Class 12 RD Sharma Solutions – Chapter 19 Indefinite Integrals – Exercise 19.10

Last Updated : 18 Mar, 2021

Question 1. Evaluate $\int x^2 \sqrt{x+2}\, dx$

Solution:

Let, $I = \int x^2 \sqrt{x+2}\, dx$ (i)

Substituting $x + 2 = t$, $x = t - 2$

$dx = dt$

Substitute the above value in eq (i)

$= \int (t - 2)^2 \sqrt{t}\, dt$

$= \int (t^2 + 4 - 4t)\, t^{1/2}\, dt$

$= \int (t^{5/2} + 4t^{1/2} - 4t^{3/2})\, dt$

Integrate the above eq then, we get

$= t^{7/2}/(7/2) + 4t^{3/2}/(3/2) - 4t^{5/2}/(5/2) + c$

$= (2/7)\, t^{7/2} + (8/3)\, t^{3/2} - (8/5)\, t^{5/2} + c$

Now, put the value of t in above eq

Hence, I =(2/7) $(x+2)^{7/2}$ + (8/3) $(x+2)^{3/2}$ – (8/5) $(x+2)^{5/2}$ + c

Question 2. Integrate ∫ $x^2/(\sqrt{x}-1)$ dx

Solution:

Let, I = ∫ $x^2/(\sqrt{x}-1)$ dx (i)

Put, x-1 = t, so the value of x=t+1

dx = dt

Put the above value in eq (i)

= ∫ $(t+1)^2/\sqrt{t}$ dt

On solving the above eq, we get

= ∫ $(t^2 + 1 + 2t)/\sqrt{t}$ dt

= ∫ $t^{3/2} + t^{-1/2} + 2t^{-1/2}$ dt

Integrate the above eq then, we get

= $(2/5)t^{5/2} + 2t^{1/2} + (4/3)t^{3/2} + c$

= $(6t^{5/2} + 30t^{1/2} + 20t^{3/2})/ 15 + c$

= $(2/15)t^{1/2} (3t^2 + 15 + 10t) + c$

= $(2/15)(x -1)^{1/2} (3(x -1)^2 + 15 + 10(x -1)) + c$

= $(2/15)(x -1)^{1/2} (3(x^2 + 1 - 2x) + 15 + 10x -10)) + c$

= $(2/15)(x -1)^{1/2} (3x^2 + 3 - 6x + 15 + 10x -10)) + c$

= $(2/15)(\sqrt{x} -1) (3x^2 + 4x + 8) + c$

Hence, I = = $(2/15)(\sqrt{x} -1) (3x^2 + 4x + 8) + c$

Solution:

Let, $I = \int x^2/(\sqrt{3x+4})\, dx$ *(i)*

Put, $3x + 4 = t$, *so the value of* $x = (t - 4)/3$

$dx = dt/3$

Put the above value in eq (i)

$= \int ((t - 4)/3)^2/\sqrt{t}\, dt/3$

$= (1/3) \int (t^2 + 16 - 8t)/ 9\sqrt{t}\, dt$

$= (1/27) \int (t^2 + 16 - 8t)/\sqrt{t}\, dt$

$= (1/27) \int (t^{3/2} + 16t^{-1/2} - 8t^{1/2})\, dt$

Integrate the above eq then, we get

$= (1/27)\ [(2/5)t^{5/2} - (16/3)t^{3/2} + 32t^{1/2}] + c$

Now put the value of t in above eq

$= (1/27)\ [(2/5)(3x + 4)^{5/2} - (16/3)(3x + 4)^{3/2} + 32(3x + 4)^{1/2}] + c$

Hence, $I = (1/27)\ [(2/5)(3x + 4)^{5/2} - (16/3)(3x + 4)^{3/2} + 32(3x + 4)^{1/2}] + c$

Question 4. Integrate $\int (2x-1)/ (x-1)^2\, dx$

Solution:

Let, $I = \int (2x-1)/ (x-1)^2\, dx$

Substituting $x-1 = t$ *and* $dx = dt$, *we get*

$= \int 2(t + 1)-1 / t^2\, dt$

$= \int (2t + 2 - 1)/ t^2\, dt$

$= \int 2t/t^2 + 1/t^2 \, dt$

$= 2\int 1/t \, dt + \int t^{-2} \, dt$

Integrate the above eq then, we get

$= 2\log |t| - t^{-1} + c$

Put the value of t in above eq

$= 2\log |x-1| - 1/(x-1) + c$

Hence, $I = 2\log |x-1| - 1/(x-1) + c$

Question 5. Integrate $\int(2x^2 + 3) \sqrt{x+2} \, dx$

Solution:

Let, $I = \int (2x^2 + 3) \sqrt{x+2} \, dx$

Substituting $x + 2 = t$ *and* $dx = dt$, *we get*

$= \int [2(t-2)^2 + 3] \sqrt{t} \, dt$

$= \int [2(t^2 + 4 - 4t) + 3] \sqrt{t} \, dt$

$= \int [2t^2 + 8 - 8t + 3] \sqrt{t} \, dt$

$= \int [2t^{5/2} + 11t^{-1/2} - 8t^{3/2}] \, dt$

Integrate the above eq then, we get

$= (4/7)t^{7/2} + (22/3)t^{3/2} - (16/5) t^{5/2} + c$

$= (4/7)(x+2)^{7/2} + (22/3)(x+2)^{3/2} - (16/5)(x+2)^{5/2} + c$

Hence, $I = (4/7)(x+2)^{7/2} + (22/3)(x+2)^{3/2} - (16/5)(x+2)^{5/2} + c$

Solution:

Let, $I = \int (x^2 + 3x + 1)/ (x+1)^2\, dx$

Substituting $x + 1 = t$ *and* $dx = dt$*, we get*

$= \int [(t - 1)^2 + 3(t - 1) + 1]/ t^2\, dt$

$= \int (t^2 + 1 - 2t + 3t - 3 + 1)/ t^2\, dt$

$= \int (t^2 + t - 1)/ t^2\, dt$

$= \int t^2/t^2 + t/ t^2 - 1/t^2\, dt$

$= \int (1 + 1/t - t^{-2})\, dt$

Integrate the above eq then, we get

$= t + \log |t| + 1/t + c$

Put the value of t in above eq

$= (x + 1) + \log |x + 1| + 1/(x+1) + c$

Question 7. Integrate $\int x^2 / (\sqrt{1-x})\, dx$

Solution:

Let, $I = \int x^2 / (\sqrt{1-x})\, dx$

Substituting $1 - x = t$ *and* $dx = - dt$*, we get*

$= \int - (1-t)^2 / \sqrt{t}\, dt$

$= - \int (1 + t^2 - 2t)/ \sqrt{t}\, dt$

$= - \int t^{-1/2} + t^{3/2} - 2t^{1/2}\, dt$

Integrate the above eq then, we get

$= - 2t^{1/2} + (2/5)t^{5/2} - (4/3)t^{3/2} + c$

$= - (30t^{1/2} + 6t^{5/2} - 20t^{3/2}) / 15 + c$

$= (-2/15)\sqrt{(1-x)}\,(15 + 3(1-x)^2 - 10(1-x)) + c$

$= (-2/15)\sqrt{(1-x)}\,(15 + 3(1 + x^2 - 2x) - 10 + 10x)) + c$

$= (-2/15)\sqrt{(1-x)}\,(5 + 3 + 3x^2 - 6x + 10x) + c$

$= (-2/15)\sqrt{(1-x)}\,(3x^2 + 4x + 8) + c$

Hence, $I = (-2/15)\,(3x^2 + 4x + 5)\sqrt{1-x} + c$

Question 8. Integrate $\int x(1-x)^{23}\,dx$

Solution:

Let, $I = \int x(1-x)^{23}\,dx$

Substituting $1 - x = t$ and $dx = -dt$, we get

$= -\int (1-t)t^{23}\,dt$

$= -\int (t^{23} - t^{24})\,dt$

$= \int (t^{24} - t^{23})\,dt$

Integrate the above eq then, we get

$= t^{25}/25 - t^{24}/24 + c$

$= (1-x)^{25}/25 - (1-x)^{24}/24 + c$

Hence, $I = (1-x)^{25}/25 - (1-x)^{24}/24 + c$