EXERCISE 14.3

1. For each of the following compound statements first identify the connecting words and then break it into component statements.

(*i*) All rational numbers are real and all real numbers are not complex.

Sol. "And". The component statements are:
All rational numbers are real.
All real numbers are not complex.

(*ii*) Square of an integer is positive or negative.

Sol. "Or". The component statements are:
Square of an integer is positive.
Square of an integer is negative.

(*iii*) The sand heats up quickly in the Sun and does not cool down fast at night.

Sol. "And", the component statements are:
The sand heats up quickly in the sun.
The sand does not cool down fast at night.

(*iv*) $x = 2$ and $x = 3$ are the roots of the equation $3x^2 - x - 10 = 0$.

Sol. "And". The component statements are:
$x = 2$ is a root of the equation $3x^2 - x - 10 = 0$
$x = 3$ is a root of the equation $3x^2 - x - 10 = 0$.

2. Identify the quantifier in the following statements and write the negation of the statements.

(*i*) There exists a number which is equal to its square.

Sol. "There exists". The negation is
There does not exist a number which is equal to its square.

(*ii*) For every real number x, x is less than $x + 1$.

Sol. "For every". The negation is

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There exists a real number x such that x is not less than $x + 1$.

(*iii*) There exists a capital for every state in India.

Sol. "There exists". The negation is

There exists a state in India which does not have a capital.

Remark: 1. There are only two quantifers:

(*i*) There exists (*ii*) For all (FOR EVERY)

2. Negation of **there exists** is there does not exist.

3. For negation of 'for all', see (*ii*) above.

3. Check whether the following pair of statements are negation of each other. Give reasons for your answer.

(*i*) $x + y = y + x$ is true for every real numbers x and y.

(*ii*) There exists real numbers x and y for which $x + y = y + x$.

Sol. No. The negation of the statement in (*i*) is "There exist real number x and y for which $x + y \neq y + x$", instead of the statement given in (*ii*).

4. State whether the "Or" used in the following statements is "exclusive" or "inclusive". Give reasons for your answer.

(*i*) Sun rises or Moon sets.

Sol. Here "Or" is exclusive because only one of the two can occur. The simultaneous occurrence of the two is not possible.

(*ii*) To apply for a driving license, you should have a ration card or a passport.

Sol. Here "Or" is inclusive because you can apply for a driving licence also when you have both the ration card and the passport.

(*iii*) All integers are positive or negative.

Sol. Here "Or" is exclusive because an integer is either positive or negative. No integer can be both positive and negative.