23.15 Q5

The point of intersection of two lines can be calculated by solving the equations

Solving $2x + 3y = 21$ and $3x - 4y + 11 = 0$, we get the point of intersection as $P(3, -5)$

Distance of P from $8x - 6y + 5 = 0$ is

Here, $a = 8$, $b = -6$, $c = 5$, $x_1 = 3$, $y_1 = 5$

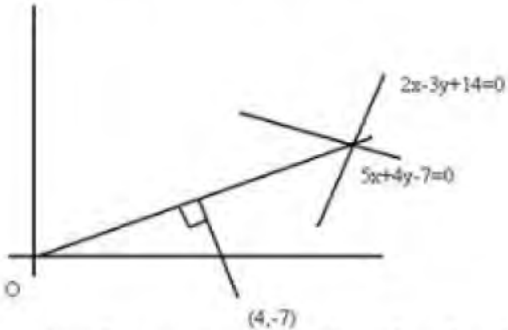
$$\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

$$\Rightarrow \frac{|8(3) - 6(-5) + 5|}{\sqrt{64 + 36}}$$

$$\Rightarrow \frac{|24 + 30 + 5|}{\sqrt{100}} = \frac{|59|}{10}$$

$$\Rightarrow \frac{59}{10}$$

Straight lines Ex 23.15 Q6



The point of intersection of the lines $2x-3y+14=0$ and $5x+4y-7=0$ can be found out by solving these equations.

Solving these equations we get, $x=-\frac{35}{23}$ and $y=\frac{252}{69}$

Equation of line joining origin and the point $\left(-\frac{35}{23}, \frac{252}{69}\right)$

is $y=mx$, where $m=\frac{\frac{252}{69}}{-\frac{35}{23}}=-\frac{12}{5}$

Therefore the equation of required line is $y=-\frac{12x}{5}$

$12x+5y=0$

Perpendicular distance from (4,-7) to $12x+5y=0$ is

$$p=\left|\frac{12(4)+5(-7)}{\sqrt{12^2+(-5)^2}}\right|=\frac{13}{13}=1$$

Straight lines Ex 23.15 Q7

Any point on x-axis is $\underset{(x_1, y_1)}{(\pm a, 0)}$

Perpendicular distance from a line $bx + ay = ab$ is

$$\left|\frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}}\right| = a$$

Where,

$$a = b,\ b = a,\ c = -ab,\ x_1 = \pm a,\ y_1 = 0$$

$$= \left|\frac{b(x) + a(0) - ab}{\sqrt{a^2 + b^2}}\right| = a$$

$$a = 0 \quad \text{or}$$

$$\frac{b(x) + a(0) - ab}{\sqrt{a^2 + b^2}} = a$$

$$\frac{b}{a}x = \pm\sqrt{a^2 + b^2} + b$$

$$x = \frac{a}{b}\left(b \pm \sqrt{a^2 + b^2}\right)$$

$$x = 0$$

RD Sharma Solutions Class 11 Maths Chapter 23 Ex 23.19

Straight lines Ex 23.19 Q1

Line through the intersection of $4x - 3y = 0$ and $2x - 5y + 3 = 0$ is

$$(4x - 3y) + \lambda(2x - 5y + 3) = 0 \qquad \text{---(i)}$$

or, $x(4 + 2\lambda) - y(3 + 5\lambda) + 3\lambda = 0$

And the required line is parallel to $4x + 5y + 6$

$\therefore$ slope of required = slope of $4x + 5y + 6 = \frac{-4}{3}$

$\therefore$ $\frac{-(4 + 2\lambda)}{-(3 + 5\lambda)} = \frac{-4}{3}$

$\Rightarrow$ $5(4 + 2\lambda) = -4(3 + 5\lambda)$

$\Rightarrow$ $20 + 10\lambda = -12 - 20\lambda$

$\Rightarrow$ $30\lambda = -32$

$\Rightarrow$ $\lambda = \frac{-16}{15}$

Putting λ in equation (i)

$$(4x - 3y) - \frac{16}{15}(2x - 5y + 3) = 0$$

$\Rightarrow$ $60x - 45y - 32x + 80y - 48 = 0$

$\Rightarrow$ $28x + 35y - 48 = 0$

Is the required line

Straight lines Ex 23.19 Q2

The equation of the required line is

$$(x + 2y + 3) + \lambda(3x + 4y + 7) = 0$$

or, $x(1 + 3\lambda) + y(2 + 4\lambda) + 3 + 7\lambda = 0$

m_1 = slope of the line $= -\left(\frac{1 + 3\lambda}{2 + 4\lambda}\right)$

The line is perpendicular to $x - y + 9 = 0$ whose slope $(m_2 = 1)$

$\therefore \quad m_1 \times m_2 = -1$

$\Rightarrow \quad -\left(\frac{1+3\lambda}{2+4\lambda}\right) \times 1 = -1$

$\Rightarrow \quad 1 + 3\lambda = 2 + 4\lambda$

$\Rightarrow \quad \lambda = -1$

$\therefore$ The required line is

$$x + 2y + 3 - (3x + 4y + 7) = 0$$

$$-2x - 2y - 4 = 0$$

or, $\quad x + y + 2 = 0$

Straight lines Ex 23.19 Q3

The required line is

$$2x - 7y + 11 + \lambda(x + 3y - 8) = 0$$

or, $\quad x(2+\lambda) + y(-7+3\lambda) + 11 - 8\lambda = 0$

(i) When the line is parallel to x-axis. It slope is 0

$\therefore \quad -\frac{(2+\lambda)}{3\lambda - 7} = 0$

$\lambda = -2$

$\therefore$ Equation of line is

$$2x - 7y + 11 - 2(x + 3y - 8) = 0$$

$$-13y + 27 = 0$$

(ii) When the line is parallel to y-axis then,

$$\frac{-1}{\text{slope}} = 0$$

i.e $\quad \frac{3\lambda - 7}{2 + \lambda} = 0$

$\lambda = \frac{7}{3}$

$\therefore$ Equation of line is

$$2x - 7y + 11 + \frac{7}{3}(x + 3y - 8) = 0$$

$$\Rightarrow \frac{6x - 21y + 33 + 7x + 21y - 56}{3} = 0$$

$\Rightarrow 6x - 21y + 33 + 7x + 21y - 56 = 0$
$\Rightarrow 13x - 23 = 0$
$\Rightarrow 13x = 23$

Straight lines Ex 23.19 Q4

The required line is

$$(2x + 3y - 1) + \lambda(3x - 5y - 5) = 0$$

or, $$x(2 + 3\lambda) + y(3 - 5\lambda) - 1 - 5\lambda = 0$$

Since this lines is equally inclined to both the axes, it slope should be 1. or -1

$\therefore \quad \frac{-2 - 3\lambda}{3 - 5\lambda} = 1 \quad$ or, $\quad \frac{-2 - 3\lambda}{3 - 5\lambda} = -1$

$\Rightarrow \quad 3 - 5\lambda = -2 - 3\lambda \quad$ or, $\quad \Rightarrow -2 - 3\lambda = -3 + 5\lambda$

$\Rightarrow \quad 5 = 2\lambda \quad$ or, $\quad \Rightarrow 1 = 8\lambda$

$\Rightarrow \quad \lambda = \frac{5}{2} \quad$ or, $\quad \Rightarrow \lambda = \frac{1}{8}$

$\therefore$ The required line is

$$2x + 3y + 1 + \frac{5}{2}(3x - 5y - 5) = 0$$
$$4x + 6y + 2 + 15x - 25y - 25 = 0$$
$$19x - 19y - 23 = 0$$

or

$$(2x + 3y + 1) + \frac{1}{8}(3x - 5y - 5) = 0$$
$$16x + 24y + 8 + 3x - 5y - 5 = 0$$
$$19x + 19y + 3 = 0$$

$\therefore$ The two possible equation are

$19x - 19y - 23 = 0 \quad$ or $\quad 19x + 19y + 3 = 0$

Straight lines Ex 23.19 Q5

The required line is

$$(x + y - 4) + \lambda(2x - 3y - 1) = 0$$

or, $x(1 + 2\lambda) + y(1 - 3\lambda) - 4 - \lambda = 0$

And it is perpendicular to $\frac{x}{5} + \frac{y}{6} = 1$

$\therefore$ (slope of required line) $\times \left(\text{slope of } \frac{x}{5} + \frac{y}{6} = 1\right) = -1$

$\Rightarrow$ $-\left(\frac{1+2\lambda}{1-3\lambda}\right) \times \frac{-6}{5} = -1$

$\Rightarrow$ $\frac{1+2\lambda}{1-3\lambda} = \frac{-5}{6}$

$\Rightarrow$ $6 + 12\lambda = -5 + 15\lambda$

$\Rightarrow$ $11 = 3\lambda$ or $\lambda = \frac{11}{3}$

$\therefore$ The required line is

$$(x + y - 4) + \frac{11}{3}(2x - 3y - 1) = 0$$

$$3x + 3y - 12 + 22x - 33y - 11 = 0$$

$$25x - 30y - 23 = 0$$

Straight lines Ex 23.19 Q6

$$x(1 + \lambda) + y(2 - \lambda) + 5 = 0$$

$\Rightarrow$ $x + x\lambda + 2y - \lambda y + 5 = 0$

$\Rightarrow$ $\lambda(x - y) + (x + 2y + 5) = 0$

$\Rightarrow$ $(x + 2y + 5) + \lambda(x - y) = 0$

This is of the form $L_1 + \lambda L_2 = 0$

So it represents a line passing through the intersection of $x - y = 0$ and $x + 2y = -5$.

Solving the two equations, we get $\left(\frac{-5}{3}, \frac{-5}{3}\right)$ which is the fixed point through which the given family of lines passes for any value of λ.

Straight lines Ex 23.19 Q7

$$(2 + k)x + (1 + k)y = 5 + 7k$$

or, $(2x + y - 5) + k(x + y - 7) = 0$

It is of the form $L_1 + kL_2 = 0$ i.e., the equation of line passing through the intersection of 2 lines L_1 and L_2.

So, it represents a line passing through $2x + y - 5 = 0$ and $x + y - 7 = 0$.

Solving the two equation we get, $(-2, 9)$. Which is the fixed point through which the given line pass. For any value of k.

Straight lines Ex 23.19 Q8

$L_1 + \lambda l_2 = 0$ is the equation of line passing through two lines. L_1 and L_2.

$\therefore \quad (2x + y - 1) + \lambda(x + 3y - 2) = 0$ is the required equation. ---(i)

or, $\quad x(2 + \lambda) + y(1 + 3\lambda) - 1 - 2\lambda = 0$

$$\frac{x}{\frac{1+2\lambda}{2+\lambda}} + \frac{4}{\frac{1+2\lambda}{1+3\lambda}} = 1$$

Area of $\Delta = \frac{1}{2} \times OB \times OA$

$$\frac{8}{3} = \frac{1}{2} \times (y \text{ intercept}) \times (x \text{ intercept})$$

$$\frac{8}{3} = \frac{1}{2} \times \left(\frac{1+2\lambda}{1+3\lambda}\right) \times \left(\frac{1+2\lambda}{2+\lambda}\right)$$

$$\frac{16}{3} = \frac{1 + 4\lambda^2 + 4\lambda}{2 + 3\lambda^2 + 7\lambda}$$

$$32 + 48\lambda^2 + 112\lambda = -3 - 12\lambda^2 - 12\lambda$$

$$60\lambda^2 + 124\lambda + 35 = 0$$

$$\lambda = \frac{-124 \pm \sqrt{(124)^2 - 4 \times 60 \times 35}}{2 \times 60}$$

$$= \frac{-124 \pm \sqrt{15376 - 8400}}{120}$$

Approximately = 1

$\therefore$ Subtituting in $(i) \Rightarrow 3x + 4y - 3 = 0,\ 12x + y - 3 = 0$

Straight lines Ex 23.19 Q9

The required line is

$$(3x - y - 5) + \lambda(x + 3y - 1) = 0$$

or, $\quad (3 + \lambda)x + (-1 + 3\lambda)y - 5 - \lambda = 0$

or, $$\frac{x}{\left(\frac{5+\lambda}{3+\lambda}\right)} + \frac{y}{\frac{5+\lambda}{3\lambda - 1}} = 1$$

And the line makes equal and positive intercepts with the line (given)

$$\therefore \quad \frac{5+\lambda}{3+\lambda} = \frac{5+\lambda}{3\lambda - 1}$$

$$3\lambda - 1 = 3 + \lambda$$

$$2\lambda = 4$$

$$\lambda = 2$$

$\therefore$ The required line is

$$3x - y - 5 + 2x + 6y - 2 = 0$$

or, $\quad 5x + 5y = 7$

Straight lines Ex 23.19 Q10

The required line is

$$x - 3y + 1 + \lambda(2x + 5y - 9) = 0$$

or, $$(1 + 2\lambda)x + (-3 + 5\lambda)y + 1 - 9\lambda = 0$$

Distance from origin of this line is

$$\left|\frac{(1+2\lambda)0 + (-3+5\lambda)0 + 1 - 9\lambda}{\sqrt{(1+2\lambda)^2 + (5\lambda - 3)^2}}\right| \qquad \left[\text{using } \frac{ax_1 + by_1 + c}{\sqrt{a^2+b^2}}\right]$$

$$\sqrt{5} = \left|\frac{1 - 9\lambda}{\sqrt{1 + 4\lambda^2 + 4\lambda + 25\lambda^2 + 9 - 30\lambda}}\right|$$

$$\Rightarrow \quad \sqrt{5} = \left|\frac{1 - 9\lambda}{\sqrt{10 + 29\lambda^2 - 26\lambda}}\right|$$

$$\Rightarrow \quad 5(10 + 29\lambda^2 - 26\lambda) = (1 - 9\lambda)^2$$

$$\Rightarrow \quad 50 + 145\lambda^2 - 130\lambda = 1 + 81\lambda^2 - 18\lambda^2$$

$$\Rightarrow \quad 64\lambda^2 - 112\lambda + 49 = 0$$

$$\Rightarrow \quad (8\lambda - 7)^2 = 0 \quad \text{or,} \quad \lambda = \frac{7}{8}$$

$\therefore$ Required line is

$$x - 3y + 1 + \frac{7}{8}(2x + 5y - 9) = 0$$

$$\Rightarrow \quad 8x - 24y + 8 + 14x + 35y - 63 = 0$$

$$\Rightarrow \quad 22x + 11y - 55 = 0$$

$$\Rightarrow \quad 2x + y - 5 = 0$$