RD Sharma Solutions Class 11 Maths Chapter 16 Ex16.5

Permutations Ex 16.5 Q1(i)

There are 12 letters in the word 'INDEPENDENCE' out of which 2 are D'S, 3 are N'S, 4 are E'S and the rest are all distinct.

so, the total number of words = $\frac{12!}{2!\,3!\,4!}$

$$= \frac{12\times11\times10\times9\times8\times7\times6\times5\times4!}{2!\,3!\,4!}$$

$$= \frac{12\times11\times10\times9\times8\times7\times6\times5}{2\times3\times2}$$

$$= 11\times10\times9\times8\times7\times6\times5$$

$$= 1663200.$$

Permutations Ex 16.5 Q1(ii)

There are 12 letters in the word 'INTERMEDIATE' out of which 2 are I'S, 2 are T'S, 3 are E'S and the rest are all distinct.

so, the total number of words

$$= \frac{12!}{2!\,2!\,3!}$$

$$= \frac{12 \times 11 \times 10 \times 9 \times 8 \times 6 \times 5 \times 4 \times 3 \times 2!}{2!\,2!\,3!}$$

$$= \frac{12 \times 11 \times 10 \times 9 \times 8 \times 6 \times 5 \times 4 \times 3}{2 \times 3 \times 2}$$

$$= 11 \times 10 \times 9 \times 8 \times 6 \times 5 \times 4 \times 3$$

$$= 19958400$$

Permutations Ex 16.5 Q1(iii)

There are 7 letters in the word 'ARRANGE' out of which 2 are A'S, 2 are R'S, and the rest are all distinct.

So, the total number of words

$$= \frac{7!}{2!\,2!}$$

$$= \frac{7 \times 6 \times 5 \times 4 \times 3 \times 2!}{2!\,2!}$$

$$= \frac{7 \times 6 \times 5 \times 4 \times 3}{2 \times 1}$$

$$= 7 \times 6 \times 5 \times 2 \times 3$$

$$= 1260$$

Permutations Ex 16.5 Q1(iv)

There are 5 letters in the word 'INDIA' out of which 2 are I'S, and the rest are all distinct.

so, the total number of

$$\text{words} = \frac{5!}{2!}$$

$$= \frac{5 \times 4 \times 3 \times 2!}{2!}$$

$$= 60$$

Permutations Ex 16.5 Q1(v)

There are 8 letters in the word 'PAKISTAN' out of which 2 are A'S, and the rest are all distinct.

So, the total number of words

$$= \frac{8!}{2!}$$

$$= \frac{8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2!}{2!}$$

$$= 8 \times 7 \times 6 \times 5 \times 4 \times 3$$

$$= 20160$$

Permutations Ex 16.5 Q1(vi)

There are 6 letters in the word 'RUSSIA' out of which 2 are S's, and the rest are all distinct.

So, the total number of words

$$= \frac{6!}{2!}$$

$$= \frac{6 \times 5 \times 4 \times 3 \times 2!}{2!}$$

$$= 6 \times 5 \times 4 \times 3$$

$$= 360$$

Permutations Ex 16.5 Q1(vii)

There are 6 letters in the word 'SERIES' out of which 2 are S's, 2 are E's and the rest are all distinct.

so, the total number of words

$$= \frac{6!}{2!\ 2!}$$

$$= \frac{6 \times 5 \times 4 \times 3 \times 2!}{2!\ 2!}$$

$$= \frac{6 \times 5 \times 4 \times 3}{2 \times 1}$$

$$= 6 \times 5 \times 2 \times 3$$

$$= 180$$

Permutations Ex 16.5 Q1(viii)

There are 9 letters in the word 'EXERCISES' out of which 3 are E's, 2 are S's and the rest are all distinct.

So, the total number of words

$$= \frac{9!}{3!\,2!}$$

$$= \frac{9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3!}{3! \times 2 \times 1}$$

$$= 9 \times 8 \times 7 \times 6 \times 5 \times 2$$

$$= 30240$$

Permutations Ex 16.5 Q1(ix)

There are 14 letters in the word 'CONSTANTINOPLE' out of which 2 are O's, 3 are N's, 2 are T's and the rest are all distinct.

So, the total number of

$$\text{words} = \frac{14!}{2!\,3!\,2!}$$

$$= \frac{14!}{2 \times 3 \times 2 \times 2}$$

$$= \frac{14!}{24}$$

Permutations Ex 16.5 Q2

There are 4 consonants in the word 'ALGEBRA'.
The number of ways to arrange these consonants = 4!

There are 3 vowels in the given word of which 2 are A's

The vowels can be arranged among themselves in $\frac{3!}{2!}$ ways.

Hence, the required number of arrangements $= 4! \times \frac{3!}{2}$

$$= 4 \times 3 \times 2 \times \frac{3 \times 2}{2}$$

$$= 72$$

Permutations Ex 16.5 Q3

In the word 'UNIVERSITY' there are 10 letters of which 2 are I's.

There are 4 vowels in the given word of which 2 are I's.

These vowels can be put together in $\frac{4!}{2!}$ ways.

Considering these 4 vowels as one letter there are 7 letters which can be arranged in 7! ways.

Hence, by fundamental principle of multiplication, the required number of arrangements is

$$= \frac{4!}{2!} \times 7!$$

$$= \frac{4 \times 3 \times 2!}{2!} \times 7 \times 6 \times 5 \times 4 \times 3 \times 2$$

$$= 4 \times 3 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2$$

$$= 60480.$$

Permutations Ex 16.5 Q4

There are $3a's$, $2b's$ and $4c's$.

So, the number of arrangements

$$= \frac{9!}{4!3!2!}$$

$$= \frac{9 \times 8 \times 7 \times 6 \times 5 \times 4!}{4! \times 3 \times 2 \times 2}$$

$$= 9 \times 4 \times 7 \times 5$$

$$= 1260.$$

Hence, the total number of arrangements are 1260.

Permutations Ex 16.5 Q5

There are 8 letters in the word 'PARALLEL' out of which A's and 3 are L's and the rest are all distinct.

So, total number of words = $\frac{8!}{2!\ 3!}$

$= \frac{8 \times 7 \times 6 \times 5 \times 4 \times 3!}{2 \times 1 \times 3!}$

$= 8 \times 7 \times 6 \times 5 \times 2$

$= 3360$

Considering all L's together and treating them as one letter we have 6 letters out of which A repeats 2 times and others are distinct. These 6 letters can be arranged in $\frac{6!}{2!}$ ways.

So, the number of words in which all L's come together

$= \frac{6!}{2!}$

$= \frac{6 \times 5 \times 4 \times 3 \times 2!}{2!}$

$= 6 \times 5 \times 4 \times 3$

$= 360$

Hence, the number of words in which all L's do not come together

$= 3360 - 360$

$= 3000.$

Permutations Ex 16.5 Q6

There are 6 letters in the word 'MUMBAI' out of which 2 are M's and the rest are all distinct.

Considering both M's together and treating as one letter we have 5 letters. These 5 letters can be arranged in 5! ways.

Hence, the total number of arrangement = 5!

$= 5 \times 4 \times 3 \times 2 \times 1$

$= 120$

Permutations Ex 16.5 Q7

Total number of digits are = 7

There are 4 odd digits 1,1,3,3 and 4 odd places $(1,3,5,7)$

So, odd digits can be arranged in odd places in $\frac{4!}{2!\ 2!}$ ways

The remaining 3 even digits 2,2,4 can be arranged in 3 even places in $\frac{3!}{2!}$ ways.

Hence, the total number of Numbers = $\frac{4!}{2!\,2!} \times \frac{3!}{2!} = \frac{4 \times 3 \times 2!}{2!\ 2!} \times \frac{3 \times 2!}{2!} = 18$

Permutations Ex 16.5 Q8

Total number of red flags = 4
Total number of white flags = 2
Total number of green flags = 3

We have to arrange 9 flags, out of which 4 are of red, 2 are white and 3 are green

So, total number of signals = $\frac{9!}{4!\,2!\,3!}$

$$= \frac{9\times8\times7\times6\times5\times4!}{4!\times2\times3\times2} = 9\times4\times7\times5 = 1260$$

Hence, total number of signals = 1260.

Permutations Ex 16.5 Q9

Total number of digits = 4

Total number of 4 digit numbers = $\frac{4!}{2!}$

But, zero cannot be first digit of the four digit numbers.

$\therefore$ Total number of 3 digit numbers = $\frac{3!}{2!}$

$\therefore$ Total number of numbers = $\frac{4!}{2!} - \frac{3!}{2!} = \frac{4\times3\times2!}{2!} - \frac{(3\times2!)}{2!}$

$$= 12 - 3 = 9$$

Hence, total number of four digit numbers = 9

Permutations Ex 16.5 Q10

There are 7 letters in the word 'ARRANGE' out of which 2 are A's 2 are R's and the rest are all distinct.

So, total number of words = $\frac{7!}{2!\,2!}$

$= \frac{7 \times 6 \times 5 \times 4 \times 3 \times 2!}{2 \times 2!}$

$= 7 \times 6 \times 5 \times 2 \times 3$
$= 1260.$

Considering all R's together and treating them as one letter we have 6 letters out of which A repeats 2 times and other are distinct. These 6 letters can be arranged in $\frac{6!}{2!}$ ways.

So, the number of words in which all R's come together = $\frac{6!}{2!}$

$= \frac{6 \times 5 \times 4 \times 3 \times 2!}{2!}$
$= 360.$

Hence, the number of words in which all R's do not came together

= Total number of words – Number of words in which all R's come together

$= 1260 - 360$

$= 900.$

Permutations Ex 16.5 Q11

Total number of digits = 5

Now, numbers greater then 50000 will have either 5 or 9 in the first place and will consist of 5 digits.

Number of numbers of which digit 5 at first place = $\frac{4!}{2!}$ $\left[\because 1 \text{ is repeated}\right]$

$= \frac{4 \times 3 \times 2!}{2!}$

$= 12.$

Number of numbers with digit 9 at first place = $\frac{4!}{2!} = 12$

Hence, the required number of numbers = $12 + 12 = 24$.

Permutations Ex 16.5 Q12

In the word 'SERIES' there are 6 letters of which 2 are S and 2 are E's.

Let us fix 5 at the extreme left and at the extreme right end. Now, we are left with 4 letters of which 2 are E's. These four letters can be arranged in $\frac{4!}{2!}$ ways.

Hence, required number of arrangements = $\frac{4!}{2!} = \frac{4 \times 3 \times 2!}{2!} = 12.$

Permutations Ex 16.5 Q13

MADHUBANI

Total number of words that ends with letter I = $\frac{8!}{2!}$

$= 8\times7\times5\times6\times4\times3$

$= 56\times30\times12$

$= 20160$

If the words starts with M and end with I , there are 7 space left for 7 letters.

Number of words that starts with M and end with I = $\frac{7!}{2!}$

$= 7\times5\times4\times3$

$= 42\times60$

$= 2520$

Number of words which do not start with M but end with I

$= 20160 - 2520$

Required number of words = 17640

Permutations Ex 16.5 Q14

Total numer of digits = 7

Since, 0 cannot be first digit of the 7 digit numbers.

$\therefore$ Number of 6 – digit

Numbers = $\frac{6!}{2!3!}$ $\left[\because \text{2comes } 2 \text{ times and } 3 \text{ comes } 3 \text{ times}\right]$

$= \frac{6\times5\times4\times3!}{2\times3!}$

$= 6\times5\times2$

$= 60.$

Now, number of 7-digit numbers = $\frac{7!}{2!3!} = \frac{7\times6\times5\times4\times3!}{2\times3!}$

$= 7\times6\times5\times2$

$= 420$

Hence, total number of numbers which is greater then 1 million = 420 – 60

= 360.

Permutations Ex 16.5 Q15

There are three copies each of 4 different books.

$\therefore$ Total number of copies = 12

$\therefore$ The number of ways in which these copies arranged in a shelf

$$= \frac{12!}{3!\,3!\,3!\,3!}$$

$$= \frac{12!}{(3!)^4}$$

Hence, required number of ways

$$= \frac{12!}{(3!)^4}$$

Permutations Ex 16.5 Q16

There are 11 letters in the word 'MATHEMATICS' out of which 2 are M's, 2 are A's, 2 are T's and the rest are all distinct.

so, the requisite number of words = $\frac{11!}{2!\,2!\,2!}$

If we fix C in the beginning, then the remaining 10 letters can be arranged in $\frac{10!}{2!\,2!\,2!}$

If we fix T in the beginning, then the remainning 10 letters can be arranged in $\frac{10!}{2!\,2!}$

Permutations Ex 16.5 Q17

Total number of molecules = 12

Now,

the chain contains 4 different molecules A, c, g, and T, and 3 molecules of each kind.

$\therefore$ the number of different arrangements = $\frac{12!}{3!\,3!\,3!\,3!}$

$$= \frac{12\times11\times10\times9\times8\times7\times6\times5\times4\times3!}{3\times2\times3\times2\times3\times2\times3!}$$

$= 369600.$

Hence, the number of different possible arrangements are = 369600.

Permutations Ex 16.5 Q18

4 red, 3 yellow and 2 green discs.

Total discs = 9

Required number of ways

$$= \frac{9!}{4!3!2!}$$

$$= \frac{9\times8\times7\times6\times5}{3\times2\times2}$$

$= 1260$

Required number of ways = 1260

Permutations Ex 16.5 Q19

Total number of digits = 7

Now,

number of 7-digit numbers = $\frac{7!}{3!\,2!}$

$= \frac{7\times6\times5\times4\times3!}{3!\times2}$

$= 7\times6\times5\times2$

$= 420$

And,0 cannot be first digit of the 7-digit numbers

$\therefore$ Number of 6-digit numbers

$= \frac{6!}{3!\,2!}$

$= \frac{6\times5\times4\times3!}{3!\times2}$

$= 6\times5\times2$

$= 60$

Hence, total number of 7-digit number which are greater than 1000000 = 420 – 60 = 360

Permutations Ex 16.5 Q20

There are 13 letters in the word 'ASSASSINATION' out of which 3 are A's, 4 are S's, 2 are I's, 2 are N's and the rest are all distinct.

Considering all S's together and treating them as one letter we have 10 letters.

These 10 letters can be arranged in $\frac{10!}{3!\,2!\,2!}$

$= \frac{10\times9\times8\times7\times6\times5\times4\times3!}{3!\times2\times2}$

$= 10\times9\times8\times7\times6\times5$

$= 151200.$

Hence, the total words are 151200

Permutations Ex 16.5 Q21

There are 9 letters in the word 'INSTITUTE' out of which 2 are I's, 3 are T's and the rest are all distinct.

$\therefore$ The total number of permutations of the letters of the word 'INSTITUTE' = $\frac{9!}{2!\,3!}$

Hence, the total number of words are $\frac{9!}{2!\,3!}$

Permutations Ex 16.5 Q22

In a dictionary the words at each stage are arranged in alphabetical order.

Starting with letter I, and arranging the other 5 letters, we obtain $5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$.
Then starting with R, and arranging the other five letters I,I,S,T,U in different ways,
we obtain $\frac{5!}{2!} = \frac{120}{2} = 60$.

Number of words beginning with S is $\frac{5!}{2!}$, but one of these words is the word SURITI itself.
So, we first find the number of words beginning with SI, SR, ST, SUI and SURI.

Number of words starting with SI = $4! = 24$

Number of words starting with SR = $\frac{4!}{2!} = 12$

Number of words starting with ST = $\frac{4!}{2!} = 12$

Number of words starting with SUI = $3! = 6$

Now, the words beginning with 'SUR' must follow.
There are $\frac{3!}{2!} = 3$ words beginning with SUR one of these words is the word SURITI.
The first word beginning which SUR is the word SURIIT and the next word is SURITI.

$\therefore$ Rank of SURITI

$= 120 + 60 + 24 + 2 \times 12 + 6 + 2$

$= 180 + 56$

$= 236.$

Permutations Ex 16.5 Q23

In a dictionary the words at each stage are arranged in alphabetical order. In the given problem we must therefore consider the words beginning with A, E, L, T in order.

'A' will occur in the first place as often as remaining 3 letters all at a time i.e A will occur in the first place the same number of times.

$\therefore$ Number of words starting with A = $3! = 6$

Number of words starting with E = $3! = 6$

Number of words begining with L is 3!, but one of these words is the word LATE itself.
The first word beginning with L is the word LATE and the next word is LATE.

$\therefore$ Rank of LATE = $2 \times 6 + 2$

$= 12 + 2$

$= 14.$

Permutations Ex 16.5 Q24

In the dictionary the words at each stage are arranged in alphabetical order. In the given problem we must therefore consider the words beginning with E, H, M, O, R, T in order. E will occur in the first place as often as there are ways of arranging the remaining 5 letters

$\therefore$ Number of words starting with $E = 5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$

Number of words starting with $H = 5! = 120$.
Number of words beginning with M is 5!, but one of these words is the word MOTHER.
So, we first find the number of words beginning with ME and MH.
Number of words starting with $ME = 4! = 4 \times 3 \times 2 \times 1 = 24$.

Now, the words beginning with 'MO' must follow.
There are 4! words beginning with MO, one of these words is the word MOTHER itself.
So, we first find the number of words beginning with MOE, MOH and MOR.

Number of words starting with $MOE = 3! = 6$
Number of words starting with $MOH = 3! = 6$
Number of words starting with $MOR = 3! = 6$

Number of words beginning with MOT is 3! but one of these words is the word MOTHER itself
So, we first find the number of words beginning with MOTE.
Number of words starting with $MOTE = 2! = 2$

Now, the words beginning with MOTH must follow.
There are 2! words beginning with MOTH, one of these words is word MOTHER itself.
The first word beginning with MOTH is the word MOTHER.

$\therefore$ Rank of MOTHER $= 2 \times 120 + 2 \times 24 + 3 \times 6 + 2 + 1$
$= 240 + 48 + 18 + 3$

$= 309$

Permutations Ex 16.5 Q25

In a dictionary the words at each stage are arranged in alphabetical order. In the given problem we must therefore consider the words beginning with a,b,c,d,e in order. 'a' will occur in the first place as often as there are ways of arranging the remaining 4 letters all at a time i.e 'a' will occur 4! times. similarly b and c will occur in the first place the same number of times

$\therefore$ Number of words starting with 'a' = $4! = 4\times3\times2\times1 = 24$
Number of words starting with 'b' = $4! = 4\times3\times2\times1 = 24$
Number of words starting with 'c' = $4! = 4\times3\times2\times1 = 24$

Number of words beginning with 'd' is 4!, but one of these words is the word debac.
So, we first find the number of words beginning with da, db, dc, and dea

Number of words starting with *da* = 3! = 6
Number of words starting with *db* = 3! = 6
Number of words starting with *dc* = 3! = 6
Number of words starting with *dea* = 2! = 2

There are 2! words beginning with deb one of these words is the word debac itself
The first word beginning with deb is the word debac.

$$\therefore \text{ Rank of debac} = 3\times24+3\times6+2+1 = 72+18+3 = 90+3 = 93$$

Permutations Ex 16.5 Q26

Total number of '+' signs = 6
Total number of '-' signs = 4

six '+' signs can be arranged in a row in $\frac{6!}{6!} = 1$ way [$\because$ All '+' signs are identical]

Now, we are left with seven places in which four different things can be arranged in ${}^7P_4$ ways but all the four '-' signs are identical, therefore, four '-' signs can be arranged in $\frac{{}^7P_4}{4!} = \frac{\frac{7!}{(7-4)!}}{4!} = \frac{7!}{3!\times4!}$

$$= \frac{7\times6\times5\times4!}{3\times2\times4!} = 7\times5 = 35$$

Hence, the required number of ways = $1\times35 = 35$.

Permutations Ex 16.5 Q27

INTERMEDIATE

$I = 2$ times, $T = 2$ times, $E = 3$ times, N, R, M, D, A

Number of letters = 12

(i) There are 6 vowels. They ocuepy even places 2nd, 4th, 6th, 8th, 10th, 12th.

After there six there are six places and 5 letters, T is 2 times.

So, number of ways for consonants = $\frac{6!}{2!}$

The total number of ways when vowels ocuepy even places

$= \frac{6!}{2!} \times \frac{6!}{2!3!}$

$= \frac{6 \times 5 \times 4 \times 3 \times 2 \times 6 \times 5 \times 4 \times 3 \times 2}{2 \times 2 \times 3 \times 2}$

$= 21600$

Required number of ways = 21600

(ii) Number of ways such that relative order of vowels and consonents do not alter

$= \frac{6!}{2! \times 3!} \times \frac{6!}{2!}$

$= 21600$

Required number of ways = 21600

Permutations Ex 16.5 Q28

In a dictionary the words at each stage are arranged in alphabetical order. In the given problem we must therefore consider the words beginning with E, H, I, N, T, Z in order.

'E' will occur in the first place as often as there are ways of arranging the remaining 5 letters all at a time i.e. E will occur 5! times. Similarly H will occur in the first place the same number of times.

$\therefore$ Number of words starting with E = $5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$

Number of words starting with H = $5! = 120$

Number of words starting with I = $5! = 120$

Number of words starting with N = $5! = 120$

Number of words starting with T = $5! = 120$

Number of words beginning with Z is 5!, but one of these words is the word ZENITH itself.

So, we first find the number of words beginning with ZEH, ZEI and ZENH

Number of words starting with ZEH = $3! = 6$

Number of words starting with ZEI = $3! = 6$

Number of words starting with ZENH = $2! = 2$.

Now, the words beginning with ZENI must follow.

There are 2! words beginning with ZENI one of these words is the word ZENITH itself.

The first word beginning with ZENI is the word ZENIHT and the next word is ZENITH.

$\therefore$ Rank of ZENITH

$= 5 \times 120 + 2 \times 6 + 2 + 2$

$= 600 + 12 + 4$

$= 600 + 16$

$= 616$

Permutations Ex 16.5 Q29

18 mice can be arranged among themselves in
$^{18}P_{18} = 18!$ ways.

There are three groups and each group is equally large.
So 18 mice are divided in three groups and they can be arranged amongst themselves inside the group.

Therefore the number of ways mice placed into three groups are

$$= \frac{18!}{6!6!6!} = \frac{18!}{(6!)^3}$$