RD Sharma Solutions Class 11 Maths Chapter 16 Ex 16.1

Permutations Ex 16.1 Q1

(i)

We have,

$$\frac{30!}{28!} = \frac{30 \times 29 \times 28!}{28!}$$

$$= 30 \times 29$$

$$= 870$$

Hence, $\frac{30!}{28!} = 870$

(ii)

We have,

$$\frac{11! - 10!}{9!} = \frac{11 \times 10 \times 9! - 10 \times 9!}{9!}$$

$$= \frac{9! \times 10 [11-1]}{9!}$$

$$= 10 \times 10$$

$$= 100$$

Hence, $\frac{11! - 10!}{9!} = 100$

(iii)

We have,

$$8! = 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$$

$$7! = 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$$

and $6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1$

$\therefore$ L.C.M. $(6!, 7!, 8!) = 8!$

Permutations Ex 16.1 Q2

L.H.S:

$$\frac{1}{9!}+\frac{1}{10!}+\frac{1}{11!}$$

$$\frac{1}{9!}+\frac{1}{10\times 9!}+\frac{1}{11\times 10\times 9!}$$

$$=\frac{11\times 10+11+1}{11\times 10\times 9!}$$

$$=\frac{110+11+1}{11!}$$

$$=\frac{122}{11!}$$

$$= \text{RHS}$$

Hence, $\frac{1}{9!}+\frac{1}{10!}+\frac{1}{11!}=\frac{122}{11!}$

Permutations Ex 16.1 Q3(i)

We have,

$$\frac{1}{4!}+\frac{1}{5!}=\frac{x}{6!}$$

$$\Rightarrow \quad \frac{1}{4!}+\frac{1}{5\times 4!}=\frac{x}{6\times 5\times 4!}$$

$$\Rightarrow \quad 4!\times\left[\frac{1}{4!}+\frac{1}{5\times 4!}\right]=\frac{x}{30}$$

$$\Rightarrow \quad 1+\frac{1}{5}=\frac{x}{30}$$

$$\Rightarrow \quad \frac{6}{5}=\frac{x}{30}$$

$$\Rightarrow \quad \frac{x}{30}=\frac{6}{5}$$

$$\Rightarrow \quad x=\frac{6\times 30}{5}$$

$$\Rightarrow \quad x = 6\times 6$$

$$\Rightarrow \quad x = 36$$

Hence, $x = 36$.

Permutations Ex 16.1 Q3(ii)

$$\frac{x}{10!} = \frac{1}{8!} + \frac{1}{9!}$$

$$\Rightarrow x = \frac{10!}{8!} + \frac{10!}{9!}$$

$$\Rightarrow x = \frac{10 \times 9 \times 8!}{8!} + \frac{10 \times 9!}{9!}$$

$$\Rightarrow x = 10 \times 9 + 10$$

$$\Rightarrow x = 100$$

Permutations Ex 16.1 Q3(iii)

$$\frac{1}{6!} + \frac{1}{7!} = \frac{x}{8!}$$

$$\Rightarrow x = \frac{8!}{6!} + \frac{8!}{7!}$$

$$\Rightarrow x = \frac{8 \times 7 \times 6!}{6!} + \frac{8 \times 7!}{7!}$$

$$\Rightarrow x = 8 \times 7 + 8$$

$$\Rightarrow x = 64$$

Permutations Ex 16.1 Q4(i)

We have,

$$5 \times 6 \times 7 \times 8 \times 9 \times 10$$

$$= \frac{10 \times 9 \times 8 \times 7 \times 6 \times 5 \times (4 \times 3 \times 2 \times 1)}{4 \times 3 \times 2 \times 1}$$

$$= \frac{10!}{4!}$$

Hence, $5 \times 6 \times 7 \times 8 \times 9 \times 10 = \frac{10!}{4!}$

Permutations Ex 16.1 Q4(ii)

We have,

$$3 \times 6 \times 9 \times 12 \times 15 \times 18$$

$$= 3 \times (3 \times 2) \times (3 \times 3) \times (3 \times 4) \times (3 \times 5) \times (3 \times 6)$$

$$= 3^6 \times [2 \times 3 \times 4 \times 5 \times 6]$$

$$= 3^6 \times (6!)$$

Permutations Ex 16.1 Q4(iii)

We have,

$$(n+1)(n+2)(n+3)\dots\dots\dots(2n)$$

$$= \frac{\left[1\times 2\times 3\times 4\dots\dots\dots\dots(n-1)n\right]\times(n+1)(n+2)\dots(2n-1)\times 2n}{\left[1\times 2\times 3\times 4\dots\dots\dots\dots\dots(n-1)n\right]}$$

$$= \frac{(2n!)!}{n!}$$

Permutations Ex 16.1 Q4(iv)

We have,

$$1\times\times 5\times 7\times 9\dots\dots\dots\dots(2n-1)$$

$$= \frac{\left[1.3.5.7.9\dots\dots(2n-1)\right].\left[2.4.6.8\dots\dots\dots(2n-2)(2n)\right]}{2.4.6.8\dots\dots(2n-2)(2n)}$$

$$= \frac{\left[1.3.5.7.9\dots\dots\dots(2n-1)\right].\left[2.4.6.8\dots\dots(2n-2)(2n)\right]}{2^n\left[1.2.3.4\dots\dots\dots((n-1)(n))\right]}$$

$$= \frac{1.2.3.4.5.6.7.8\dots\dots\dots(2n-2)(2n-1)(2n)}{2^n.n!}$$

$$= \frac{(2n)!}{2^n.n!}$$

$$\therefore\quad 1.3.5.7.9\dots\dots\dots(2n-1) = \frac{(2n)!}{2^n.n!}$$

Permutations Ex 16.1 Q5

(i) LHS = $(2+3)!$

$= 5!$

$= 5\times4\times3\times2\times1$

$= 120$

and, RHS $= 2!+3!$

$= 2\times1+3\times2$

$= 2\times1+3\times2\times1$

$= 2+6$

$= 8$

$\therefore \quad 120 \neq 8$

$\therefore \quad (2+3)! \neq 2!+3!$

So, it is false.

(ii) LHS $= (2\times3)!$

$= 6!$

$= 6\times5\times4\times3\times2\times1$

$= 720$

and, RHS $= 2!\times3!$

$= 2\times1\times3\times2$

$= 12$

$\therefore \quad 720 \neq 12$

$\therefore \quad (2\times3)! \neq 2!\times3!$

Hence, it is false.

Permutations Ex 16.1 Q6

LHS $= n!+(n+1)!$

$= n!+(n+1)(n+1-1)!$

$= n!+(n+1)n!$

$= n!(1+n+1)$

$= n!(n+2)$

$=$ LHS

$\therefore \quad n!(n+2) = n!+(n+1)!$

Hence, proved

Permutations Ex 16.1 Q7

We have,

$$(n+2)! = 60[(n-1)!]$$

$(n+2)(n+1)(n)(n-1)! = 60[(n-1)!]$

$\Rightarrow \quad (n+2)(n+1)n = 60$

$\Rightarrow \quad (n+2)(n+1)n = 5\times4\times3$

$\therefore \quad n = 3$ [By comparing]

Hence, $n = 3$

Permutations Ex 16.1 Q8

We have,

$$(n+1)! = 90[(n-1)!]$$

$\Rightarrow \quad (n+1) \times n \times (n-1)! = 90[(n-1)!]$

$\Rightarrow \quad n(n+1) = 90$

$\Rightarrow \quad n^2 + n = 90$

$\Rightarrow \quad n^2 + n - 90 = 0$

$\Rightarrow \quad n^2 + 10n - 9n - 90 = 0$

$\Rightarrow \quad n(n+10) - 9(n+10) = 0$

$\Rightarrow \quad (n-9)(n+10) = 0$

$\Rightarrow \quad n - 9 = 0 \qquad [\because n + 10 \neq 0]$

$\Rightarrow \quad n = 9$

Hence, $n = 9$

Permutations Ex 16.1 Q9

We have,

$$(n+3)! = 56[(n+1)!]$$

$\Rightarrow \quad (n+3) \times (n+2) \times (n+1)! = 56[(n+1)!]$

$\Rightarrow \quad (n+2)(n+3) = 56$

$\Rightarrow \quad n^2 + 3n + 2n + 6 = 56$

$\Rightarrow \quad n^2 + 5n + 6 - 56 = 0$

$\Rightarrow \quad n^2 + 5n - 50 = 0$

$\Rightarrow \quad n^2 + 10n - 5n - 50 = 0$

$\Rightarrow \quad n(n+10) - 5(n+10) = 0$

$\Rightarrow \quad (n+10)(n-5) = 0$

$\Rightarrow \quad n - 5 = 0 \qquad [\because n + 10 \neq 0]$

$\Rightarrow \quad n - 5 = 0$

$\Rightarrow \quad n = 5$

Permutations Ex 16.1 Q10

We have,

$$\frac{\frac{(2n)!}{3!(2n-3)!}}{\frac{n!}{2!(n-2)!}} = \frac{44}{3}$$

$$\Rightarrow \quad \frac{(2n)!\times 2!(n-2)!}{3!(2n-3)!\times n!} = \frac{44}{3}$$

$$\Rightarrow \quad \frac{(2n)(2n-1)(2n-2)(2n-3)!\times 2!(n-2)!}{3\times 2!(2n-3)!\times n(n-1)(n-2)!} = \frac{44}{3}$$

$$\Rightarrow \quad \frac{2n(2n-1)(2n-2)}{3n(n-1)} = \frac{44}{3}$$

$$\Rightarrow \quad \frac{2(2n-1)\times 2(n-1)}{3(n-1)} = \frac{44}{3}$$

$\Rightarrow \quad 4(2n-1) = 44$

$\Rightarrow \quad 2n - 1 = 11$

$\Rightarrow \quad 2n = 12$

$\Rightarrow \quad n = 6$

$\therefore \quad n = 6$

Permutations Ex 16.1 Q11(i)

We have,

$$\text{LHS} = \frac{n!}{(n-r)!}$$

$$= \frac{n(n-1)(n-2)(n-3)\ldots(n-r+2)(n-r+1)(n-r)!}{(n-r)!}$$

$$= n(n-1)(n-2)(n-3)\ldots(n-r+2)(n-r+1)$$

$$= n(n-1)(n-2)(n-3)\ldots((n-(r-2))(n-(r-1))$$

$$= n(n-1)(n-2)(n-3)\ldots(n-(r-1))$$

$= \text{RHS}$

$\therefore \quad \text{LHS} = \text{RHS}$

Hence proved

Permutations Ex 16.1 Q11(ii)

We have,

$$\text{LHS} = \frac{n!}{(n-r)!\,r!} + \frac{n!}{(n-r+1)!\,(r-1)!}$$

$$= \frac{n!}{(n-r)!\,r \times [(r-1)!]} + \frac{n!}{(n-r+1)[(n-r)!](r-1)!}$$

$$= \frac{n!}{(n-r)! \times (r-1)!}\left[\frac{1}{r} + \frac{1}{n-r+1}\right]$$

$$= \frac{n!}{(n-r)! \times (r-1)!}\left[\frac{n-r+1+r}{r(n-r+1)}\right]$$

$$= \frac{n!}{(n-r)! \times (r-1)!}\left[\frac{n+1}{r(n-r+1)}\right]$$

$$= \frac{(n+1) \times n!}{(n-r+1) \times (n-r)! \times r \times (r-1)!}$$

$$= \frac{(n+1)!}{(n-r+1)! \times r!}$$

$$= \frac{(n+1)!}{r!\,(n-r+1)!}$$

$$= \text{RHS}$$

$\therefore$ LHS = RHS

Hence proved

Permutations Ex 16.1 Q12

We have,

$$\text{LHS} = \frac{(2n+1)!}{n!}$$

$$= \frac{(2n+1)[1.2.3.4.5.6.7.8\ldots(2n-1)2n]}{n!}$$

$$= \frac{[1.3.5.7\ldots\ldots(2n-1)\times(2n+1)][2.4.6.8\ldots(2n-2)2n]}{n!}$$

$$= \frac{[1.3.5.7\ldots\ldots(2n-1)(2n+1)]\times 2^n\,[1.2.3.4\ldots\ldots(n-1)n]}{n!}$$

$$= \frac{[1.3.5.7\ldots\ldots(2n-1)(2n+1)]2^n \times n!}{n!}$$

$$= 2^n\,[1.3.5.7\ldots.(2n-1)(2n+1)]$$

= RHS

Hence proved