In the dictionary the words at each stage are arranged in alphabetical order. In the given problem we must therefore consider the words beginning with E, H, M, O, R, T in order. E will occur in the first place as often as there are ways of arranging the remaining 5 letters

$\therefore$ Number of words starting with $E = 5! = 5\times4\times3\times2\times1 = 120$

Number of words starting with $H = 5! = 120.$
Number of words beginning with M is 5!, but one of these w... word MOTHER.
So, we first find the number of words beginning with ME
Number of words starting with $ME = 4! = 4\times3\times2\times1 = 2$...

Now, the words beginning with 'MO' must follow...
There are 4! words beginning with MO, one o... ...self.
So, we first find the number of words begi...

Number of words starting with MOE ...
Number of words starting with MO...
Number of words starting with...

Number of words beginni... ...words is the word MOTHER itself
So, we first find the n... ...
Number of words startin...

Now, the words beginning wit...
There are 2! words beginning wit... of these words is word MOTHER itself.
The first word beginning with MOTH... ...word MOTHER.

$\therefore$ Rank of MOTHER $= 2\times120 + 2\times24 + 3\times6 + 2 + 1$
$= 240 + 48 + 18 + 3$

$= 309$

Permutations Ex 16.5 Q25

In a dictionary the words at each stage are arranged in alphabetical order. In the given problem we must therefore consider the words beginning with a,b,c,d,e in order. 'a' will occur in the first place as often as there are ways of arranging the remaining 4 letters all at a time i.e 'a' will occur 4! times. similarly b and c will occur in the first place the same number of times

$\therefore$ Number of words starting with'a' = $4! = 4\times3\times2\times1 = 24$
Number of words starting with 'b' = $4! = 4\times3\times2\times1 = 24$
Number of words starting with 'c' = $4! = 4\times3\times2\times1 = 24$

Number of words beginning with 'd' is 4!, but one of these words is the word debac.
So, we first find the number of words beginning with da, db, dc, and dea

Number of words starting with *da* = 3! = 6
Number of words starting with *db* = 3! = 6
Number of words starting with *dc* = 3! = 6
Number of words starting with *dea* = 2! = 2

There are 2! words beginning with deb one of these words is the word debac itself
The first word beginning with deb is the word debac.

$\therefore$ Rank of debac $= 3\times24+3\times6+2+1$
$= 72+18+3$
$= 90+3$

$= 93$

Permutations Ex 16.5 Q26

Total number of '+' signs = 6
Total number of '-' signs = 4

six '+' signs can be arranged in a row in $\frac{6!}{6!} = 1$ way [$\because$ All '+' signs are identical]

Now, we are left with seven places in which four different things can be arranged in ${}^7P_4$ ways but all the four '-' signs are identical, therefore, four '-' signs can be arranged in $\frac{{}^7P_4}{4!} = \frac{\frac{7!}{(7-4)!}}{4!} = \frac{7!}{3!\times4!}$

$= \frac{7\times6\times5\times4!}{3\times2\times4!} = 7\times5 = 35$

Hence, the required number of ways = $1\times35 = 35$.

Permutations Ex 16.5 Q27

INTERMEDIATE

$I = 2$ times, $T = 2$ times, $E = 3$ times, N, R, M, D, A

Number of letters = 12

(i) There are 6 vowels. They ocuepy even places 2nd, 4th, 6th, 8th, 10th, 12th.

After there six there are six places and 5 letters, T is 2 times.

So, number of ways for consonants = $\frac{6!}{2!}$

The total number of ways when vowels ocuepy even places

$$= \frac{6!}{2!} \times \frac{6!}{2!3!}$$

$$= \frac{6 \times 5 \times 4 \times 3 \times 2 \times 6 \times 5 \times 4 \times 3 \times 2}{2 \times 2 \times 3 \times 2}$$

$$= 21600$$

Required number of ways = 21600

(ii) Number of ways such that relative order of vowels and consonents do not alter

$$= \frac{6!}{2! \times 3!} \times \frac{6!}{2!}$$

$$= 21600$$

Required number of ways = 21600

Permutations Ex 16.5 Q28

In a dictionary the words at each stage are arranged in alphabetical order. In the given problem we must therefore consider the words beginning with E, H, I, N, T, Z in order.

'E' will occur in the first place as often as there are ways of arranging the remaining 5 letters all at a time i.e. E will occur 5! times. Similarly H will occur in the first place the same number of times.

$\therefore$ Number of words starting with E = $5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$

Number of words starting with H = 5! = 120

Number of words starting with I = 5! = 120

Number of words starting with N = 5! = 120

Number of words starting with T = 5! = 120

Number of words beginning with Z is 5!, but one of these words is the word ZENITH itself.

So, we first find the number of words beginning with ZEH, ZEI and ZENH

Number of words starting with ZEH = 3! = 6

Number of words starting with ZEI = 3! = 6

Number of words starting with ZENH = 2! = 2.

Now, the words beginning with ZENI must follow.

There are 2! words beginning with ZENI one of these words is the word ZENITH itself.

The first word beginning with ZENI is the word ZENIHT and the next word is ZENITH.

$\therefore$ Rank of ZENITH

$$= 5 \times 120 + 2 \times 6 + 2 + 2$$

$$= 600 + 12 + 4$$

$$= 600 + 16$$

$$= 616$$

Permutations Ex 16.5 Q29

18 mice can be arranged among themselves in

${}^{18}P_{18} = 18!$ ways.

There are three groups and each group is equally large.
So 18 mice are divided in three groups and they can be arranged amongst themselves inside the group.

Therefore the number of ways mice placed into three groups are

$$= \frac{18!}{6!6!6!} = \frac{18!}{(6!)^3}$$